S13. Group Theory

Organizers:

- Gustavo A. Fernández-Alcober (University of the Basque Country, Spain)
- Jon González-Sánchez (University of the Basque Country, Spain)
- Patrizia Longobardi (University of Salerno, Italy)
- Mercede Maj (University of Salerno, Italy)

Invited speakers:

1. Ilya Kazachkov (University of Oxford, United Kingdom)
   Elementary equivalence of groups: a survey and examples
2. Primož Moravec (University of Ljubljana, Slovenia)
   Bogomolov multipliers
3. Marta Morigi (University of Bologna, Italy)
   Coverings of word values
4. Lucía Sanus (University of Valencia, Spain)
   On the prime graph of finite groups

Other speakers:

1. Milagros Arroyo-Jordá (Polytechnic University of Valencia, Spain)
   Conditional permutability of subgroups
2. Azam Babai (Amirkabir University of Technology, Iran)
   On the prime graph associated with a finite group
3. Elena Couselo (University of Oviedo, Spain)
   Generators for the representation of Reed-Solomon group codes
4. Gabriella D'Este (University of Milan, Italy)
   A personal survey on recent and less recent results on tilting theory
5. Khadijeh Fathalikhani (University of Kashan, Iran)
   On the symmetries of Fibonacci and Lucas cubes

6. María José Felipe (Polytechnic University of Valencia, Spain)
   Conjugacy class sizes and normal subgroups

7. Oihana Garaialde (University of the Basque Country, Spain)
   Cohomology of the sporadic group J_2 over F_3

8. Şükran Gül (Middle East Technical University, Turkey)
   Beauville structures in powerful p-groups and regular p-groups

9. Dessislava Kochloukova (State University of Campinas, Brazil)
   Sigma invariants for Artin groups

10. Leire Legarreta (University of the Basque Country, Spain)
    Characterization of some infinite groups with restricted centralizers

11. Emma Leppälä (University of Oulu, Finland)
    Nilpotent loops and groups

12. Conchita Martínez-Pérez (University of Zaragoza, Spain)
    Bredon cohomological dimension for Coxeter groups

13. Carmela Musella (University of Naples Federico II, Italy)
    On soluble groups of infinite rank

14. Péter P. Pálfy (Alfréd Rényi Institute of Mathematics, Hungary)
    Elliptic curves and p-groups

15. Ralph Stöhr (University of Manchester, United Kingdom)
    On torsion in free central extensions of infinite groups

16. Antonio Tortora (University of Salerno, Italy)
    On Engel groups satisfying an identity

Elementary equivalence of groups: a survey and examples

Ilya Kazachkov

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The model-theoretic notion of elementary equivalence can be likened to isomorphism in algebra. In this talk I shall present a number of results addressing the problem of classification of groups elementarily equivalent to a given one, survey the main ideas that go into proofs and present open problems in this area.

Bogomolov multipliers

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The Bogomolov multiplier is a group theoretical invariant isomorphic to the unramified Brauer group of a given quotient space, and represents an obstruction to Noether's problem on rationality of fixed fields. In this talk we survey some recent results regarding Bogomolov multipliers. We describe a homological version of the Bogomolov multiplier, which relies on universality of certain commutator relations in groups. We prove a Hopf-type formula, find a five term exact sequence corresponding to this invariant, and describe the role of the Bogomolov multiplier in the theory of central extensions. An algorithm for computing the Bogomolov multiplier is developed.

Coverings of word values

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A word w in n variables is an element of the free group on n generators. If G is a group, we may see w as a function $w : G^n \to G$, and we denote by G_w the set of values taken by w. The verbal subgroup $w(G)$ is the subgroup generated by G_w. We will address the following question: if G_w is contained in the union of a finite number of subgroups of G all satisfying some property, what information can we deduce about $w(G)$? Particular emphasis will be put on the case when G is a profinite group and w is an outer commutator word.

On the prime graph of finite groups

Lucía Sanus

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Let G be a finite group; we denote by Irr(G) the set of all irreducible complex characters of G, and write

$$\mathrm{cd}(G) = \{\chi(1) \mid \chi \in \mathrm{Irr(G)}\}$$

for the set of the degrees of such characters. The character degree graph $\Delta(G)$ is thus defined as the graph with vertex set the set $\rho(G)$ of all the primes that divide some $\chi(1) \in \mathrm{cd}(G)$, and two distinct primes p and q are adjacent if and only if pq divides some degree in $\mathrm{cd}(G)$. In the literature, we find many results that show the relationships between the properties of $\Delta(G)$ and the structure of the group G.

It is well known that, whenever $\Delta(G)$ is connected, the diameter of $\Delta(G)$ is at most 3. The aim of this talk is to present some new results about the finite solvable groups for which the diameter of this graph attains the upper bound.

This is joint work with Carlo Casolo and Silvio Dolfi (University of Florence, Italy), and with Emanuele Pacifici (University of Milan, Italy).

