

New isoperimetric inequalities with densities arising in reaction-diffusion problems

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In joint works with X. Ros-Oton and J. Serra, the study of the regularity of stable solutions to reaction-diffusion problems has led us to certain Sobolev and isoperimetric inequalities with weights.

We will present our results in these new isoperimetric inequalities with the best constant, that we establish via the ABP method. More precisely, we obtain a new family of sharp isoperimetric inequalities with weights (or densities) in open convex cones of \mathbf{R}^n . Our results apply to all nonnegative homogeneous weights satisfying a concavity condition in the cone. Surprisingly, even that our weights are not radially symmetric, Euclidean balls centered at the origin (intersected with the cone) minimize the weighted isoperimetric quotient. As a particular case of our results, we provide with new proofs of classical results such as the Wulff inequality and the isoperimetric inequality in convex cones of Lions and Pacella.

Furthermore, we also study the anisotropic isoperimetric problem for the same class of weights and we prove that the Wulff shape always minimizes the anisotropic weighted perimeter under the weighted volume constrain.