S11. Geometric Analysis

Organizers:

- Barbara Nelli (Universitá dell'Aquila, Italy)
- Magdalena Rodríguez (Universidad de Granada, Spain)

Speakers:

1. Theodora Bourni (Freie Universität Berlin, Germany)
   "Null mean curvature" flow and marginally outer trapped surfaces
2. Sebastien Cartier (Université Paris-Est, France)
   Saddle towers in Heisenberg space
3. Ana Hurtado (Universidad de Granada, Spain)
   Estimates of the first Dirichlet eigenvalue from exit time moment spectra
4. Debora Impera (Universitá degli Studi di Milano-Bicocca, Italy)
   Potential theory for manifolds with boundary and applications to controlled mean curvature graphs
5. Miguel Manzano (Universitá Roma Tre, Italy)
   Mean curvature vs. bundle curvature
6. Laurent Mazet (Université Paris-Est, France)
   Minimal surfaces in complete hyperbolic 3-manifolds with finite volume
7. Pablo Mira (Universidad Politécnica de Cartagena, Spain)
   Constant mean curvature spheres in homogeneous three-spheres
8. Tommaso Pacini (Squola Normale Superiore, Italy)
   Coupled flows, convexity and calibrations
9. Giuseppe Tinaglia (King's College London, United Kingdom)
   On the topology of the limits of a sequence of embedded minimal disks
10. Francisco Torralbo (Katholieke Universiteit Leuven, Belgium)
    Minimal surfaces in Riemannian product spaces

"Null mean curvature" flow and marginally outer trapped surfaces

Theodora Bourni

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In this talk we discuss a new second order parabolic evolution equation for hypersurfaces in space-time initial data sets, that generalizes mean curvature flow (MCF). In particular, the 'null mean curvature' - a space-time extrinsic curvature quantity - replaces the usual mean curvature in the evolution equation defining MCF. This flow is motivated by the study of black holes and mass/energy inequalities in general relativity. We present a theory of weak solutions using level-set methods and an appropriate variational principle, and outline a natural application of the flow as a parabolic approach to finding outermost marginally outer trapped surfaces (MOTS), which play the role of quasi-local black hole boundaries in general relativity.

This is joint work with Kristen Moore.

Saddle towers in Heisenberg space

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Saddle towers are minimal surfaces with $2n$ planar ends, $n \geq 2$, that can be seen as the desingularization of n minimal planes intersecting along a geodesic. In Heisenberg 3-space, we construct *periodic* Saddle towers, meaning that the ends are distributed at constant angle π/n.

The key point is the construction of a suitable barrier, which is obtained by deforming a minimal entire graph, with prescription of a periodic asymptotic behavior. Such a deformation is maid possible by the extension of the mean curvature operator of a certain family of minimal immersed disks, to control the behavior at infinity of these immersions.

Estimates of the first Dirichlet eigenvalue from exit time moment spectra

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2000 Mathematics Subject Classification. Primary 58C40; Secondary 53C20

We compute the first Dirichlet eigenvalue of a geodesic ball in a rotationally symmetric model space in terms of the moment spectrum for the Brownian motion exit times from the ball. As an application of the model space theory we prove lower and upper bounds for the first Dirichlet eigenvalues of extrinsic metric balls in submanifolds of ambient Riemannian spaces which have model space controlled curvatures. Moreover, from this general setting we thereby obtain new generalizations of the classical and celebrated results due to McKean and Cheung–Leung concerning the fundamental tones of Cartan-Hadamard manifolds and the fundamental tones of submanifolds with bounded mean curvature in hyperbolic spaces, respectively.

Potential theory for manifolds with boundary and applications to controlled mean curvature graphs

Debora Impera

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We characterize the Neumann-parabolicity of manifolds with boundary in terms of a new form of the classical Ahlfors maximum principle and of a version of the so called Kelvin-Nevanlinna- Royden criterion. The motivation underlying this study is to obtain new information on the geometry of graphs with prescribed mean curvature inside a Riemannian product. In this direction two kind of results will be presented: height estimates for constant mean curvature graphs parametrized over unbounded domains in a complete manifold, which extend results by A. Ros and H. Rosenberg, [3], and slice type results for graphs whose superlevel sets have finite volume.

Finally, the use of the Ahlfors maximum principle allows us to establish a connection between the Neumann parabolicity and the Dirichlet parabolicity commonly used in minimal surface theory. In particular we will be able to give a deterministic proof of a result by R. Neel, [2].

This is a joint work with S. Pigola and A. G. Setti, [1].

[1] D. Impera, S. Pigola, A.G. Setti, Potential theory for manifolds with boundary and applications to controlled mean curvature graphs; http://arxiv.org/abs/1303.2853.

[2] Neel, R., Brownian motion and the parabolicity of minimal graphs; http://arxiv.org/abs/0810.0669.

[3] Ros, A., Rosenberg, H., Properly embedded surfaces with constant mean curvature, *Amer. J. Math.* **132** (2010), 1429–1443.

