Integration of a Heuristic Global Optimum Test and DIRECT Algorithm for Mutual Information Based Image Registration

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Abstract

The goal of image registration is to align two or more images of the same scene. Among a large number of image registration techniques, maximization of mutual information (MMI) has become the most popular method for parametric image registration problem. As implied by its name, (global) maximization is the major component of this technique. The purpose of this study is to design a robust yet efficient global optimizer based on DIRECT algorithm and a heuristic global optimum test for this problem. In this paper, we employ a heuristic global optimum test algorithm to determine when to stop the DIRECT algorithm and show how to integrate them effectively. The efficacy of the developed optimization scheme is illustrated by registering two pairs of remote sensing images.

Keywords: Image registration, DIRECT algorithm, Mutual information, global optimum test.

1. Introduction

Image registration is a fundamental task in image processing and computer vision to align two or more images that might be taken at different times, from different sensors or from different viewpoints. Mutual information (MI) based image registration technique has gained substantial attention recently and is the leading technique for multimodal image registration. In this paper, we focus on the optimization aspect of MI based image registration and present a global optimization scheme based on DIRECT algorithm [1] and a heuristic global optimum test [2]. Existing global optimizers can be made very robust but with the price of very low efficiency by employing a very strict stopping criterion. In this paper, we seek ways to improve the efficiency of DIRECT algorithm without sacrificing its robustness for MI based parametric image registration problem.

For a fully automated, robust, and efficient image registration system, a mechanism checking whether the current best solution is the global optimum is helpful. To distinguish the global optimum from local ones for image registration application, a heuristic test has been proposed [2]. In this paper, we develop an effective way to incorporate this global optimum test into DIRECT algorithm to determine when to terminate the DIRECT algorithm. Besides, if there are many deep local optima, DIRECT algorithm will keep sampling the neighboring areas of those local optima until it flees from those deep local optima. This makes DIRECT algorithm highly inefficient. In this paper, we also propose a method to avoid it to make the optimization process very efficient.

This paper is organized as follows. In Section 2, we briefly review the MI based image registration, the employed heuristic global optimum test, and the DIRECT algorithm. In Section 3, we describe how to combine the global optimum test with the DIRECT algorithm to improve both its robustness and efficiency for MI based image registration. Two experimental results using remote sensing images are presented in Section 4. Finally, this paper is summarized in Section 5.

2. Brief review of related topics

2.1. Mutual information based image registration

Given two images U and V, the MI between them can be calculated through the following formula:

$$MI(U,V) = H(U) + H(V) - H(U,V) \qquad (1)$$

Where $H(U)$ and $H(V)$ denote the entropy of U and V respectively, and $H(U,V)$ the joint entropy of U and V. They are calculated through

$$H(U) = -\sum_{u} P_U(u) \log P_U(u) \qquad (2)$$

$$H(V) = -\sum_{v} P_V(v) \log P_V(v) \qquad (3)$$

$$H(U,V) = -\sum_{u,v} P_{U,V}(u,v) \log P_{U,V}(u,v) \qquad (4)$$

Where $P_U(u), P_V(v)$ are the probability mass functions (normalized histograms) of images U and V and $P_U(u,v)$ is the joint probability mass function (normalized joint histogram) of images U and V. Once the joint histogram of images U and V, denoted by $h(u,v)$, is obtained, they can be calculated directly as

$$P_{U,V}(u,v) = \frac{h(u,v)}{\sum_{u,v} h(u,v)} \quad (5)$$

$$P_U(u) = \sum_{v} P_{U,V}(u,v) \quad (6)$$

$$P_V(v) = \sum_{u} P_{U,V}(u,v) \quad (7)$$

MMI criterion states that two image U and V are registered if the MI between them reaches its maximum. Mathematically, this can be expressed as:

$$\alpha^* = \arg\max(MI(U(\tilde{x}), V(T_\alpha(\tilde{x})))) \quad (8)$$

where $\tilde{x}$ represents the coordinates of a pixel in U and the transformation model is represented by T with associated parameters α. For more information about MI based image registration, readers are referred to [3, 5].

