S21. Model theory and applications

Organizers:

- Enrique Casanovas (Universitat de Barcelona, Spain)
- Carlo Toffalori (Università degli Studi di Camerino, Italy)

Speakers:

1. Elías Baro (Universidad Complutense, Spain)
   Cartan subgroups of groups definable in o-minimal structures
2. Andrés Cordón (Universidad de Sevilla, Spain)
   Existentially closed models in the framework of first order arithmetic
3. Paola D'Aquino (Università Napoli 2, Italy)
   Recursively saturated real closed fields
4. Antongiulio Fornasiero (Università Napoli 2, Italy)
   Topology in definably complete structures
5. Sonia L'Innocente (Università degli Studi di Camerino, Italy)
   Rings of definable scalars of some sl_3-modules
6. Vincenzo Mantova (SNS Pisa and Università degli Studi di Camerino, Italy)
   Raising to powers on the unit circle
7. Daniel Palacín (Universität Münster, Germany)
   On superstable expansions of free abelian groups
8. Françoise Point (F. N. R. S - F. R. S, France)
   Separably closed fields and contractive Ore modules
9. Giuseppina Terzo (Università Napoli 2, Italy)
   Zero sets of exponential polynomials
10. Frank O. Wagner (Université Lyon 1, France)
    A Fitting Theorem for Simple Theories

Cartan subgroups of groups definable in o-minimal structures

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A subgroup Q of a group G is called Cartan if

(1) Q is nilpotent and maximal with this property among subgroups of G, and

(2) For any subgroup $H \leq Q$ which is normal in Q and of finite index in Q, the normalizer $N_G(H)$ of H in G contains H as a finite index subgroup.

If G is a real connected Lie group then the Cartan subgroups have a deep relation with the Cartan subalgebras of its Lie algebra and their behaviour is well-known. For example, there is a finite number of Cartan subgroups up to conjugacy and the union of all Cartan subgroups is a dense subset of the group.

On the other hand, a foundational result of Pillay assures that any group definable in an o-minimal structure can be equipped with a definable manifold structure. Since then many resemblances between definable groups and Lie groups have been proved. However, the use of the Lie algebra in the o-minimal setting has some limitations. In [**?**] we study Cartan subgroups of o-minimal groups applying techniques similar to those of groups of Finite Morley Rank. In this talk I will survey what is known about Cartan subgroups of groups definable in o-minimal structures.

[1] Elias Baro, Eric Jaligot, Margarita Otero, Cartan subgroups of groups definable in o-minimal structures, *J. Inst. Math. Jussieu* (2014), to appear.

Existentially closed models in the framework of first order arithmetic

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Existentially closed models of arithmetic theories were investigated in the seventies in a number of works including [1, 2, 3, 4]. At that time a systematic study of the fragments of Peano arithmetic was still yet to come and the authors opted for working over strong arithmetic theories. However, most of those classic results remain true when working over (moderately) weak arithmetic theories.

In this talk we present an updated account of that classic work as well as we give some improvements of already known results that we have obtained recently. Topics will include definability of the set of standard integers, recursive saturation, distribution of definable elements and connections with important open problems in the field of fragments of arithmetic.

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[1] Goldrei, D.C., Macintyre, A., Simmons, H., The forcing companions of number theories, *Israel J. Math.* **14** (1973), 317–337.
[2] Hirschfeld, J., Wheeler, W., *Forcing, Arithmetic, Division Rings*, Lecture Notes in Mathematics **454**, Springer, 1975.
[3] Macintyre, A., Simmons, H., Algebraic properties of number theories, *Israel J. Math.* **22** (1975), 7–27.
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Recursively saturated real closed fields

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Recursive saturation is ubiquitous in non standard models of Peano Arithmetic. Countable recursively saturated real closed fields have been characterized as those which admit an integer part which is a model of Peano Arithmetic. I will present a characterization of recursively saturated real closed fields and ordered divisible abelian groups in terms of valuation theory.

This is a joint work with S. Kuhlmann and K. Lange.

Topology in definably complete structures

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Let M be a Definably Complete (DC) expansion of a valued field. In [3] we showed how to extend several results from real analysis to functions definable in M (for instance, Lebesgue's result that a monotonic real function is differentiable almost everywhere). When M is o-minimal, several authors [1, 5, 2] extended to M several tools from algebraic and differential topology. In particular, they showed how to define the topological degree of a continuous definable map: as in the classical case, it is a well-defined integer number, which is invariant under definable homotopies. From this fact, Brouwer's fixed point theorem for definable continuous maps follows easily. When M is not o-minimal, but only DC, the degree of a definable map is no longer an integer, but an element of a suitable "ring of cardinalities", defined in a way similar to the Grothendieck ring of M of [4]. Again, the degree of a map well-defined and invariant under definable homotopies. Under a suitable pigeon-hole assumption, such ring is nontrivial, and hence, as usual, Brouwer's fixed point theorem for definable maps follows.

[1] Berarducci, A., Otero, M., Intersection theory for o-minimal manifolds, *APAL* **107** (2001), 87-119.
[2] Edmundo, M., Woerheide, A., Comparation theorems for o-minimal singular (co)homology, *Trans. Amer. Math. Soc.* **360** (2008), 4889–4912.
[3] Fornasiero, A., Hieronymi, P., A fundamental dichotomy for definably complete expansions of ordered fields, *J. Symbolic Logic*, to appear.
[4] Krajicek, J., Scanlon, T, Combinatorics with definable sets: Euler characteristics and Grothendieck rings, *Bull. Symbolic Logic* **6** (2000), 311–330.
[5] Peterzil, Y., Starchencko, S., Computing o-minimal topological invariants using differential topology, *Trans. Amer. Math. Soc.* **359** (2007), 1375–1401.

