Resolution Based on Six Lattice-valued First-order Logic $L_6F(X)$

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Abstract

Resolution-based automated reasoning theory is an important and active research field in artificial intelligence. It is not only used to judge the satisfiability of logic formula, but also widely applied to areas such as artificial intelligence, logic programming, problem solving and question answering systems, database theory and so on. With the development of classical and non-classical logic, resolution theory and method based on different logic system has been discussed widely and deeply. In this paper, resolution-based automated reasoning method in a six lattice-valued first-order logic on lattice implication algebra is focused. It is based on resolution principle on Six Lattice-valued First-order Logic $L_6F(X)$. In the presented paper, some necessary preliminaries including some necessary definition, resolution principle on $L_6F(X)$ and related soundness and completeness are given first. Then resolution method on $L_6F(X)$ is given. Because $L_6F(X)$ is a non-chain, non-boolean and non-well-ordered algebra structure, the research of resolution method will be helpful support for the application of intelligent reasoning system based on lattice-valued logic which includes incomparable information.

1. Introduction

Since its introduction in 1965, automated reasoning based on Robinson's[7] resolution rule has been extensively studied [1, 6, 8] in the context of finding natural and efficient proof systems to support a wide spectrum of computational tasks. They are widely applied to areas such as artificial intelligence, logic programming, problem solving and question answering systems, database theory, and so on. As the existence of uncertainty in real world, it is difficult and unbefitting to design any intelligent system based on traditional logic. In other words, any intelligent system that is designed to work in this real world must be able to make decisions in an uncertain environment. With the development of fuzzy logic, expert system and knowledge engineering, especially since non-classical logics became a considerable formal tool for computer science and artificial intelligence, the area of automated reasoning based on non-classical logic (especially multi-valued logic and fuzzy logic) has drawn the attention of many researches.

Lattice-valued logic is an important case of multi-valued logic. In [9, 10, 18], lattice-valued logic and its resolution is discussed deeply. However, it is based on Kleene's implication. Since 1993, LIA[17] and some related discussion on logic system based on LIA is introduced and discussed. The related work is summarized and presented in [14, 15]. In addition to the above results, α-resolution principle based on lattice-valued propositional logic $LP(X)$ and α-resolution principle based on first-order lattice-valued logic $LF(X)$ are given in [14, 15], which are fundamental contributions to resolution-based automated reasoning methods on Lattice-valued logic, which is based on LIA [17]. All related research results are summarized in [12].

$LP_6(X)$ as a non-chain, non-boolean and non-well-ordered algebra structure, the research of resolution principle and algorithm based on it will help the realization on intelligent system which includes incomparable element. In [3, 4], resolution principle and its algorithm in six lattice-valued proposition logic $LP_6(X)$ are discussed. In [5], resolution principle based on six lattice-valued first-order logic $L_6F(X)$ is discussed. However, how to realize resolution based on $L_6F(X)$ is an important research field. The problem is tried to be solved is this paper.

The paper is structured as follows.some necessary preliminaries including some necessary definition, resolution principle on $L_6F(X)$ and related soundness and completeness are given first. Then resolution method on $L_6F(X)$ is given in section 3.

2. Preliminaries

Firstly we give some definitions and results:

Definition 2.1 *[12] Let $(L, \wedge, \vee, ', I, O)$ be a bounded lattice with order-reversing involution " $'$ ", $\rightarrow: L \times L \rightarrow L$ be a mapping. $(L, \wedge, \vee, ', \rightarrow, I, O)$ is called a lattice implication algebra if the following conditions hold for any $x, y, z \in L$:*

- $(I1)$ $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$*(exchange property);*
- $(I2)$ $x \rightarrow x = I$ *(identity);*
- $(I3)$ $x \rightarrow y = y' \rightarrow x'$*(contraposition);*
- $(I4)$ $x \rightarrow y = y \rightarrow x = I$ *implies* $x = y$*;*
- $(I5)$ $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$ *;*
- $(I6)$ $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$ *;*
- $(I7)$ $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$.

Example 2.1 *There is the following six element lattice L_6 $= \{I, \alpha, \beta, \gamma, \delta, O\}$, its "$\vee$", "$\wedge$" operation as shown in $HASSE$ figure L_6, its "$\longrightarrow$", "$'$" operation is defined as*

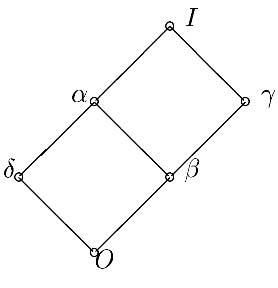

Figure 1. L_6

the table 1,2:

then (L_6, $\wedge$, $\vee$, $'$, I, O) is a lattice implication algebra.

