The Rebirth of Solved Games

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Abstract

There has been a tremendous advancement witnessed in the computer technology industry in recent years. With such progress, many of the traditional two-person complete information games have been solved. The game of tic-tac-toe is one of such games and it is well known that when both players select their best moves, the game always ends in a draw. In the game of hex, Black has always a winning strategy and for small board's size it has been completely solved. In this paper we have explored the possibility of reviving a strongly solved game, such as tic-tac-toe and hex by adding some stochastic elements to it. In our attempt we have altered the rule and allowed both players to make their moves in synchronized manners.

Keywords: Games; Solved; Synchronized tic-tac-toe (game); Synchronized hex (game)

1. Introduction

In our previous study we have analyzed the uncertainties that games have from the view point of "Games = Uncertainty"[Iida 2005]. Uncertainties in terms of (1) The difficulties of choice, (2) the outcome, and (3) game-theoretic value. Each one of them corresponds to the following: the difficulties of selecting the best moves, maintaining the equilibrium, seesaw game, and reaching the end of the game after both players performed optimally.

From the perspective of enjoying games it is clear that the second uncertainty is very important [Iida et al. 2003]. The uncertainty concerning the outcome of the game emphasizes the importance of the process which reveals the outcome of the game gradually as the match progresses. Since this process is reflected on the value of $\sqrt{B}/D$ it is possible to utilize this value as the index of game refinement. B indicates the average number of possible moves and D means the game length. Generally when the game is solved, the first and the third uncertainties are no longer factors [Allis et al. 1991]. In other words, winning strategies and game-theoretic values of games are revealed.

However, the reason we can still enjoy playing games is due to the fact that the second type of uncertainties still remain. When there is enough of all three types of uncertainties and when they are in harmony games become almost irresistible. In such games we are able to see the fairness in them.

Fairness has been pointed out as an essential concept of games [Iida 2004a][Iida 2004b]. Fairness in term of the outcome of games is expressed by the "winning percentage difference between White (First Player) and Black (Second Player)" and "game-theoretic values". The former is a practical fairness and the latter can be said as fairness in theory. When the difference in the winning percentages between White and Black is too large the game can be identified as the game of determining the turn of play. Therefore, it is desirable that games do not give any advantages to White nor Black. In the same manner, if White always wins in the game-theoretic value, that game can be considered as the game of determining the turn to play. Therefore, in order to maintain the fairness, the game-theoretic value must be even.

In complex games such as chess, shogi and Go, the practical fairness is already in place. The history of games has been paved by a series of evolutionary transformations in order to maintain the fairness and as the result games have been refined [Iida 2004b].
Then how about with simple games such as checkers or Othello? With recent advancements made by computers, various games have been solved and this has revealed game-theoretic values of some games that have been popular [Herik et al. 2002]. Clearly people tend to lose interest as games get solved.

It is possible to do away with the games which have been solved. However, what is the likeliness that they can be revived? And what would it take to do so?
In this paper we demonstrate with a two-person perfect information game which always has players alternating turns, as the subject of our study. However, in our study we eliminate turns, allowing both players to play their moves at the same time, adding a stochastic element in attempt to revive the game. We specifically focus on the significance of games which have been solved, as well as reviving them.

2 Revival Method

This study focuses on two specific games: the extremely simple "tic-tac-toe" which qualifies as strongly solved as a test bed and "hex". In tic-tac-toe when both players select their best moves the outcome is always a draw, instead in hex the first player has always a winning strategy. However, by implementing the new rule of "allowing both players to play their moves simultaneously" or more accurately, in a "synchronized" manner, we demonstrate that it might be possible to revive simple games. In order to compare the traditional tic-tac-toe and hex with the synchronized version, we focus on the statistical data of "game depth", "branching factor", and "draw ratio".

2.1 Special Rules

We introduce the rules used in our synchronized games.

- **LV0 rule**. *It is the normal way to play.*
- **LV1 rule**. *At each turn, both players execute their moves contemporaneously, if they select the same square both marks will be indicated there.*
- **LV2 rule**. *Both players are to announce and execute two consecutive moves. The first one is selected from the possible moves permitted by normal tic-tac-toe (hex). Of course, the second move of every player must be different from his/her first move; moreover if the second move of a player is equal to the first move of his opponent, the second move of this player is not allowed to be executed.*

When both players form three marks in row (or a chain between the respective side of the board in the game of hex) simultaneously it is considered as a draw.

