S18. Mathematical Aspects and Applications of Fractional Differential Equations

Organizers:

- Carlota Cuesta (University of the Basque Country (UPV/EHU), Spain)
- Gianni Pagnini (BCAM – Basque Center for Applied Mathematics, Spain)

Speakers:

1. Franz Achleitner (Vienna University of Technology, Austria)
   Travelling waves for a non-local Korteweg–de Vries–Burgers equation
2. Maxim Dolgushev (University of Freiburg, Germany)
   Anomalous dynamics of semiflexible polymers
3. Roberto Garra (University of Rome La Sapienza, Italy)
   Fractional Klein–Gordon equation and related processes
4. Agnieszka Jurlewicz (Wrocław University of Technology, Poland)
   Anomalous diffusion subordination scenarios related with continuous-time random walks
5. Christian Kuehn (Austrian Academy of Sciences/Vienna University of Technology, Austria)
   Operator Perturbations of Dynamical Systems
6. József Lőrinczi (Loughborough University, UK)
   Some results on fractional eigenvalue problems
7. Enrico Scalas (University of Sussex, UK)
   On the compound fractional Poisson process
8. Noèlia Viles (University of Barcelona, Spain)
   Functional limit theorems for the quadratic variation of a continuous time random walk and for certain stochastic integrals

Travelling waves for a non-local Korteweg–de Vries–Burgers equation

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We study travelling wave solutions of a Korteweg–de Vries–Burgers equation with a non-local diffusion term. This model equation arises in the analysis of a shallow water flow by performing formal asymptotic expansions associated to the triple-deck regularisation (which is an extension of classical boundary layer theory). The resulting non-local operator is of fractional type with order between 1 and 2. Travelling wave solutions are typically analysed in relation to shock formation in the full shallow water problem. We show rigorously the existence of these waves. In absence of the dispersive term, the existence of travelling waves and their monotonicity was previously established. In contrast, travelling waves of the non-local KdV–Burgers equation are not in general monotone, as is the case for the corresponding classical (or local) KdV–Burgers equation. This requires a more complicated existence proof compared to the previous work. Moreover, the travelling wave problem for the classical KdV-Burgers equation is usually analysed via a phase-plane analysis, which is not applicable here due to the presence of the non-local diffusion operator. Instead, we apply fractional calculus results available in the literature and a Lyapunov functional. In addition we discuss the monotonicity of the waves in terms of a control parameter and prove their dynamic stability in case they are monotone.

Anomalous dynamics of semiflexible polymers

Maxim Dolgushev

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Polymers being giant molecules consisting of a huge number of monomers are spectacular materials. Thus, the motion of each single monomer inside the polymer is much influenced by the other monomers, leading to a so-called anomalous dynamic behavior, which can be described by means of fractional dynamical equations (e.g. Langevin, Fokker–Planck) [1]. The classical approach for modeling polymers is the formalism of generalized Gaussian structures (GGS) [2]. While the GGS-approach allows to model polymers with arbitrary architectures, it does not include some important aspects, such as hydrodynamic interactions and semiflexibility. This contribution is particularly focused on the extension of GGS–approach to model semiflexible polymers and on their dynamics [3, 4].

[1] I. M. Sokolov, J. Klafter, A. Blumen, Fractional kinetics, *Physics Today* **55** (2002), 48–54.

[2] A. A. Gurtovenko, A. Blumen, Generalized Gaussian Structures: Models for polymer systems with complex topologies, *Adv. Polym. Sci.* **182** (2005), 171–182.

[3] M. Dolgushev, A. Blumen, Dynamics of semiflexible treelike polymeric networks, *J. Chem. Phys.* **131** (2009), 044905.

[4] M. Dolgushev, A. Blumen, Dynamics of discrete semiflexible chains under dihedral constraints: Analytic results, *J. Chem. Phys.* **138** (2013), 204902.

Fractional Klein–Gordon equation and related processes

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A fractional formulation of the one-dimensional Klein–Gordon equation is studied by means of generalized fractional integrals in the sense of McBride [1]. Indeed, by using a series of simple transformations of variables, this equation can be reduced to a fractional hyper-Bessel-type equation that can be explicitly solved by using the theory of fractional power of operators. Exact solutions are found in terms of multi-index Mittag–Leffler functions. A telegraph-type process related to the solution of the fractional Klein–Gordon equation is studied. We also treat the two-dimensional case and related processes. We then discuss other mathematical approaches to fractional hyper-Bessel-type equations by means of fractional Hadamard derivatives and sequential operators involving Caputo derivatives. Also in this case exact solutions are found, that can be of interest for applications in mathematical physics.

[1] A. C. McBride, A theory of fractional integration for generalized functions, *SIAM J. Math. Anal.* **6** (1975), 583–599.

Anomalous diffusion subordination scenarios related with continuous-time random walks

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Continuous time random walks (CTRWs), in which random jumps are separated by random waiting times, are used in physics to model diffusing particles. We present different types of decoupled and coupled CTRW models. We show that the corresponding CTRW scaling limits have a form of time-changed processes, whose densities solve an anomalous diffusion equations (involving space and/or time fractional derivatives). We study properties of the limiting processes; especially focused on the case of the Brownian motion subordinated by some operational-time process. We present also some applications of the considered models.

Operator perturbations of dynamical systems

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In this talk I shall discuss as a main result the existence, uniqueness and stability proof for travelling waves in bistable reaction-diffusion systems involving Riesz–Feller operators. The technique is based upon a sub- and super-solution method, a-priori integral estimates of Riesz–Feller operators, far-field control and several tools from Levy processes. Furthermore, I shall outline a more general framework for dynamical systems analysis of fractional and other non-local partial differential equations

Some results on fractional eigenvalue problems

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In this talk I will discuss an example of a fractional Schrödinger operator whose eigenvalue equation can be solved exactly in the sense that the spectrum and the Fourier transform of the eigenfunctions can be determined in terms of explicit formulae. This seems to be the first example of this kind in the literature. I will present further details (spectral gap, heat trace behaviour, full asymptotic expansions etc.), and explain the relevance of this equation in mathematical physics. If time allows, I will present also some results on related fractional equations.

On the compound fractional Poisson process

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The compound fractional Poisson process (CFPP) is a random walk subordinated to a fractional Poisson process (FPP). The latter is a simple generalization of the Poisson process where waiting times between events follow a Mittag–Leffler distribution. Several results on both CFPP and FPP will be presented related to applications in different fields of science.

Functional limit theorems for the quadratic variation of a continuous time random walk and for certain stochastic integrals

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A continuous time random walk (CTRW) can be formally defined as a random walk subordinated to a counting renewal process. CTRWs became a widely used tool for describing random processes that appear in a large variety of physical models and also in finance. The main motivation of our work comes from the physical model given by a damped harmonic oscillator subject to a random force (Lévy process) studied in the paper of Sokolov et al. [1]. We study the convergence of a class of stochastic integrals with respect to the compound fractional Poisson process. Under proper scaling and distributional hypotheses, we establish a functional limit theorem for the integral of a deterministic function driven by a time-changed symmetric α-stable Lévy process with respect to a properly rescaled continuous time random walk in the Skorokhod space equipped with the Skorokhod M_1-topology. The limiting process is the corresponding integral but with respect to a time-changed α-stable Lévy process where the time-change is given by the functional inverse of a β-stable subordinator.

[1] I. M. Sokolov, W. Ebeling, B. Dybiec, Harmonic oscillator under Lévy noise: Unexpected properties in the phase space, *Phys. Rev. E* **83** (2011), 041118.