$\longrightarrow$	O	γ	α	δ	β	I
O	I	I	I	I	I	I
γ	δ	I	α	δ	α	I
α	β	γ	I	α	γ	I
δ	γ	γ	I	I	γ	I
β	α	I	I	α	I	I
I	O	γ	α	δ	β	I

Table 1. implication

x	O	γ	α	δ	β	I
x'	I	δ	β	γ	α	O

Table 2. complementary

2.1 Language

The definition of the alphabet of $LF(X)$, the set T of terms of $LF(X)$, the set F of well-formed formulas(wff) is given in [12] and is omitted here. The notions of free and bound variables are similar to those of classical first-order logic.

Note 2.1 *In this paper, we will discuss resolution based on Lattice-valued First-order Logic $L_6F(X)$ whose truth-valued field is L_6 presented in figure 1.*

2.2 Related concepts on resolution

The definition of $generalized-literal, generalized-prenex$ normal form, $generalized - Skolem$ standard form of formula G is given in [12] and is omitted here.

Other concepts in $LF(X)$, such as interpretation, $H - interpretation$, ground term, ground term set, ground atom, ground atom set, ground literal, ground clause, ground clause set, ground instance, instance, $H - field$ of a clause set, substitution, unifier, and most general unifier, etc., are formally the same as the corresponding concepts in classical logic.

Definition 2.2 *[11] Let $(L, \vee, \wedge, ', \longrightarrow, O, I)$ is a LIA, A $\subseteq L$ is a implication filter of L, if $\forall$ $x \in L$, $x \in A$ if and only if $x' \notin A$, then A is called a ultrafilter of L.*

2.3 Resolution principle using ultrafilter in $L_6F(X)$

Definition 2.3 *[5] Let $G \in L_6F(X)$, J is ultrafilter of L_6, then*

1. *if there is an interpretation $I_D =< D, \mu_D, \nu_D >$, such that $\nu_D(G) \in J$, then G is $J - satisifable$;*
2. *if for any interpretation $I_D =< D, \mu_D, \nu_D >$, such that $\nu_D(G) \in J$, then G is $J - true$;*

3. *if there is an interpretation* $I_D =< D, \mu_D, \nu_D >$*, such that* $\nu_D(G) \notin J$*, then* G *is* $J - false$*.*

Lemma 2.1 *[5] Let* $\theta \in L$*,*$G, G^* \in LF(X)$*, and* G^* *is* $generalized - Skolem$ *standard form of a formula* G*, then* G *is* $\theta - false$ *iff* G^* *is* $\theta - false$*.*

Lemma 2.2 *[5] Let* $G, G^* \in L_6F(X)$*, and* G^* *is* $generalized - Skolem$ *standard form of a formula* G*, then* G *is* $J - false$ *iff* G^* *is* $J - false$*.*

Lemma 2.3 *[5] Let* $G, G^* \in L_6F(X)$*, and* G^* *is* $generalized - Skolem$ *standard form of a formula* G*, if interpretation* $I_D =< D, \mu_D, \nu_D > \theta - satisfies\ G^*$*, then the* $H - interpreation\ I_H =< H, \mu_H, \nu_H >$ *of* G^* *corresponding to* I_D *also* $\theta - satisfies\ G^*$*.*

Lemma 2.4 *[5] Let* $G, G^* \in L_6F(X)$*,* J *is ultrafilter of* L_6*, and* G^* *is* $generalized - Skolem$ *standard form, if interpretation* $I_D =< D, \mu_D, \nu_D > J - satisfies\ G^*$*, then the* $H - interpreation\ I_H =< H, \mu_H, \nu_H >$ *of* G^* *corresponding to* I_D *also* $J - satisfies\ G^*$*.*

Lemma 2.5 *[5] Let* $G, G^* \in L_6F(X)$*, and* G^* *is* $generalized - Skolem$ *standard form of a formula* G*.* G^* *is* $J - false$ *iff there exist* $k \in N$*, such that for any adjoint* ν_H *of every element in* L^K*.*

Lemma 2.6 *[5] In* $L_6F(X)$*,* G^* *is* $generalized - Skolem$ *standard form of a formula* G*,* G^* *is* $J - false$ *iff there is a finite ground instance set* G^{*0} *of* G^**, such that* G^{*0} *is* $J - false$*.*

Definition 2.4 *[5] Let* C_1^0 *and* C_2^0 *be two generalized-clauses without common variables. For any generalized-clauses* C_1 *and* C_2*, if there exists a substitution* ε *such that the following conditions hold:*

1. $C_1^\varepsilon = C_1^0$*,*$C_2^\varepsilon = C_2^0$*.*
2. $C_1 = C_1^* \vee p_1 \vee p_{11} \vee \cdots \vee p_{1k_1}$*,* $C_2 = C_2^* \vee p_2 \vee p_{21} \vee \cdots \vee p_{2k_2}$*, satisfy*