2.2 Experiment Method

For tic-tac-toe we prepare computer players with various depths of search and they do not incorporate any evaluation function. Through many self-play experiments, we obtained statistics such as the average "game length", "number of possible moves", and "draw percentages". Each player, according to its ability, is to forfeit when it realizes defeat. Concerning player models we followed the semi-random self-play idea [Kajihara et al. 1999]. The two players are distinguished as max and min player. Let assume that max player plays a move at a given position. A semi-random self-play is defined below by **R1** and **R2:**

- **R1**. *Generate all possible moves and try a game-tree search to look ahead by a given search depth. If there is a winning move (by which the max player is able to reach a winnig position), then choose it.*
- **R2**. *Remove losing moves (after which the min player is able to reach his winning position) from the list of candidates at a position considered. If the list is not empty, select a move among the list at random. Otherwise, select a move at random among all possible moves.*

Each player, according to its ability, is to forfeit when it realizes defeat. Of course, when we want to apply semi-random self-play method using LV1 or LV2 rule we have to take some precautions. If max's first (second) move is equal to min's first (second) move then both marks will be indicated. Moreover, if max's second move is equal to min's first move then max's second move will not be allowed to be excuted. Simmetrically, if min's second move is equal to max's first move then min's second move will not be allowed to be excuted. The "depth of search" which corresponds to the player's strength is divided into four levels (0, 2, 4, and 6). However, since the synchronized tic-tac-toe forces both players to make their moves in the synchronized manner, the "depth of search" is even-numbered because in reality it is equivalent to two moves made at once. For hex we implement computer players with a full-random strategy to be able to increase the complexity of the game changing the board's size.

3 Results and Comparison

By implementing the synchronized move rule, the advantage associated with making the first move ceases to exist, therefore players are unable to anticipate the opponent's move completely. This can be interpreted as synchronized tic-tac-toe (hex) being successful in reviving the traditional version. The results for tic-tac-toe using LV0, LV1, and LV2 rules are compared in Table 1 and plotted in Figure 1, 3, and 5; the results for hex are compared in Table 1 and plotted in Figure 2, 4, and 6.

3.1 Discussion

To evaluate the effectiveness of synchronized tic-tac-toe (hex) in reviving the age old game, we list the characteristics of what we believe of intriguing games:

1. Smaller the $\sqrt{B}/D$ the better.
2. Larger the B^D the better.
3. Smaller the draw percentage and greater the game length is better

When all three criteria are satisfied the game can be considered as "revived".

At the search depth of six, the draw ratio for both "LV1" and "LV2" is 100%, however it is less than 50% for "LV2". "LV2" offers the lowest chance of draw out of all three, except when the depth of search is at "0". This is the result of having to select two consecutive moves at once, making it more difficult to predict the opponent's moves, hence resulting in the less chance of games ending in a draw. The reason for the decrease in draw ratio as the number of moves players have to determine in consecutiveness increases, is because of the enormous disadvantage players face when they are not allowed to make the second of the two moves when it is not included as one of the possible moves.

We focus on the values of $\sqrt{B}/D$ which are used as the index for game refinement. $\sqrt{B}/D$ decreases as "LV1" is changed to "LV2". This indicates that the game's outcome is more dependent on skill. Therefore, it is reasonable to assume that as the number of consecutive moves selected at once increases the games becomes more skill-dependent. However, it is also possible that √B/D decreased as the result of the game length increasing because of the implementation of the new rule of players not allowed to make the second of the two consecutive moves if it is not among the legal moves.

4 Future Works

In this paper we have studied the changes in tic-tac-toe and hex's game characteristics by introducing the "synchronized" version. It was confirmed that it prevented both the "unfair advantage of the First player" and the "total prediction of opponent's moves", while discovering the characteristics of the "LV2 Rule" to decrease the draw ratio. In the future we will apply this to games that are more complex than tic-tac-toe and hex. As the next step we have started working on "checkers" as the next test bed and also make further attempts to revive more complex games, such as chess and shogi thereafter.

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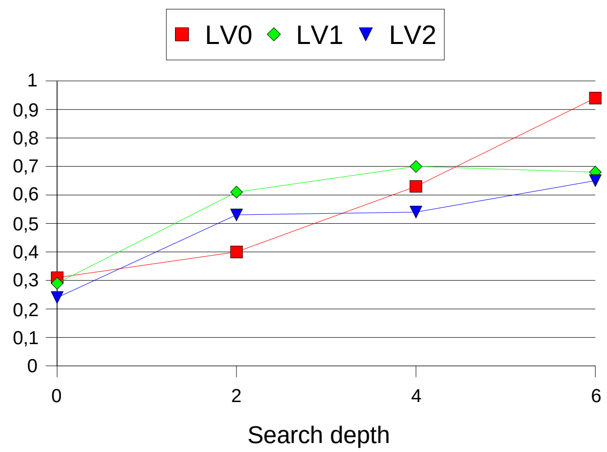
Table 1: Synchronized tic-tac-toe and hex variants compared

