S2. Algebraic Geometry in Applications and Algorithms

Organizers:

- A. Alzati (Università degli Studi di Milano, Italy)
- M. Bertolini (Università degli Studi di Milano, Italy)
- J.R. Sendra (Universidad de Alcalá, Spain)
- C. Turrini (Università degli Studi di Milano, Italy)

Speakers:

1. GianMario Besana (De Paul University, USA)
   Critical loci for projective reconstruction from multiple views: a class of determinantal varieties arising in Computer Vision
2. Cristiano Bocci (Università di Siena, Italy)
   Advances in model identifiability: Bernoulli, tensors and beyond
3. Marta Casanellas Rius (Universitat Politècnica de Catalunya, Spain)
   Applying algebraic geometry to phylogenetics
4. Laureano González Vega (Universidad de Cantabria, Spain)
   Polynomial Algebra by Values: Geometric Problems involving Curves and Surfaces
5. Chris Peterson* (Colorado State University, USA), Bruce Draper, Michael Kirby, Tim Marrinan
   Flags, Grassmannians, and Schubert Varieties in Computer Vision and Signal Detection
6. Tony Shaska (Oakland University, USA)
   Families of genus two curves with many elliptic subcovers
7. Carlos Villarino Cabellos*, J.R. Sendra (Universidad de Alcalá, Spain)
   Hypercircles and ultraquadrics: a tool for simplifying coefficients in rational parametrizations
8. Franz Winkler (Johanner Kepler Universität Linz, Austria)
   Algebro-Geometric Methods for Solving Differential Equations

Critical loci for projective reconstruction from multiple views: a class of determinantal varieties arising in computer vision

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Projective reconstruction of three dimensional scenes from multiple two dimensional images is a classical problem in computer vision, from which multi-view geometry has developed as a fruitful application of classical projective/affine algebraic geometry. The interpretation of some dynamic and segmented scenes in a higher dimensional context has further extended the interaction between vision and algebraic geometry. It is well known in classical multi-view geometry that there exist special configurations of camera positions and sample scene points that, for intrinsic geometric reasons, prevent the reconstruction algorithm to be successful. This talk will explore the same phenomenon in the context of projective reconstruction from multiple views in higher dimensions. An interesting family of determinantal varieties arises as a result of this investigation. Their essential algebro-geometric properties are presented and their relevance to applications is also discussed.

Advances in model identifiability: Bernoulli, tensors and beyond

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In this talk I will show the application of the concept of weakly-defective varieties to two different problems: tensors decomposition and identifiability of statistical models. After the definition of identifiable projective variety I will show the links between identifiability and weak-defectivity. Then, using an inductive lemma for identifiability and working mainly in the case of Segre products, I will show some recent result that improves previous bounds on decomposition of tensors and identifiability of Bernoulli and other statistical models.

Applying algebraic geometry to phylogenetics

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A new emerging field that applies algebraic geometry and commmutative algebra to phylogenetics has been arising during the last decade. Phylogenetics studies ancestral relationships among living species, and it does so by modeling evolution. It turns out that most of the evolutionary models used in phylogenetics can be seen as algebraic varieties. Biologist gave the name "phylogenetic invariants" to the equations of these varieties, and attempted to use them in order to reconstruct evolution.

Although many mathematicians are working on obtaining phylogenetic invariants, biologists are not really using them. The main reason is that, up to now, there did not exist accurate methods based on invariants that could outperform classical reconstruction methods in a broad range of situations. But nowadays all theoretical results that could interest biologists have already been proven, so it is time to provide accurate phylogenetic reconstruction methods based on them. In this talk we will present an improvement of a method proposed by N. Eriksson that may finally convince biologists about the usefulness of phylogenetic invariants. Using data simulated under different settings, we will compare its accuracy against classical phylogenetic reconstruction methods.

This is a joint work with J. Fernández-Sánchez.

Polynomial Algebra by Values: Geometric Problems involving Curves and Surfaces

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A new family of algorithms is presented for solving geometric problems involving algebraic curves and surfaces presented either parametrically or defined by its implicit equations (see [1, 2]). The main difference between our approach and those previously used is the fact that the algebraic manipulation of polynomials (and their roots) is replaced by computations with numerical matrices (and their eigenvalues). This is motivated by the fact that we prefer to avoid the computation of determinants of polynomial matrices and to deal with numerical matrices of moderate size instead of high degree polynomials. This is typically a better strategy, from the numerical point of view, when the coefficients of the polynomials defining the considered curves and surfaces are floating point real numbers.

In our approach, the involved polynomials are presented by their values and they can be easily evaluated at any desired point with a reasonable computational cost. It will be shown also that dealing with polynomials presented in this way requires to rewrite the way of computing, by values, tools from Elimination Theory such as resultants and subresultants and to replace the computation of the roots of polynomial matrices by solving generalized eigenvalue problems.

[1] Diaz-Toca, G. M., Fioravanti, M., Gonzalez–Vega, L., Shakoori, A., Using implicit equations of parametric curves and surfaces without computing them: polynomial algebra by values, *Comput. Aided Geom. Design* **30** (2013), 116–139.

[2] Corless, R. M., Diaz-Toca, G. M., Fioravanti, M., Gonzalez–Vega, L., Rua, I. F., Shakoori, A., Computing the topology of a real algebraic plane curve whose defining equations are available only "by values", *Comput. Aided Geom. Design* **30** (2013), 675–706.