2.2. DIRECT algorithm

The DIRECT optimization algorithm was first introduced in [1], motivated by a modification to Lipschitzian optimization. It is a deterministic sampling method to find the global minimum of a multivariate function subject to sample bounds. The main idea of DIRECT algorithm in 1D case is outlined below.

Suppose that d_1, .., d_p are the p different interval sizes at the start of an iteration. Suppose also that $f(c_j)$ denotes the smallest of the mid-point function values in the intervals of size d_j and c_j is the corresponding midpoint. The DIRECT algorithm will divide into three parts the interval containing c_j if it is determined to be potentially optimal according to the following definition

Definition 2.1 Let $\varepsilon > 0$ be a positive constant and let $f_{\min}$ be the current best function value. Interval j is said to be potentially optimal if there exists some rate of change constant $\hat{K} > 0$ such that

$$f(c_j) - \hat{K} d_j \le f(c_i) - \hat{K} d_i, \quad \forall i, \quad \textit{and}$$

$$f(c_j) - \hat{K} d_j \le f_{\min} - \varepsilon |f_{\min}|$$

In this definition, c_j is the center of interval j, and d_j is a measure for the size of interval. Jones et. al. [1] chose to use the distance from c_j to its endpoints as the measure.

Fig. 1 graphically illustrates the determination of potentially optimal intervals. Each dot represents an interval. The horizontal coordinate of each dot corresponds to the interval size, and the vertical coordinate of each dot is the value of the function evaluated at the interval's center.

The ideas outlined above can be extended easily to produce a version of DIRECT for multidimensional

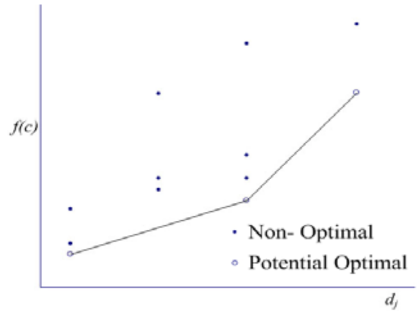


Fig. 1: Set of potentially optimal intervals

problems. The original search region is now a hyper-rectangle rather than a line segment. Once a sub-hyper-rectangle has been identified as potentially optimal hyper-rectangle by Definition 2.1, DIRECT divides this hyper-rectangle into smaller hyper-rectangles.

2.3. A heuristic global optimum test

Like all the existing global optimization methods, DIRECT algorithm needs an effective stopping criterion. Ideally, the sampling process should not be terminated until the global optimum is reached. In this paper, we use the heuristic global optimum test proposed in [2] to determine whether the global optimum is reached or not. The basic idea of the test is the following: if two images, one is denoted as floating image and the other as the reference image, are registered by a set of global transformation parameters, the same set of global transformation parameters should be also held for the registration of any sub-images of the two images. Fig. 2 gives the implementation of this heuristic global optimum test algorithm [2].

In Fig. 2, α is the transformation parameter set that results in an optimum found by any optimizer using the whole floating image. The goal is to test whether α is the global optimum. α_i, $i = 1 \ldots k$, is the parameter set found by any local optimizer using the i^{th} partition of the floating image for registration. In this paper, we developed a method to integrate the global optimum test into the DIRECT algorithm effectively for MI based image registration

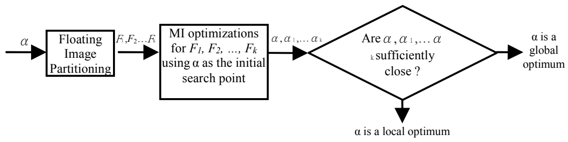


Fig. 2: Block diagram of the heuristic global optimum test.

