S14. Harmonic Analysis

Organizers:

- Albert Mas (Universitat Autònoma de Barcelona, Spain)
- Marco Peloso (Università degli Studi di Milano, Italy)

Speakers:

1. Vasilis Chousionis (University of Helsinki, Finland)
   Square functions and uniform rectifiability
2. Leonardo Colzani (Università degli Studi di Milano, Italy)
   Localization and convergence of Fourier series and integrals
3. Javier Duoandikoetxea (Universidad del País Vasco/Euskal Herriko Unibertsitatea, Spain)
   A comparison of the characterizations of A_∞
4. Bruno Franchi (Università degli Studi di Bologna, Italy)
   Differential forms in Heisenberg groups and div-curl systems
5. Gustavo Garrigós (Universidad de Murcia, Spain)
   Maximal functions, weights and convergence to initial data of certain Poisson equations
6. Jose María Martell (ICMAT, Universidad Autónoma de Madrid, Spain)
   The Dirichlet problem for elliptic systems in the upper-half space
7. Alessio Martini (Christian-Albrechts-Universität zu Kiel, Germany)
   Hypoelliptic operators and sharp multiplier theorems
8. Joan Mateu (Universitat Autònoma de Barcelona, Spain)
   Euler equation and vortex patches
9. Carlos Pérez (Universidad de Sevilla, Spain)
   Around a conjecture of E. Sawyer for the Hilbert Transform and weights
10. Ezequiel Rela (Universidad de Sevilla, Spain)
    Optimality of weighted estimates without examples

11. Fulvio Ricci (Scuola Normale Superiore di Pisa, Italy)
Hardy and uncertainty inequalities on stratified groups

12. Ignacio Uriarte-Tuero (Michigan State University, USA)
Two weight norm inequalities for singular and fractional integral operators in $\mathbb{R}^n$

13. Maria Vallarino (Politecnico di Torino, Italy)
Schrödinger equations on Damek-Ricci spaces

Square functions and uniform rectifiability

Vasilis Chousionis

Department of Mathematics and Statistics, University of Helsinki, Finland
chousion@mappi.helsinki.fi

We characterize uniform rectifiability via square functions. In particular we show that an Ahlfors-David n-dimensional measure μ on $\mathbb{R}^d$ is uniformly n-rectifiable if and only if for any ball $B(x_0, R)$ centered at $\operatorname{supp}(\mu)$,

$$\int_0^R \int_{x\in B(x_0,R)} \left| \frac{\mu(B(x,r))}{r^n} - \frac{\mu(B(x,2r))}{(2r)^n} \right|^2 d\mu(x)\, \frac{dr}{r} \leq c\, R^n.$$

This can be realized as a square functions analogue of Preiss theorem which characterizes rectifiability in terms of the existence of densities.

The talk is based on a recent joint work with J. Garnett, T. Le and X. Tolsa.

Localization and convergence of Fourier series and integrals

Leonardo Colzani

Dipartimento di Matematica, Universit degli Studi di Milano Bicocca, via Cozzi 55, 20125 Milano Italia
leonardo.colzani@unimib.it

We present some multidimensional analogues of classical results on the convergence and localization of Fourier expansions, and the dimension of the sets of points where convergence or localization may fail. The Riesz means are defined by the Fourier integrals

$$S_R^\alpha f(x) = \int_{|\xi|<R} (1 - |R^{-1}\xi|^2)^\alpha f(\xi) exp(2\pi i x\xi) d\xi.$$

Under suitable assumptions on the parameters $d, p, \alpha, \beta, \gamma$, the Riesz means with index α of functions with β derivatives in $L^p(\mathbb{R}^d)$ converge pointwise, with possible exception of sets of points with Hausdorff dimension at most γ. Similarly, under suitable assumptions, for functions vanishing in an open set localization holds at all points in the open set, with possible exceptions of small dimension. Some of these results extend to more general eigenfunction expansions.

[1] Colzani, L., Fourier expansions of functions with bounded variation of several variables, *Trans. Amer. Math. Soc.* **358** (2006), 5501–5521.

[2] Colzani, L., Meaney, C., Prestini, E., Almost everywhere convergence of inverse Fourier transforms, *Proc. Amer. Math. Soc.* **134** (2006), 1651–1660.

[3] Colzani, L., Volpi, S., Pointwise convergence of Bochner-Riesz means in Sobolev spaces, *Trends in harmonic analysis* (M. A. Picardello ed.), Springer INdAM Ser., 3, Springer, Milan, 2013, 135–146,

[4] Colzani, L., Gigante, G., Volpi, S., Equiconvergence theorems for Sturm Liouville expansions and sets of divergence for Bochner Riesz means in Sobolev spaces, *J. Fourier Anal. Appl.* **19** (2013), 1184–1206.