LV	Characteristics	0	2	4	6	4	8	12	16
0									
	B^D	567486	115702	5587	512	8.78E+06	7.88E+43	7.10E+122	1.93E+250
1									
	B^D	3.09E+06	10793	4569	8545	1.44E+07	1.99E+44	2.44E+123	7.69E+250

	B^D	38.93E+06	40001	37859	11439	5.29E+07	3.80E+45	1.51E+125	1.12E+253

Figure 1: √B/D for tic-tac-toe



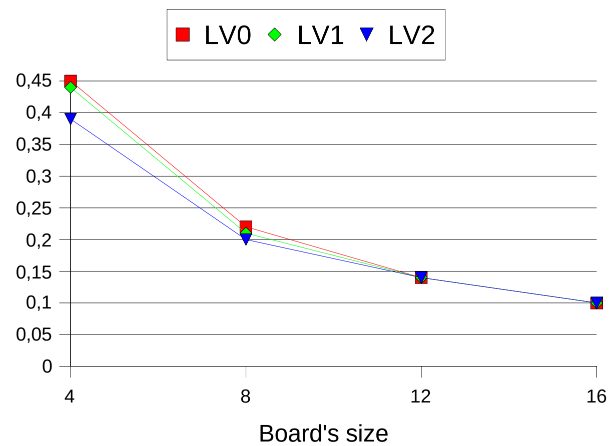


Figure 3: D•log(B) for tic-tac-toe

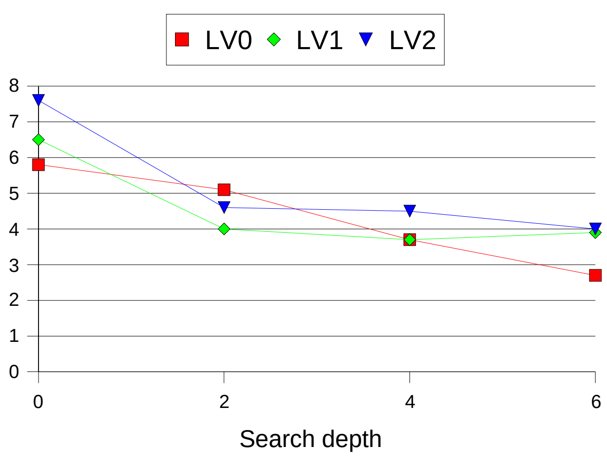


Figure 4: D•log(B) for hex

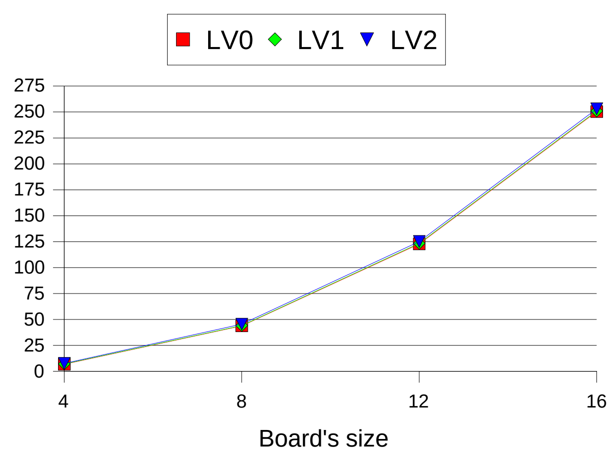


Figure 5: Draw ratio for tic-tac-toe

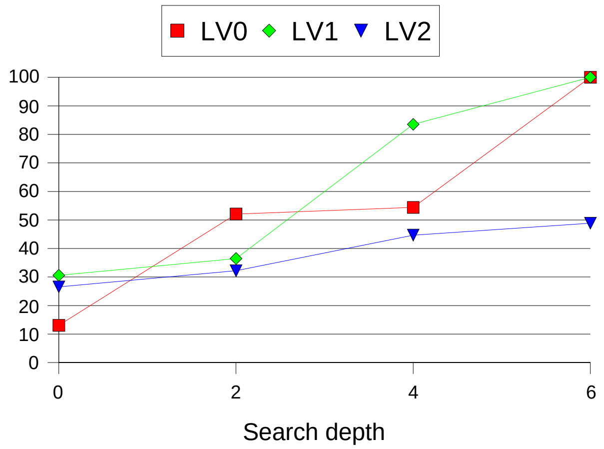


Figure 6: Draw ratio for hex