Conditional permutability of subgroups

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Factorized groups in which certain subgroups of the corresponding factors permute have been widely investigated over the last years. A first step leading this research was the consideration of mutual and total permutability to provide conditions for a product of supersoluble groups to be supersoluble. Later on this study was extended within the context of classes of groups. Also new contributions have been achieved by considering a weaker condition of subgroup permutability, namely conditional permutability. The aim of this talk is to collect recent results on structural properties of such factorized groups and their relation with certain classes of groups.

Work in collaboration with Paz Arroyo-Jordá, Ana Martínez-Pastor (Polytechnic University of Valencia, Spain) and M. Dolores Pérez-Ramos (University of Valencia, Spain).

On recognition of finite groups via their prime graph

Azam Babai

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We construct the prime graph of G, which is denoted by $\Gamma(G)$, as follows: the vertex set of this graph is the set of prime divisors of $|G|$ and two distinct vertices p and q are joined by an edge if and only if G contains an element of order pq. A finite group G is said to be recognizable by its prime graph if, whenever H is another finite group, the equality $\Gamma(G) = \Gamma(H)$ implies that $G \cong H$. In this talk, we want to discuss about recognizability of finite groups by their prime graph.

Generators for the representation of Reed-Solomon group codes

Elena Couselo

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A new technique for the representation of Reed-Solomon codes is introduced. The previously used techniques consisted in representing them as kernels of some homomorphisms, that is, they are defined by parity-check relations. In the new method, generators for the ideals of the group ring of an elementary abelian p-group over a finite field of characteristic p presenting the codes are explicity specified.

A personal survey on recent and less recent results on tilting theory

Gabriella D'Este

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I will present some results on tilting theory obtained by looking at the underlying abelian groups of rings, modules, bimodules and morphisms. These abelian groups will always be vector spaces over some algebraically closed field K, and their dimension will be countable and almost always finite, as in the classical situation considered by S. Brenner and M.C.R. Butler in "Generalizations of the Bernstein-Gelfand-Ponomarev reflection functors", Springer LNM 832 (1980). We will show that both discrete and continuous properties show up in a surprising way. Indeed, on the one hand, "simple" and combinatorial objects may have unexpected concealed topological properties. On the other hand, "non-simple" objects do not always have complicated topological properties.

On the symmetries of Fibonacci and Lucas cubes

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In this talk, the orbits of the Fibonacci and the Lucas cubes under the action of their automorphism groups on the set of their vertices will be investigated. We will be concerned with the number of these orbits and their sizes. Some relations to other combinatorial objects will also be presented.

This is joint work with Ali Reza Ashrafi (University of Kashan, Iran), and with Jernej Azarija, Sandi Klavžar, and Marko Petkovšek (Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia).

Conjugacy class sizes and normal subgroups

María José Felipe

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It is known that the structure of a finite group is strongly controlled by the set of its conjugacy class sizes. Let G be a finite group and N a normal subgroup of G. Since N is union of conjugacy classes of G, it is natural to wonder what information on the structure of N can be obtained from the G-class sizes of N, that is, the sizes of the conjugacy classes in G contained in N. We would like to point out that there is no relation between the cardinal of the set of the conjugacy class sizes of N and the cardinal of the set of its G-class sizes. However, several results have put forward that the G-class sizes, or even the G-class sizes of certain subsets of elements of a normal N, such as the p'-elements for some prime p or the prime-power order elements, also influence on its structure. We present some recent researches concerning this topic.

Cohomology of the sporadic group J_2 over $\mathbb{F}_3$

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We present a spectral sequence for fusion systems. We also describe the cohomology ring $H^*(J_2;\mathbb{F}_3)$ both as a subring of $H^*(3_+^{1+2};\mathbb{F}_3)$ and with abstract presentation.

This is joint work with Antonio Díaz (University of Málaga, Spain).

Beauville structures in powerful p-groups and regular p-groups

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Given a group G and two elements $x, y \in G$, we denote by $\sum(x, y)$ the union of all conjugacy classes of all powers of x, y, and xy. Then G is called a *Beauville group* of unmixed type if the following two conditions hold:

(1) G is a 2-generator group.

(2) G has two sets of generators $\{x_1, y_1\}$ and $\{x_2, y_2\}$ such that $\sum(x_1, y_1) \cap \sum(x_2, y_2) = 1$.

In this case, $\sum(x_1, y_1)$ and $\sum(x_2, y_2)$ are said to form a *Beauville structure* for G. It is known that an abelian finite p-group is a Beauville group if and only if it is isomorphic to $C_{p^n} \times C_{p^n}$, where $p \geq 5$ and $n \geq 1$. In this talk we will discuss the conditions under which a 2-generator powerful or regular p-group is a Beauville group. It turns out that these conditions are similar to the conditions for an abelian p-group to be a Beauville group.

This is joint work with Gustavo A. Fernández-Alcober (University of the Basque Country, Spain).