[5] Colzani, L., Gigante, G., Vargas, A., Localization for Riesz means of Fourier expansions, *Trans. Amer. Math. Soc.*, to appear.

A comparison of the characterizations of A_∞

Javier Duoandikoetxea

Departamento de Matemáticas, University of the Basque Country UPV/EHU, Apdo. 640, 48080 Bilbao, Spain
javier.duoandikoetxea@ehu.es

A weight w is in A_∞ if the Hardy-Littlewood maximal operator is bounded on $L^p(w)$ for some $p \in (1, \infty)$, that is, if w is in A_p for some p. There are a number of characterizations of A_∞ and it is customary to choose one of them as the definition of the class (not the same one for all authors) and to prove that the others are equivalent. Esssential to the proof of some of the equivalences is that we are considering the basis of cubes (or Euclidean balls) of $\mathbf{R}^n$. The equivalence can fail for a different basis.

In this work we consider a general basis and define the classes of weights corresponding to the different characterizations of the usual A_∞. We prove that in general there are two independent chains of inclusions and that they can be strict. The basis of $(0, +\infty)$ consisting of the intervals of the form $(0, b)$ $(b > 0)$ provides the counterexamples for those inclusions that are strict. In the case of the usual A_∞ weights we are able to obtain new characterizations.

This is joint work with Francisco J. Martín-Reyes (Universidad de Málaga, Spain) and Sheldy Ombrosi (Universidad Nacional del Sur, Bahía Blanca, Argentina).

Differential forms in Heisenberg groups and div-curl systems

Bruno Franchi

Department of mathematics, University of Bologna, Piazza di Porta S. Donato, 5, 40126 Bologna, Italy
bruno.franchi@unibo.it

In this talk we present a result proved in collaboration with Annalisa Baldi (Bologna). We prove a family of inequalities for differential forms in Heisenberg groups (Rumin's complex), that are the natural counterpart of a class of div-curl inequalities in de Rham's complex proved by Lanzani & Stein and Bourgain & Brezis.

Maximal functions, weights and convergence to initial data of certain Poisson equations

Gustavo Garrigós

University of Murcia, Spain
gustavo.garrigos@um.es

Consider the Poisson equation $u_{tt} = \mathcal{L}u$, with $\mathcal{L}$ the Hermite or the Ornstein-Uhlenbeck operator in $\mathbf{R}^d$. We look for very general conditions on the initial datum f, so that $u(t,x) = e^{-t\sqrt{-\mathcal{L}}}f(x)$ converges a.e. to $f(x)$.

When $v(x)$ is a weight in A_p, this is classically obtained from the $L^p(v)$ boundedness of the associated maximal operators

$$\mathcal{M}f(x) = \sup_{t>0} |u(t,x)|.$$

However, such convergence also holds with less restrictive conditions, such as boundedness from $L^p(v) \to L^p(u)$, for some other weight $u(x)$, of a *local* maximal operator $\mathcal{M}_a f = \sup_{0<t<a} |u(t,x)|$ for some $a > 0$. This produces a larger class of functions than classical A_p theory.

In this work we solve this version of the 2-weight problem, and as a consequence characterize the weights $v(x)$ for which $u(t,x) \to f(x)$ a.e. for all $f \in L^p(v)$.

The proofs are based on factorization techniques developed by Rubio de Francia, and a careful analysis of the involved Poisson kernels in the Hermite and Ornstein-Uhlenbeck settings.

The results are part of the joint work [1].

[1] G. Garrigós, S. Hartzstein, T. Signes, J. L. Torrea, B. Viviani, Pointwise convergence to initial data of heat and Laplace equations (2014), preprint. Available at webs.um.es/gustavo.garrigos.

The Dirichlet problem for elliptic systems in the upper-half space

José María Martell

ICMAT CSIC-UAM-UC3M-UCM, Consejo Superior de Investigaciones Científicas, C/ Nicolás Cabrera, 13-15, E-28049 Madrid, Spain
chema.martell@icmat.es

Let L be an arbitrary second-order, homogeneous, elliptic system, with constant complex coefficients (e.g., the Laplacian or the Lamé system of elasticity). Given $1 < p < \infty$, consider the L^p-Dirichlet problem in the upper-half space:

$$\begin{cases} u \in C^\infty(\mathbb{R}^n_+), \\ Lu = 0 \ \text{ in } \ \mathbb{R}^n_+, \\ \mathcal{N}u \in L^p(\mathbb{R}^{n-1}), \\ u\big|^{\text{n.t.}}_{\partial\mathbb{R}^n_+} = f \in L^p(\mathbb{R}^{n-1}), \end{cases}$$

where $\mathcal{N}u$ is the nontangential maximal function of u, and $u\big|^{\text{n.t.}}_{\partial\mathbb{R}^n_+}$ is the nontangential limit of u on the boundary of the upper half-space.