Rings of definable scalars of some sl(3)-modules

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In the paper [1], Herzog investigated the ring of definable scalars of the finite dimensional representations of the Lie algebra sl(2) of the 2x2 traceless matrices over the complex field. This is the ring of definable actions on the category of finite-dimensional sl(2)-modules, that is, the ring to which the action of the universal enveloping algebra, $U = U(\mathrm{sl}(2))$, on these modules extends in a definable way. Herzog showed that this ring is von Neumann regular and is a universal localization of U. This work inspired further investigations, in particular on rings of definable scalars of Verma modules [**?**] and on $U(\mathrm{sl}(2), q)$)-modules (where q is not a root of unity) [2]. It is natural to ask what happens when sl(2) is replaced by other simple Lie algebras, in particular by sl(3). We are able to obtain similar results if we restrict to the representations which are contained in, or whose dual is contained in, the natural representation of sl(3) on the polynomial ring on three generators.

[1] I. Herzog, The pseudo-finite dimensional representations of $sl(2, k)$, *Selecta Math. (N.S.)*, **7** (2001), 241–290.
[2] I. Herzog, S. L'Innocente, The Nonstandard quantum plane, *Ann. Pure Appl. Logic*, **156** (2008), no. 1, 78–85
[3] S. L'Innocente, M. Prest, Rings of definable scalars of Verma modules, *J. Algebra Appl.*, **6** (2007), no. 5, 779–787
[4] S. L'Innocente, M. Prest, Rings of definable scalars of some $sl(3)$-modules, in preparation.

Raising to powers on the unit circle

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In the last years, progress has been made towards understanding reducts of exponential fields, namely real and complex fields together with irrational power functions (see e.g. [2, 3, 4]). As shown by Bays, Kirby and Wilkie [1], the corresponding statement of Schanuel's Conjecture becomes a theorem for sufficiently generic powers.

I will present the special case of raising to imaginary powers on the unit circle, where, even in presence of instability, the techniques of Hrushovski amalgamation yield a natural axiomatisation of the theory and its near-model-completeness, again for sufficiently generic powers.

This is joint work with Boris Zilber.

[1] Martin Bays, Jonathan Kirby, and Alex J. Wilkie, A Schanuel property for exponentially transcendental powers, *Bull. London Math. Soc.* **42** (5) (2010), 917–922.

[2] Gareth O. Jones and Tamara Servi, On the decidability of the real field with a generic power function, *J. Symbolic Logic* **76** (4) (2011), 1418–1428.

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[4] Boris Zilber, The theory of exponential sums, 2011; http://people.maths.ox.ac.uk/zilber/revisited.pdf.

On superstable expansions of free abelian groups

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In this talk we present a proper expansion of the group of integers whose first-order theory is superstable of infinite Lascar rank. In fact, we also prove that the group of integers has no proper expansion of finite Lascar rank.

This is a joint work with Rizos Sklinos.

Separably closed fields and contractive valued modules

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We will recall the notion of valued modules over the skew polynomial ring $K[t;\sigma]$ of difference operators introduced by O. Ore, where K is a (valued) field and σ a distinguished endomorphism of K. We will axiomatize a class of abelian structures which contains the class of additive reducts of valued separably closed fields of positive characteristic and fixed finite imperfection degree. We will determine the completions of that class, using an effective quantifier elimination result. Part of the work consists in translating in this module context, certain properties of valued separably closed fields.

This is a joint work with Luc Bélair.

Zero sets of exponential polynomials

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In 2004 Zilber [3] introduced new exponential fields with the final goal of understanding the complex exponential field. These are algebraically closed fields of characteristic 0, with an exponentiation satisfying Schanuel's conjecture and allowing solutions of systems of exponential equations. Zilber gives an axiomatization of the class of his fields. Moreover, he proves a categoricity result for this class of structure, i.e. he proves that for each uncountable cardinality there is a unique field satisfying his axioms, and poses the conjecture that the unique field of cardinality $2^{\aleph_0}$ is the complex exponential field. In support of his conjecture in [2] we continue the research programme of comparing the complex exponential field with Zilber exponential field. Using diophantine geometry, we prove various properties about zero sets of exponential functions, which are proved for $\mathbf{C}$ using analytic function theory [1], among this a version of the Identity Theorem.

[1] L. Ahlfors, *Complex Analysis*, 3rd edition, McGraw-Hill, 1979.

[2] P. D'Aquino, A. Macintyre and G. Terzo, Comparing C and Zilber's exponential fields: zero set of exponential polynomials, *J. Inst. Math. Jussieu*, to appear.

[3] B. Zilber, Pseudo-exponentiation on algebraically closed fields of characteristic zero, *Ann. Pure Appl. Logic* **132** (2004), 67–95.

A Fitting Theorem for Simple Theories

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I shall show that in a group type-definable in a simple theory the Fitting subgroup, i.e. the group generated by all normal nilpotent subgroups, is itself nilpotent. The proof uses the ideas from the MC-case, plus the notion of FC-centraliser, FC-solubility and FC-nilpotency introduced in the 50s by Haimo.