3. Proposed integrated system

3.1. The architecture

The procedure of integrating the reviewed global optimum test and DIRECT algorithm for MI based image registration is shown in Fig. 3. The inputs are the two images to be registered and the upper bound and lower bound of each transformation parameter that defines the search space. The DIRECT algorithm is then launched to search in the specified domain for the maximum value of the MI similarity measure between the two images. The next block is a mechanism used to check whether the DIRECT algorithm is approaching a local optimum. If it is determined to be approaching a local optimum, we switch the optimization process from DIRECT algorithm to any local optimizer to accelerate the convergence. Once a local optimum is reached, the global optimum test is invoked to determine whether the local optimum is the desired global optimum. The whole optimization process stops when a local optimum is determined to be the global optimum by the global optimum test. Otherwise, a neighborhood of the local optimum will be determined and marked so that the DIRECT algorithm will no longer evaluate the objective function within the marked region. The predefined searching space excluding the marked neighborhoods of found local optima will be the search space for the next iteration. This mechanism prevents the DIRECT algorithm from being trapped in a local optimum for long.

3.2. Regional Optimum Detection

The DIRECT algorithm is said to be approaching a regional optimum if the following criterion is satisfied:

$$|X_{n+1} - X_n| \le D_x \quad and \quad |Y_{n+1} - Y_n| \le D_y$$

for successive 5 iterations. In the above criterion, (X_i, Y_i) are the coordinates of the ith sampled point in 2D space involving x and y translations. D_x and D_y are the predefined distance of 2 consecutive iterations. Our experiments show that the values for D_x and D_y can range from 5 to 20. For 3D cases involving rotation and two translations, the criterion can be extended as

$$|R_{n+1} - R_n| \le D_{Rotation} \quad , |X_{n+1} - X_n| \le D_x \quad , \quad |Y_{n+1} - Y_n| \le D_y$$

After regional optimum detection, the system then switches from DIRECT algorithm to a local optimizer which can converge to the local optimum quickly. The result (the parameter set α) is then passed to the global optimum test. In our implementation, simplex search algorithm [4] is used whenever a local optimizer is required.

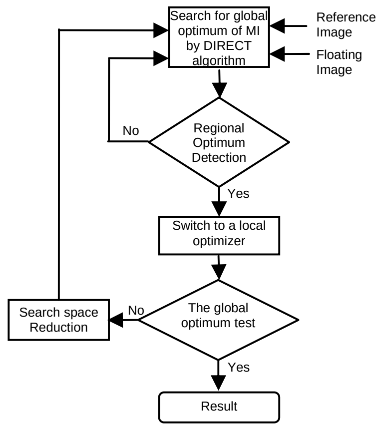


Fig. 3: Block diagram of the proposed integrated system for MI based image registration

3.3. Search space reduction

The neighborhood of a found local optimum to be excluded from the search space is determined using the rules specified in Tables 1 and 2 for 2D and 3D cases respectively. (α is the parameter set of the local optimum).Those values are obtained empirically based on images of size 512 by 512 pixels.

Table 1: Eliminated region in 2D case

	X-Displacement	Y-Displacement
Upper Bound	$\alpha_{X\text{-}Displacement}$+15	$\alpha_{Y\text{-}Displacement}$+15
Lower Bound	$\alpha_{X\text{-}Displacement}$-15	$\alpha_{Y\text{-}Displacement}$-15

Table 2: Eliminated region in 3D case

	Rotation	X-Displacement	Y-Displacement
Upper Bound	$\alpha_{Rotation}$+2	$\alpha_{X\text{-}Displacement}$+15	$\alpha_{Y\text{-}Displacement}$+15
Lower Bound	$\alpha_{Rotation}$-2	$\alpha_{X\text{-}Displacement}$-15	$\alpha_{Y\text{-}Displacement}$-15

4. Experimental results

In this section, we compare the performance of the proposed optimization method with that of using DIRECT algorithm alone. For the latter case, the stopping criterion is the following: if the current best parameter set remains the same after T iterations, then the program terminates. Notice that in practice, it is hard to determine the T value. In our experiments, we show the results from the smallest acceptable T that produces correct solution.