As known from the seminal work of S. Agmon, A. Douglis, and L. Nirenberg, L has a Poisson kernel P^L, an object whose properties mirror the most basic characteristics of the classical harmonic Poisson kernel. Taking its convolution with f, we obtain a solution of the previous problem. Hence, the difficulty in proving well-posedness comes down to proving uniqueness. In the well-known case of the Laplacian, this is done by employing the maximum principle for harmonic functions, a tool not available in the case of systems. We overcome this difficulty by constructing an appropriate Green function and using a version of the divergence theorem.

Our methods are flexible and allows us to establish well-posedness of the Dirichlet problems with non-smooth boundary data in some other classes of functions (e.g., variable exponent Lebesgue spaces, Lorentz spaces, Zygmund spaces, as well as their weighted versions). We present a general result, written in the framework of Köthe function spaces, establishing that the well-posedness of the corresponding boundary value problems is equivalent to the boundedness of the Hardy-Littlewood maximal function.

Joint work with D. Mitrea, I. Mitrea, and M. Mitrea.

Hypoelliptic operators and sharp multiplier theorems

Alessio Martini

Mathematisches Seminar, Christian-Albrechts-Universität zu Kiel, Ludewig-Meyn-Str. 4, 24106 Kiel, Germany
martini@math.uni.kiel.de

Let L be the Laplacian on $\mathbf{R}^n$. The investigation of necessary and sufficient conditions for an operator of the form $F(L)$ to be bounded on L^p in terms of "smoothness properties" of the spectral multiplier F is a classical research area of harmonic analysis, with long-standing open problems (e.g., the Bochner-Riesz conjecture) and connections with the regularity theory of PDEs.

In settings other than the Euclidean, particularly in the presence of a sub-Riemannian geometric structure, the natural substitute L for the Laplacian need not be an elliptic operator, and it may be just hypoelliptic. In this context, even the simplest questions related to the L^p-boundedness of operators of the form $F(L)$ are far from being completely understood.

I will present some recent results, obtained in joint works with Detlef Müller (Kiel) and Adam Sikora (Sydney), dealing with the case of sublaplacians on 2-step stratified (Carnot) groups, and with Grushin operators.

[1] Martini, A., Sikora, A., Weighted Plancherel estimates and sharp spectral multipliers for the Grushin operators, *Math. Res. Lett.* **19** (2012), 1075–1088.

[2] Martini, A., Müller, D., A sharp multiplier theorem for Grushin operators in arbitrary dimensions (2012), *Rev. Mat. Iberoam.*, to appear; http://arxiv.org/abs/1210.3564.

[3] Martini, A., Müller, D., L^p spectral multipliers on the free group $N_{3,2}$, *Studia Math.* **217** (2013), 41–55.

[4] Martini, A., Spectral multipliers on Heisenberg–Reiter and related groups, *Ann. Mat. Pura Appl.* (2014), doi:10.1007/s10231-014-0414-6.

[5] Martini, A., Müller, D., Spectral multiplier theorems of Euclidean type on new classes of 2-step stratified groups (2013), *Proc. Lond. Math. Soc.*, to appear; http://arxiv.org/abs/1306.0387.

Euler equation and vortex patches

Joan Mateu

Departament de Matemàtiques, Universitat Autònoma de Barcelona,
80193 Bellaterra, Spain
`mateu@mat.uab.cat`

We will talk about the existence of rotating vortex patches for the Euler equation. Firstly, we will recall the existence result in the simply connected case. Then we will present the existence of rotating V-states in the doubly connected case. The proofs in this case follow by bifurcation from the annuli.

This is a joint work with F. de la Hoz, T. Hmidi and J. Verdera.

A conjecture of E. Sawyer for the Hilbert transform and weights

Carlos Pérez

Institute of Mathematical Sciences, University of Seville
carlosperez@us.es

Muckenhoupt and Wheeden in [2] and later on Sawyer in [5], gave one-dimensional highly nontrivial extensions of the fundamental weak type $(1, 1)$ property of the maximal function involving weights. These results were conjectured to be true for the Hilbert transform and for the maximal function in higher extensions. These conjectures were proved in [1] an extended in different directions. In this lecture we will survey about these results and present new more precise quantitatives estimates involving the A_p or A_∞ constants of the weights involved.

This is part of joint work with S. Ombrosi and J. Recchi ([4], [3]).

[1] D. Cruz-Uribe, SFO, J.M. Martell and C. Pérez, Weighted weak-type inequalities and a conjetura of Sawyer, *Int. Math. Res. Not.* **30** (2005), 1849–1871.