[1]Partially supported by the Spanish "Ministerio de Economía y Competitividad" and by the European Regional Development Fund (ERDF), under the Project MTM2011–25816–C02-02

Flags, Grassmannians, and Schubert Varieties in Computer Vision and Signal Detection

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Given a finite collection of subspaces of $\mathbf{R}^n$, perhaps of differing dimensions, one can attempt to capture structure in the collection via objects such as Schubert varieties and/or flags of vector spaces (i.e. a nested sequence of vector spaces). This talk will describe several algorithms and applications where Grassmann and Flag representations have been utilized to detect, classify, and magnify weak signals that lie in a collection of digital signals.

Families of genus two curves with many elliptic subcovers.

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The space $\mathcal{L}_d$ of degree d coverings from a genus 2 curve C to an elliptic curve E is an algebraic 2-dimensional irreducible locus in $\mathcal{M}_2$ when $d \equiv 1 \mod 2$. Genus 2 curves with this property have been studied extensively in the XIX century. In the last decade of the XX century there was renewed interests on the topic coming from interests from number theory, cryptography, mathematical physics, solitons, differential equations, etc. These spaces for $d = 3, 5$ were explicitly computed by this author in [2] and [3].

In [1] such genus two curves were used for factorization of large numbers. Although the arithmetic of C is more complicated than that on an elliptic curve, the author shows that this is balanced by the fact that each computation on C essentially corresponds to a pair of computations carried out on the two elliptic curves E_1 and E_2.

In this talk we construct a family of genus 2 curves which have 4 elliptic subcovers such that two of them are of degree 2 and the other two of degree 3. We explicitly determine all genus 2 curves, defined over $\mathbb{C}$, which have simultaneously degree 2 and 3 elliptic subcovers. The locus of such curves has three irreducible 1-dimensional genus zero components in $\mathcal{M}_2$. For each component we find a rational parametrization and construct the equation of the corresponding genus 2 curve and its elliptic subcovers in terms of the parameterization. Such families of genus 2 curves are determined for the first time. Furthermore, we prove that there are only finitely many genus 2 curves (up to $\mathbb{C}$-isomorphism) defined over $\mathbb{Q}$, which have degree 2 and 3 elliptic subcovers also defined over $\mathbb{Q}$.

[1] R. Cosset, Factorization with genus 2 curves, *Math. Comp.* **79** (2010), 270, 1191-1208.

[2] T. Shaska, Genus 2 fields with degree 3 elliptic subfields, *Forum Math.* **16** (2) (2004), 263–280.

[3] K. Magaard, T. Shaska, H. Völklein, Genus 2 curves that admit a degree 5 map to an elliptic curve, *Forum Math.* **21** (3) (2009), 547–566.

Hypercircles and ultraquadrics: a tool for simplifying coefficients in rational parametrizations

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The notion of hypercircle was introduced in [4] to approach the problem of computing an optimal field of parametrization for rational curves given parametrically. Similarly, in [1], the notion was extended to the case of optimal fields of parametrizations of unirational varieties. In this talk we will describe these concepts as well as their main properties, and we will see how they can be used to solve the complex-real case for ruled surfaces and swung surfaces (see [2, 3]).

[1] C. Andradas, T. Recio, J. R. Sendra, and L. F. Tabera, On the simplification of the coefficients of a parametrization, *J. Symbolic Comput.* **44** (2) (2009) 192–210.

[2] C. Andradas, T. Recio, L. F. Tabera, J. R. Sendra, and C. Villarino, Reparametrizing Swung Surfaces over the Reals. *Appl. Algebra Engrg. Comm. Comput.* **25** (2014), 39–65.

[3] C. Andradas, T. Recio, L. F. Tabera, J. R. Sendra, and C. Villarino, Proper real reparametrization of rational ruled surfaces, *Comput. Aided Geom. Design* **28** (2) (2011), 102–113.

[4] T. Recio, J. R. Sendra, L. F. Tabera, and C. Villarino, Generalizing circles over algebraic extensions, *Math. Comp.* **79** (270) (2010), 1067–1089.

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Algebro-Geometric Methods for Solving Differential Equations

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We discuss recently developed methods and algorithms for creating explicit symbolic solutions for algebraic differential equations. An algebraic differential equation is a polynomial equation in the unknown function y and its derivatives. There are complete algorithms for deciding the existence of rational solutions of ordinary algebraic differential equations of order 1, cf. [1, 2, 3]. First steps have been taken towards extending the method to higher order ODEs, to different classes of solution functions cf. [5], and to partial differential equations. Geometric transformation groups leaving the differential solvability invariant may lead to more efficient solution stategies; cf. [4].

All these methods are based on the parametrization of a low dimensional algebraic variety, the so-called solution variety of the given algebraic differential equation.

[1] Feng, R., Gao, X.-S., A polynomial time algorithm for finding rational general solutions of first order autonomous ODEs, *J. Symb. Comp.* **41/7** (2006), 739–762.

[2] Ngô, L.X.C., Winkler, F., Rational general solutions of first order non-autonomous parametrizable ODEs, *J. Symb. Comp.* **45/12** (2010), 1426–1441.

[3] Ngô, L.X.C., Winkler, F., Rational general solutions of planar rational systems of autonomous ODEs, *J. Symb. Comp.* **46/10** (2011), 1173–1186.

[4] Ngô, L.X.C., Sendra, J.R., Winkler, F., Classification of algebraic ODEs with respect to their rational solvability, *Contemp. Math.* **572** (2012), 193–210.

[5] Grasegger, G., Winkler, F., Symbolic solutions of first order algebraic ODEs, *Proc. RICAM Workshop on Computer Algebra and Polynomials* (J. Gutierrez, J. Schicho, M. Weimann eds.), Springer LNCS, 2014.

[1]Partially supported by the Spanish "Ministerio de Economía y Competitividad" under the Project MTM2011–25816–C02-01