Two experimental results are shown in Figs. 4 and 5. The images used for the first experiment are all [512×512] (Fig. 6 (a)(b)) and the images size used for the second experiment are [512×512] for the floating image (Fig. 7(b)) and [2048×2048] for the reference image (Fig. 7(a)). The search space for the first case is -250~255 for vertical and horizontal displacements, and rotation is set from -45~45 degree. For the second experiment, the search space is -200~100 for the vertical displacement, -330~230 for the horizontal displacement, and 0 ~ 90 degree for the rotation angle. Scaling factors were obtained from the associated header files. From Fig.4, we observe that the global optimum can be reached and identified successfully using the proposed optimization method in about 3000 function evaluations. On the other hand, the global optimum can not be reached using DIRECT algorithm alone if T is less than 60 and this results in total more than 14000 function evaluations. And from Fig.5, the global optimum can also be determined successfully using the proposed optimization method in about 3000 function evaluations but about 20,000 function evaluations are required using DIRECT algorithm alone.

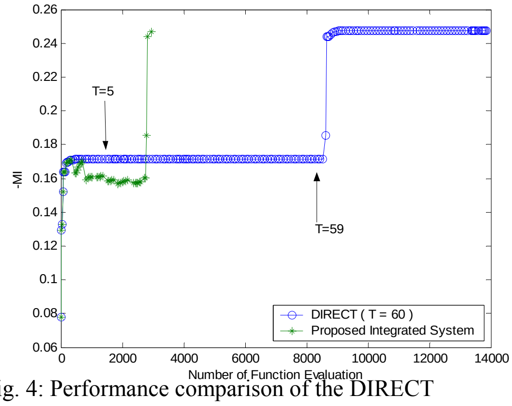


Fig. 4: Performance comparison of the DIRECT algorithm and the proposed integrated system for the first

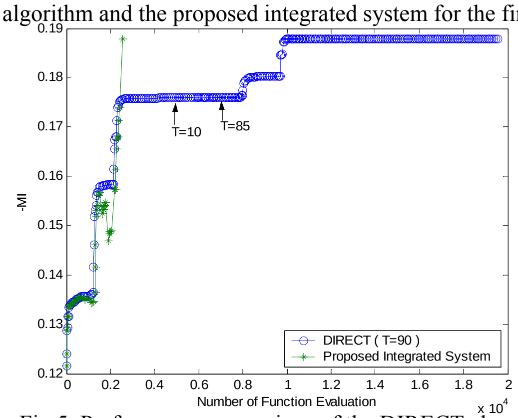
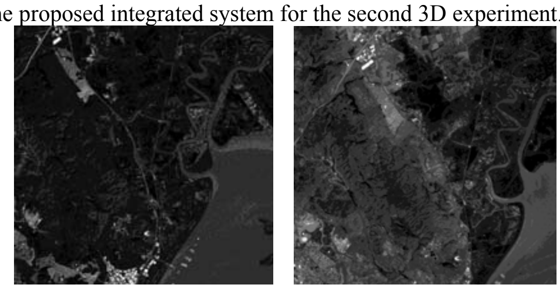
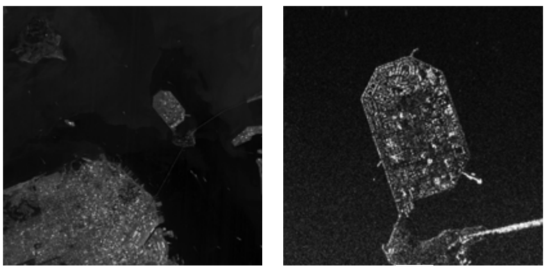


Fig 5: Performance comparison of the DIRECT algorithm and the proposed integrated system for the second 3D experiment.



(a) (b)

Fig 6: Image pair used in the first experiment



(a) (b)

Fig 7: Image pair used in the second experiment.

5. Summary

In this paper, we demonstrate how to integrate the heuristic global optimum test described in [2] into DIRECT algorithm for parametric image registration application. The efficacy of the proposed optimization approach is evidenced by two pairs of remote sensing images. Our experimental results show that by integrating the global optimum test into the DIRECT algorithm, the efficiency of DIRECT algorithm can be dramatically improved.

6. References

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