[2] B. Muckenhoupt and R. Wheeden, Some weighted weak-type inequalities for the Hardy-Littlewood maximal function and the Hilbert transform, *Indiana Univ. Math. J.* **26** (1977), 801–816.

[3] S. Ombrosi, C. Pérez and J. Recchi, Quantitative weighted mixed weak-type inequalities for classical operators, in preparation.

[4] J. Recchi, *Estimaciones Cuantitativas en Análisis Armónico,* PhD thesis, Universidad Nacional del Sur (Bahía Blanca, Argentina), May 2014.

[5] E. Sawyer, A weighted weak type inequality for the maximal function, *Proc. Amer. Math. Soc.* **93** (1985), 610–614.

Optimality of weighted estimates without examples

Ezequiel Rela

Departamento de Análisis Matemático, University of Seville, Spain
erela@us.es

We present a general approach for proving the optimality of the exponents on weighted estimates. We show that if an operator T satisfies a bound like

$$\|T\|_{L^p(w)} \leq c\,[w]_{A_p}^{\beta} \qquad w \in A_p,$$

then the optimal lower bound for β is closely related to the asymptotic behaviour of the unweighted L^p norm $\|T\|_{L^p}$ as p goes to 1 and $+\infty$.

By combining these results with the known weighted inequalities, we derive the sharpness of the exponents, without building any specific example, for a wide class of operators including maximal-type, Calderón–Zygmund and fractional operators. In particular, we obtain a lower bound for the best possible exponent for Bochner-Riesz multipliers. We also present a new result concerning a continuum family of maximal operators on the scale of logarithmic Orlicz functions. Further, our method allows to consider in a unified way maximal operators defined over very general Muckenhoupt bases.

This is a joint work with Teresa Luque and Carlos Pérez from University of Seville.

Hardy and uncertainty inequalities on stratified groups

Fulvio Ricci

Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126 Pisa, Italy
fricci@sns.it

We present results obtained jointly with P. Ciatti and M. Cowling.

The starting point is the proof of L^p-boundedness of the operators $T_\alpha : f \mapsto |\cdot|^{-\alpha} L^{-\alpha/2} f$, where $|\cdot|$ is a homogeneous norm, $0 < \alpha < Q/p$, and L is a sub-Laplacian, with a control on the constants as $\alpha \to 0$ which allows to deduce a logarithmic uncertainty inequality. We also deduce a general version of the Heisenberg–Pauli–Weyl inequality, relating the L^p norm of a function f to the L^q norm of $|\cdot|^\beta f$ and the L^r norm of $L^{\frac{\delta}{2}} f$.

Two weight norm inequalities for singular and fractional integral operators in R^n

Eric T. Sawyer, Chun-Yen Shen, Ignacio Uriarte-Tuero*

Department of Mathematics, Michigan State University, 619 Red Cedar Rd., East Lansing, MI 48824, USA
ignacio@math.msu.edu

I will report on recent advances on the topic, related to proofs of T1 type theorems with side conditions and related counterexamples.

Joint work with Eric Sawyer and Chun-Yen Shen.

Schrödinger equations on Damek–Ricci spaces

Maria Vallarino

Dipartimento di Scienze Matematiche, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy
maria.vallarino@polito.it

Let S be a Damek–Ricci space. We discuss the dispersive properties of the Schrödinger propagator $e^{it\Delta}$ associated with the Laplace–Beltrami operator Δ on S and deduce Strichartz estimates for the solution of the nonhomogeneous linear Schrödinger equation associated with Δ. These results were obtained in a joint work with J.Ph. Anker and V. Pierfelice [1].

We then consider a distinguished Laplacian $\mathcal{L} = -\sum_{i=1}^{n} X_i^2$, where $\{X_i\}_{i=1}^{n}$ is a basis of left invariant vector fields of the Lie algebra of S: we show that the Schrödinger propagator $e^{it\mathcal{L}}$ associated with $\mathcal{L}$ does not satisfy a L^1–L^∞ dispersive estimates. However we show that a dispersive estimate for the spectrally localized Schrödinger propagator associated with $\mathcal{L}$ holds. This implies local in time Strichartz estimates for the solution of the nonhomogeneous linear Schrödinger equation associated with $\mathcal{L}$ with fractional loss of derivatives.

This is a joint work with M. Peloso [2].

[1] J.-Ph. Anker, V. Pierfelice, M. Vallarino, Schrödinger equations on Damek-Ricci spaces, *Comm. Partial Differential Equations* **36** (2011), no. 6, 976–997.

[2] M. Peloso, M. Vallarino, The Schrödinger equation associated with a distinguished Laplacian on Damek-Ricci spaces, preprint.