Mean curvature vs. bundle curvature

José M. Manzano

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In this talk we will consider a large class of 3-manifolds, namely, those which admit a Riemannian submersion over a surface such that the fibers of the submersion are the integral curves of a unit Killing vector field [2]. These 3-manifolds will be called Killing submersions and can be either Riemannian or Lorentzian (when the Killing vector field is assumed to be timelike). We will see that a natural geometric function in the base of the submersion, called bundle curvature, characterizes completely the geometry of the 3-manifolds.

In 1970 Calabi proved a remarkable correspondence between minimal surfaces in the Euclidean space $\mathbb{R}^3$ and maximal spacelike surfaces in the Minkowski space $\mathbb{L}^3$. Using the fact that the mean curvature of a surface transversal to the Killing vector field in a Killing submersion admits a divergence-type equation, we will generalize the aforementioned Calabi's correspondence to a correspondence between

(a) Mean curvature H graphs in Riemannian Killing submersions over some surface with bundle curvature τ.

(b) Mean curvature τ spacelike graphs in Lorentzian Killing submersions over the same surface with bundle curvature H.

Here τ and H are arbitrary smooth functions, which leads to a quite general result with applications, among other, to the existence of solutions for the prescribed mean curvature equation in $\mathbb{R}^3$ or to the existence of complete spacelike surfaces in some spacetimes admitting unit Killing vector fields.

This is a joint work with Hojoo Lee [1].

[1] Lee, H., Manzano, J.M., Generalized Calabi's correspondence and complete spacelike surfaces (2013); http://arxiv.org/abs/1301.7241.

[2] Manzano, J.M., On the classification of Killing submersions and their isometries, *Pacific J. Math.*, to appear; http://arxiv.org/abs/1211.2115.

Minimal surfaces in complete hyperbolic 3-manifolds with finite volume

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In this talk, I will present some recent results about the existence and the behaviour of proper minimal surfaces in complete hyperbolic 3-manifolds with finite volume.

This is a joint work with P. Collin, L. Hauswirth and H. Rosenberg.

Constant mean curvature spheres in homogeneous three-spheres

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A famous theorem by Hopf states that any immersed topological sphere of constant mean curvature in a 3-dimensional space of constant curvature is a totally umbilical round sphere. In this talk we will explain the generalization of this theorem to the case where the ambient space is a homogeneous manifold diffeomorphic to S^3. We will prove that for any value H there is a unique (up to isometries) immersed sphere of mean curvature H in any such manifold, and we will study its most important geometric properties.

This is a joint work with William H. Meeks, Joaquin Pérez and Antonio Ros.

Coupled flows, convexity and calibrations

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Mean curvature flow provides a canonical way to deform a given submanifold, so as to minimize its volume. It is a purely Riemannian concept, but in Kähler manifolds it has the interesting property that it preserves the class of Lagrangian submanifolds, which originates within symplectic geometry. Recent joint work with J. Lotay (UCL) indicates that the links between these two geometries lie much deeper, and are manifestations of a more general theory concerning totally real submanifolds in complex geometry. The aim of the talk will be to present an overview of these interactions, and possible applications.

On the topology of the limits of a sequence of embedded minimal disks.

Giuseppe Tinaglia

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The work of Colding-Minicozzi gives that a sequence of embedded minimal disks converges, up to a subsequence, to a minimal lamination away from a closed set of singular points. In several examples of Colding-Minicozzi and others, the leaves of such lamination are disks, while Hoffman-White recently produced examples where some of the leaves are annuli. In this talk I will describe several results on the topology of the leaves of such lamination in a manifold that admits an isoperimetric inequality for minimal surfaces. For instance, each leaf has genus zero.

This is joint work with Bernstein.

Minimal surfaces in Riemannian product spaces

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2010 Mathematics Subject Classification. Primary 53C42 Secondary 53C40

A general study of minimal surfaces of the Riemannian product of two spheres $\mathbb{S}^2 \times \mathbb{S}^2$ is presented. The most relevant contribution will be the establishment of a local geometric correspondence between (non-complex) minimal surfaces of $\mathbb{S}^2 \times \mathbb{S}^2$ and a certain pair of minimal surfaces of the sphere $\mathbb{S}^3$. This correspondence also allows us to link minimal surfaces in $\mathbb{S}^3$ and in the Riemannian product $\mathbb{S}^2 \times \mathbb{R}$. If time allows we will also discuss a similar relation between maximal surface in anti-De Sitter 3-space and minimal surfaces in $\mathbb{H}^2 \times \mathbb{H}^2$ with applications to $\mathbb{H}^2 \times \mathbb{R}$.

This is a joint work with F. Urbano, [1].

[1] Torralbo, F., Urbano, F., Minimal surfaces in $\mathbb{S}^2 \times \mathbb{S}^2$, *J. Geom. Anal.*; doi: 10.1007/s12220-013-9460-3.