S19. Mathematics at school: teachers, students, technology and assessment

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- Angel Gutiérrez (Universidad de Valencia, Spain)

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    The meta-didactical transposition to analyse teachers' professional development

An example of meta-didactical transposition from France: the Pairform@nce project

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Pairform@nce (and since 2014 m@gistère) are French national teacher education programs from the ministry of education whose objectives are to provide in-service teachers training through hybrid sessions. They address all class levels, from primary to secondary and all topics. The implemented principles of teacher education are the design of resources and peer collaboration using both face to face and on-line work. The meta-didactical transposition offers a framework allowing to analyse existing in-training sessions but also a guide to construct such sessions. The institutional aspects are particularly important to consider because of the necessary dialog between different levels of institutions: the local education authority level, the institutional context of teachers in their schools and the national curricula. Meta-didactical praxeologies and double dialectics offer tools to analyse the evolution and the dynamic of professional knowledge during in-training sessions; for example, the necessity for trainers, in hybrid sessions, to encourage and to guide trainees in the accomplishing of a particular task has to be analysed both in term of task and technique but also in term of justification and founding principles of the task: The trainees praxeologies enrich themselves from the theoretical point of view when trainers' praxeologies feed on empirical data. The concept of brokering is directly linked to the boundaries objects that occur during sessions and participates to the global landscape of the analysis as well as it allows to build milestones giving structure to in-training sessions.

The SRP-TPD project: a proposal for teachers' professional development from the Anthropological theory of the didactic

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Many investigations in mathematics education assume the necessity to foster the teaching of *mathematics as a modelling tool*. At the same time, they highlight important objective difficulties that hinder any proposal of implementing modelling activities in current educational systems, mainly enclosed in a 'monumentalistic paradigm' where mathematical contents appear as "works to visit" more than tools to provide answers to questions. To approach this far-reaching problem and help move towards the new paradigm of 'questioning the world', recent research carried out in the frame of the anthropological theory of the didactic proposes a new teaching device called 'study and research paths' (SRP) based on the long-term inquiry of generating problematic questions. However, designing new teaching devices is not enough for this change to happen. Among many challenges that should be faced, an important one is related to teachers' current knowledge and their mathematical and didactic tools to face this change.

We start from some particular cases of SRP that have been designed, implemented and analysed in our previous research, in early childhood, secondary and tertiary levels. We explore how these SRP could be used to design teacher professional development programmes (both pre-service and in-service training). The aim is twofold: on the one hand, to identify teachers' professional questions that could lead to a meaningful development of their praxeological equipment; on the other hand, enriching teachers' mathematical experiences with inquiry and modelling processes. In this talk we present the framework, principles and phases for the design of teacher professional development study and research paths, with preliminary results.

National and international assessments as a tool for teacher training

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Assessment has been described as "*a way of understanding a child in order to make informed decisions about the child*" ([4]) and the teaching-learning process, and it is commonly intended as a key task for the teacher. In most theoretical frameworks for assessment the first element is Diagnostic Assessment ([1]). The process of actual assessment managed by the teacher, and in particular the role of diagnostic assessment in it, is very often based on mechanical procedures and on implicit philosophies ([5]). Standardised Testing ([3]) in mathematics may play a crucial role in this diagnostic process, even if many theoretical issues must be discussed ([2]). We describe a teacher training program model which integrates working on Standardised assessments' explicit frameworks, test results and National Curricula in order to make explicit the teacher's own framework.

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Standardized test in Switzerland: discussion of process and results

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The standardized tests use and their role in the education systems are currently the focus of a discussion in different parts of the world. The use of standardized tests to monitor the system has in fact shown, for example, on one hand the tendency to improve the performance of the students (in this sense can be read in the reports of some analysis by NCES) on the other to a narrowing of the curriculum and the elimination of more complex tasks ([1]).

In Switzerland the discussion is vibrant and connected with the harmonization process involving several cantons in Switzerland, and that concerns compulsory schools from 4 to 15 years of age (HarmoS Concordat of 14 June 2007; http://www.edk.ch/dyn/23222.php). In 2011 to identify a common disciplinary background were defined which were the fundamental competencies in mathematics. These fundamental competencies are the first national training standards for compulsory education and represent an important contribution to the harmonization of the objectives of the stages of training at the national level.

In particular, as regards the Ticino since 2010 a project with the aim of producing and administering a standardized test ([2]) to evaluate mathematical competencies in the fourth class of primary school has been running. In our intervention we present: the development of the harmonization process; the principal step of test building and the main results.

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The MDT Model as a tool to highlight fundamental features of teacher education processes

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We will show how the MDT model could become a suitable tool to describe, analyse and understand what happens during teacher education programs, to highlight foreseen or unexpected dynamics and to compare different programs. Starting from the analysis of specific examples, some critical phenomena of teacher education programs will be discussed referring to the MDT Model with the aim of highlighting fundamental aspects, such as the role played by meta-didactical praxeologies and their evolution over time, the dynamics between internal and external components and the double dialectic ([1] and [2]).