   (a)
   $$\{p_1, p_{11}, \cdots, p_{1k1}\} =$$
   $$\{q|q\text{is a g-literal in } C_1, q^\varepsilon = q_1^\varepsilon\},$$
   and
   $$\{p_2, p_{21}, \cdots, p_{2k2}\} =$$
   $$\{q|q\text{is a g-literal in } C_2, q^\varepsilon = q_2^\varepsilon\},$$
   so
   $$C_1^0 = C_1^{*\varepsilon} \vee p_1^\varepsilon, C_2^0 = C_2^{*\varepsilon} \vee p_2^\varepsilon.$$

   (b) There exist a generalized-literal $p, p = p_2\prime$*;*

   (c) p_1^ε *and* $p\varepsilon$ *has the most general unifier* σ*. Then the generalized-clauses*
   $$C_1^{0*}\varepsilon \vee C_2^{0*} = R_J(C_1^0, C_2^0)$$
   is called an $J - resolvent$ *of* $(C_1^0$ *and* $C_2^0)$*;* p_1^ε *and* p_2^ε *are called an* $J - resolution$ *pair, denoted by* $(p_1^\varepsilon, p_2^\varepsilon) - J$*, here*

   i.
   $$C_1^{0\sigma} = C_1^{0*\sigma} \vee p_1^{\varepsilon\sigma} \vee q_{11}^{\varepsilon\sigma} \vee \cdots \vee q_{1t1}^{\varepsilon\sigma},$$
   and
   $$\{p_1^{\varepsilon\sigma}, q_{11}^{\varepsilon\sigma}, \cdots, q_{1t1}^{\varepsilon\sigma}\}$$
   $$= \{q|q\text{is a g-literal in} C_1^0, q^\sigma = p_1^{\varepsilon\sigma}\}.$$
   Hence
   $$C_1^{0\sigma} = C_1^{0*\sigma} \vee p_1^{\varepsilon\sigma}.$$

   ii.
   $$C_2^{0\sigma} = C_1^{0*\sigma} \vee p_2^{\varepsilon\sigma} \vee q_{21}^{\varepsilon\sigma} \vee \cdots \vee q_{2t2}^{\varepsilon\sigma},$$
   and
   $$\{p_2^{\varepsilon\sigma}, q_{21}^{\varepsilon\sigma}, \cdots, q_{2t2}^{\varepsilon\sigma}\}$$
   $$= \{q|q\text{is a g-literal in} C_2^0, q^\sigma = p_2^{\varepsilon\sigma}\}.$$
   Hence
   $$C_1^{0\sigma} = C_1^{0*\sigma} \vee p_2^{\varepsilon\sigma}.$$

3 Resolution Method based on $LF(X)$

Theorem 3.1 *In* $L_6F(X)$*, if* C_1^0 *and* C_2^0 *are instances of generalized-clauses* C_1 *and* C_2 *respectively,* C^0 *is an* $J - resovent$ *of* C_1^0 *and* C_2^0*, then there exists an* $J - resolvent$ C *of* C_1 *and* C_2 *such that* C^0 *is an instance of* C*.*

Theorem 3.2 *(quasi-completeness theorem) In* $L_6F(X)$*,* G^* *is* $generalized - Skolem$ *standard form of a formula* G*. If* G^* *is* $J - false$ *and there is* $J - complementary$ *literals in ground instance of* G^**, then there exist a resolution deduction from* G^* *to* $J - box$ *clause.*

Theorem 3.3 *(quasi-soundness theorem) In* $L_6F(X)$*,* G^* *is* $generalized - Skolem$ *standard form of a formula* G*. If there exist* $J - complementary$ *literals in ground instance of* G^**, and there exist a resolution deduction from* G^* *to* $J - box$ *clause, then* G^* *is* $J - false$*.*

Lemma 3.1 $S, S' \in L_6F(X)$*, if* $S \geq S'$ *and* S*is* $J - false$*,then* S' *is* $J - false$*; if* $S \leq S'$ *and* S *is* $J - satisfiable$ *, then* S' *is* $J - satisfiable$*.*

Theorem 3.4 $S, S' \in L_6F(X)$*, if* $S \geq S'$*,if there exist a* $J - refutation\ \omega$ *of* S*, then* S' *is* $J - false$*; if* $S \leq S'$*,and there doesn't exist* $J - refutation\ \omega$ *of* S'*, ,then* S *is* $J - satisfiable$

4. Conclusion

In this paper, resolution method based on resolution principle using ultrafilter based on a six lattice-valued first-order logic $L_6F(X)$ is investigated. It will be helpful for realization for intelligent system includes incomparable element. However, it is only an attempt to solve resolution realization on computer. More efficient method will be discussed in further study.

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