Sigma invariants for Artin groups

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We discuss the first dimensional Sigma invariant of Artin group based on a graph with fundamental group infinite cyclic group. The Sigma invariants were first defined for finitely generated metabelian groups by Bieri and Strebel and later generalised for all finitely generated groups by Bieri, Neumann, Renz, Strebel. In the special case of right angle Artin groups the Sigma invariants are calculated in all dimensions by Meier, Meiner, Van Wyk.

This is a joint work with Kisnney Almeida (Brazil).

Characterization of some infinite groups with restricted centralizers

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What can be said about an infinite group G if the centralizer of every non-central/non-normal subgroup $\langle g \rangle$ is not much larger than $\langle g \rangle$? In order to formalise this question more precisely, we call a group G an FCI-group if the index $|C_G(g) : \langle g \rangle|$ is finite for every $\langle g \rangle \not\trianglelefteq G$, and a BCI-group if all these indices are bounded. Now focusing on locally finite FCI-groups, we show how all these groups can be constructed starting from an infinite Dedekind group D of finite 2-rank and an appropriate power automorphism of D of finite order. Similarly, starting from a Dedekind group D which is the direct product of finitely many p-groups of finite rank and an appropriate power automorphism of D of any order, we also characterize all locally nilpotent FCI-groups. Finally, trying to avoid the presence of Tarski groups, we deal with locally graded BCI-groups, and we prove that, in the periodic case, they must be necessarily locally finite groups.

This is joint work with Gustavo A. Fernández-Alcober (University of the Basque Country, Spain), and with Antonio Tortora and Maria Tota (University of Salerno, Italy).

Nilpotent loops and groups

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A groupoid Q with a neutral element is a loop, if the equations $ax = ya = b$ have unique solutions x and y for each a and b. If we add associativity, we get a group. We define two permutation groups associated to loops, the multiplication group and inner mapping group of a loop. Many properties of loops can be investigated through these groups. Here we focus on the nilpotency of loops, which is defined analogously to the nilpotency in group theory. We present fundamental results concerning the relation between nilpotency of the two groups and nilpotency of the corresponding loop. We also introduce some recent results regarding the nilpotency class of certain loops.

Bredon cohomological dimension for Coxeter groups

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Using compactly supported cohomology we obtain a formula for the Bredon cohomological dimension of certain groups. As an application we show that for some families of groups including Coxeter groups the Bredon and the virtual cohomological dimensions coincide.

This is a joint work with Dieter Degrijse.

On soluble groups of infinite rank

Carmela Musella

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A group G is said to have finite (Prüfer) rank if there exists a positive integer r such that all finitely generated subgroups of G can be generated by at most r elements; otherwise, if such r does not exist, the group is said to have infinite rank. In a series of recent papers it has been proved that the subgroups of infinite rank of a group of infinite rank have the power to influence the structure of the whole group and to force also the behaviour of the subgroups of finite rank, at least in the case of (generalized) soluble groups. In this talk, we discuss some new contributions to this topic.

Elliptic curves and p-groups

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We will discuss two enumeration problems concerning finite p-groups.

(1) What is the number of (isomorphism types of) groups of order p^n? Higman's PORC conjecture states that for a fixed n there exist a number M and finitely many polynomials such that the number of groups of order p^n is the value of one of these polynomials at p, and the suitable polynomial is chosen according to the residue class of p modulo M.

(2) What is the number of conjugacy classes in the group of n-by-n upper unitriangular matrices over the q-element field? Another conjecture of Higman states that for a fixed n it is a polynomial in q.

Recent results of du Sautoy and Vaughan-Lee related to problem (1), and Halasi and the speaker related to problem (2) might indicate that both conjectures will turn out to be false for large values of n. Both results are based on encoding some elliptic curves into the structure of certain p-groups.

On torsion in free central extensions of infinite groups

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Let G be a group given by a free presentation $G = F/R$ and consider the quotient

$$F/[R', F] \tag{1}$$

where R' denotes the commutator subgroup of R. This quotient is a free central extension of F/R'. While F/R' is always torsion-free, elements of finite order may occur in $R'/[R', F]$, the centre of (1). The torsion subgroup $t(R'/[R', F])$ has exponent dividing 4 (Kuzmin 1982). In fact, if G has no elements of order 2, it is isomorphic to the homology group $H_4(G, \mathbb{Z}_2)$ (Stöhr 1987). Very little was known in the case where the group G does contain elements of order 2. The talk is on recent results in the case where G has as many elements of order 2 as possible, namely, the case where G is an elementary abelian 2-group.

This is joint work with Maria Alexandrou.

On Engel groups satisfying an identity

Antonio Tortora

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It is well known that there exist counterexamples to the general Burnside problem, even among residually finite groups. However, by a deep result of Zelmanov, any residually finite group which is periodic and satisfies some nontrivial identity, is locally finite. We will discuss an analogous result in the realm of Engel groups and, in particular, some consequences depending on its proof.