[1] Aldon, G., Arzarello, F., Cusi, A., Garuti, R., Martignone, F., Robutti, O., Sabena, C., Soury-Lavergne, S. (2013). The meta-didactical transposition: a model for analysing teachers education programs. In Lindmeier, A. M., Heinze, A. (eds.). *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 97-124. Kiel, Germany: PME.

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The Realistic Mathematics Education (RME)

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Mathematics education has to prepare students for society, work and further study. But international studies show that much of what is taught in school seems to be lost when you assess it not immediately after the lessons or in different contexts. An example is a question about fractions in TIMSS 2003 for grade 8 students: *A scoop holds 1/5 kg of flour. How many scoops are needed to fill a bag with 6 kg of flour?* The international average of a full credit for this item was 38%.

Such low scores must have us reflect on what we teach and how we teach it. I will not argue that citizens in current society should be able to solve this question by heart, but you would expect that it is possible to solve it with pen and paper. The steps underlying the calculation are rather elementary and fundamental for proportional and algebraic reasoning in a wide range of topics in mathematics education at secondary school. An approach to mathematics education that tries to provide for learning trajectories that support students in understanding and tracing concepts and skills is RME. This approach starts with rich contexts that ask for mathematical organization. Well-chosen problems offer opportunities for students to develop informal, highly context-specific models and solving strategies. These informal solving procedures then function as foothold inventions for formalization and generalization. As a consequence, during the activities the model and the situation being modelled co-evolve. Modelling in this view is a process of reorganizing both activities and the situation and drives the learning process of the students.

In the Netherlands, RME influenced the current textbooks in primary education. The full credit score of Dutch students on the above item was 74%. I will not argue that this was fully caused by RME, but it strengths us in the feeling that this approach contributes to the quality of mathematics education. I'll present examples of RME in different mathematical topics to illustrate the need for approaches to mathematics education that provide for an understanding supporting everyday-life and workplace use of concepts and skills, as well as an understanding needed for further study.

Unfolding 3D stories from 2D mathematical diagrams using dynamic geometry

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Exploring and discovering relationships between parts of geometric 2D and 3D figures is a crucial step in the study of spatial geometry. Among teachers and students there is the general reputation that spatial geometry is difficult because it is difficult to *see*. Seeing in 3D involves visual challenges, which concern the fact that we have to do with bi-dimensional diagrams that embodied three-dimensional entities. The perception of the third dimension is related to the way we perceive the shape of space around us, and it is, in turn, a matter of how the eye measures reality and virtual reality. The regions around a moving perceiver give different kinds of perceptual spaces, opening room for different natures of their geometries. Moreover, the teaching of space geometry has revealed coding problems, in terms of knowing versus seeing, due to the conflict between what one knows of a 3D object and what one sees of it. In our study, the visual and cognitive potential of interlacing related but different objects is offered by the use of a dynamic geometry software (Cabri 3D). Seeing how definitions and properties captured in one object are related to those in a second object allows students to move back and forth between plane and space. Through rotational movement and redefinition of points afforded by the DGS, learners can take on multiple perspectives, as if they were taking on positions from which to view the object, as projecting themselves both beyond and around it. This engagement with the diagrams effects kinds of vision that push towards looking for similarities and differences, invariants and changes, and speaks to students' enhanced problem solving skills ([1]).

[1] Ferrara, F. and Mammana, M.F. (2013). Close your eyes and see... An approach to spatial geometry. Paper presented at the *Eight Congress of European Research in Mathematics Education* (CERME 8).

The use of technology to teach mathematically talented students

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The use of technology is more widespread everyday in schools in our countries. An advantage of using software in the classrooms is that teachers have the possibility to personalize the tasks posed to their pupils to adapt the tasks to each pupil's mathematical ability or interest. This may be of particular interest for those teachers with mathematically gifted pupils, who require faster and deeper teaching. In particular, 3-dimensional dynamic geometry software has recently entered into the scene and is gaining presence in the classrooms. But its use in the mathematics classes is not free of difficulties, some of them inherent to representations of 3-dimensional objects and students' difficulties to visualize their characteristics, other induced by the software itself due to its ways of use. This fact is raising a number of important questions to researchers in mathematics education. In this presentation we analyze the activity of a mathematically gifted student solving a sequence of tasks for the learning of concepts relative to relationships between straight lines and planes in space. The tasks were posed in a Cabri 3d environment. We shall present fragments of student's activity while solving the tasks and shall provide an interpretation, from the perspective of the instrumental genesis theory, of the conflicts arisen while solving the tasks as a consequence of the way the software represents the planes on the computer screen, which is imitated by the student.

The results reported in this presentation are part of the research project "Analysis of learning processes by primary and middle school mathematically gifted students in contexts of rich mathematical activities", funded by the Spanish Ministry of Economy and Competitivity (EDU2012-37259).

Standardized assessments and the teaching and learning of mathematics: some facts and proposals

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I will summarize some facts about the situation in Spain in the context of the new education law, LOMCE. Taking this as a starting point I will also make some proposals. The ideas expressed in the talk will come from of the work developed by the Education Commission of the Royal Spanish Mathematical Society (RSME) during these last years about this topic.

Theoretical approaches to mathematics education: the construct of rationality and its integration with other theories

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Habermas' construct of rational behaviour was conceived to deal with the complexity of discursive practices according to three interrelated points of view concerning: knowledge at issue (epistemic rationality); action and its goals (teleological rationality); discourse and its choices (communicative rationality). As such, Habermas' construct seemed to be suitable for an adaptation to mathematical activities like proving and modelling that move along between the poles of epistemic validity, problem solving strategic choices and communicational requirements.

Conceived as a tool to analyze and evaluate students' behaviour in proving activities according to researchers' (and teachers') expectations, the adaptation of Habermas' rational behaviour developed as a toolkit with various applications, taking advantage also from the integration with other constructs, in and/or outside research in mathematics education.

We will present two of such integrations, with the double aim of presenting the most recent results related to the construct of rationality and providing examples of integration of theoretical approaches in mathematics education. Firstly, we illustrate the first use of the adapted construct of rationality and the subsequent integration with Toulmin's model of argumentation. Such an integration will be shown to be efficient to plan and analyze students' argumentative approach to the culture of theorems, in geometry and in elementary theory of numbers. A special attention will be devoted to the issue of using the integrated model as a guide for task design. Secondly, we present two inter-related perspectives: the adaptation of Habermas' construct to the case of school mathematics argumentation and its use to analyze the emergence of validity conditions for mathematical productions while considering classroom social contexts. To this aim, the construct of rational behaviour is integrated with Sfard and Kieran's focal analysis.

Design and analysis of tasks: An onto-semiotic approach

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The Onto-semiotic Approach (OSA) is a systemic and integrative approach to research in mathematical education developed by Godino and Batanero. The OSA focus on the formulation of an ontology of mathematical objects taking into account three aspects of mathematics, as socially shared resolution of problems, as symbolic language and as a logically organized conceptual system. Taking the problem situation as the primitive notion, the concepts of practice, object and meaning are defined, with the aim of making visible and operative, the referred triple character of mathematics on the one hand, and on the other, the personal and institutional genesis of mathematical knowledge, as well as its mutual interdependence.

In mathematical practice, various types of objects intervene (symbols, graphs, definitions, propositions, etc.) which are represented in written, oral, graphic or even in sign language form. From the systems of operative and discursive mathematical practices, new objects emerge that give us indications on the structure and organization of these systems. These objects can be 'institutional objects', shared by an institution, or 'personal objects', including cognitive constructions (conceptions, internal representations, etc.).

For a finer analysis of the mathematical activity, it is necessary to take into account six types of primary entities: problem situations, languages in its various registers, concepts, propositions, procedures, and arguments. These six objects relate to each other by forming epistemic configurations (networks of institutional objects) and cognitive configurations (networks of personal objects).

Mathematical tasks are central to learning because the tasks transmit messages about what mathematics is and what doing mathematics involves and the systematic approach to tasks design and analyze play critical roles in mathematics education. In this communication we present some examples of tasks design and analysis, using the OSA.

Technologies for the learning of algebra with dyscalculic students

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We discuss the difficulties students encounter in algebra (in particular for the notions of unknown and variable) considering in particular those students affected by developmental dyscalculia (DD) ([2]). To identify and categorize such DD student's difficulties we consider Kieran's model ([3]) that classifies three levels of activities: generational, transformational and global meta-level. So far, the research in mathematics education has focused mainly on the difficulties of the DD students in performing transformational ([**?**]) rather than generational or global meta-level activities.

Some studies ([4], [1]) show that DD students present difficulties in algebraic manipulation of expressions or propositions when they cannot associate the appropriate mathematical meaning. According to this, we formulate this research question: how can DD students give meaning to those notions both in transformational, generational and global meta-level activities?

The expressed hypothesis is that, some of the difficulties encountered by DD students in generational and global meta-level activities could be due to the lack of meaning given to the algebraic notions. Our educational hypothesis is that DD students could grasp meaning of algebraic notions exploiting different kinds of representations based on visual non-verbal, kinaesthetic and auditory representational systems. For this reason, our aim is to analyze the potential of the software AlNuSet in designing activities that enable DD students to cope with difficulties highlighted in our research.

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The MDT Model and its main components

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A new descriptive and interpretative model (the *Meta-Didactical Transposition* model - MDT, [1]) for studying the transposition from research to teaching practice during teacher education programs will be introduced. The model, based on the Anthropological Theory of Didactics ([3], [4]), considers some main variables in the teacher education processes (community of teachers, of researchers, role of the institutions) and accounts for their mutual relationships and evolution over time. The rationale of the MDT model will be pointed out in order to discuss its compatibility with respect to other theoretical models existing in the literature ([2], [5]), and its applicability in different contexts.

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