S27. Recent Developments in Commutative Algebra

Organizers:

- Caviglia Giulio (Purdue University, USA)
- Joan Elias (University of Barcelona, Spain)
- Philippe Gimenez (University of Valladolid, Spain)
- Maria Evelina Rossi (University of Genoa, Italy)

Speakers:

1. Bruno Benedetti (Freie Universität Berlin, Germany)
Convexity and shellability

2. Holger Brenner (Universität Osnabrueck, Germany)
The case of ADE-singularities

3. Emanuela De Negri (Università di Genova, Italy)
A Gorenstein simplical complex for symmetric minors

4. Elisa Gorla (Institut de Mathématiques, Neuchätel, Switzerland)
General sections of curves in positive characteristic

5. Julio Moyano Fernández (Universität Osnabrueck, Germany)
Hilbert regularity of Z-graded modules over polynomial rings

6. Marta Narváez (Universitat de Barcelona, Spain)
Equidistribution of Galois orbits of points of small height

7. Claudia Polini (University of Notre Dame, USA)
Iterated socles and Hilbert functions

8. Eduardo Sáenz de Cabezón (Universidad de La Rioja, Spain)
The structure of the Pommaret-Seiler resolution for monomial ideals

9. M. Liana Sega (University of Missouri, USA)
Generalized Koszul properties of commutative local rings

10. N.V. Trung (Institute of Mathematics, Hanoi, Vietnam)
Associated primes of powers of edge ideals

11. Matteo Varbaro (University of Genoa, Italy)
On the dual graph of a Cohen-Macaulay algebra

12. Santiago Zarzuela (Universitat de Barcelona, Spain)
On the divisors of a module, their Rees algebras and blow up

Convexity and shellability

Bruno Benedetti

Institute of Computer Science, Freie Universität Berlin, Germany
bruno@zedat.fu-berlin.de

An abstract simplicial complex is called "convex" if it admits a geometric realization in some R^k that is convex. In other words, convex complexes are just linear subdivisions of convex polytopes. Shellability is a combinatorial property of simplicial complexes, corresponding to a nice ordering of the facets. In combinatorial commutative algebra, shellable complexes are of interest because their Stanley-Reisner rings are all Cohen-Macaulay.

In the talk, we review the known relations between the two notions, and add a new one: Every convex complex becomes shellable after two barycentric subdivisions.

This is joint work with Karim Adiprasito.

Symmetric signature: The case of ADE-singularities

Holger Brenner

Department of Mathematics, Universität Osnabrück, Germany
holger.brenner@uni-osnabrueck.de

The F-signature (or minimal Hilbert-Kunz multiplicity) is a measure for singularities in positive characteristics, which is based on the asymptotic splitting behavior of the ring viewed over itself via powers of the Frobenius. In an attempt to develop a characteristic-free notion, we introduce the symmetric signature of a local ring, which is defined by looking at the asymptotic splitting behavior of the symmetric powers of the top-dimensional syzygy module of the maximal ideal. As a first test case we look at the two-dimensional ADE-singularities and prove using representation theory that the symmetric signature is $1/|G|$, which coincides with the F-signature.

This is a report on ongoing joint work with Alessio Caminata.

A Gorenstein simplical complex for symmetric minors

Emanuela De Negri

Dipartimento di Matematica, Università di Genova, Italy
denegri@dima.unige.it

Let $X = (x_{ij})$ be the $n \times n$ generic symmetric matrix and let $S = K[x_{ij} \mid 1 \leq i \leq j \leq n]$ be the associated polynomial ring over a field K. Denote by I_t the ideal generated by the t-minors of X. The ring S/I_t is a Cohen-Macaulay normal domain and it is Gorenstein if and only if $n - t$ is even. It is well known that the t-minors of X are a Gröbner bases with respect to the lexicographic order with $x_{11} > x_{12} > \cdots > x_{1n} > x_{22} > \cdots > x_{nn}$. The corresponding initial ideal is square-free and Cohen-Macaulay, but it is not Gorenstein.

The question when Gorenstein ideals have initial ideals which are square-free and Gorenstein have been answered affirmatively in several instances, for example for ideals of minors of generic matrices and for ideals of Pfaffians of skew symmetric matrices.

So far for the ideal I_t (with $n - t$ even) only the case $t = 2$ has been treated. In this talk the case $t = n - 2$ is considered. We prove that the $(n-2)$-minors form a Gröbner basis of I_{n-2} with respect to a suitable reverse lexicographic order and that the corresponding initial ideal $\mathrm{in}(I_{n-2})$ is square-free and Gorenstein. Furthermore the simplicial complex associated to $\mathrm{in}(I_{n-2})$ belongs to the combinatorially interesting class of r-covered vertex sets. It also turns out that the Betti number of I_{n-2} and $\mathrm{in}(I_{n-2})$ actually coincide.

This is a joint work with A. Conca and V. Welker.

Gorenstein liaison for toric ideals of graphs

Elisa Gorla

Mathematics Department, University of Neuchatel, Switzerland.
elisa.gorla@unine.ch

I will report on joint work with A. Constantinescu, on the G-liaison class of toric ideals of graphs. Our main result says that toric ideals of bipartite graphs belong to the G-liaison class of a complete intersection.

Hilbert regularity of Z-graded modules over polynomial rings

Julio José Moyano Fernández

Departamento de Matemáticas, Escuela Superior de Tecnología y Ciencias Experimentales, Universidad Jaume I, Spain
moyano@uji.es

Let M be a finitely generated Z-graded module over the standard graded polynomial ring $R = K[X_1, ..., X_n]$ with K a field, and let $H(t) = Q(t)/(1-t)^d$ be the Hilbert series of M. We introduce the Hilbert regularity of M as the lowest possible value of the Castelnuovo-Mumford regularity for an R-module with Hilbert series H. Our main result is an arithmetical description of this invariant which connects the Hilbert regularity of M to the smallest k such that the power series $Q(1-t)/(1-t)^k$ has no negative coefficients. Finally we give an algorithm for the computation of the Hilbert regularity and the Hilbert depth of an R-module.

This is a joint work with W. Bruns and J. Uliczka.

Equidistribution of Galois orbits of points of small height

Carlos D'Andrea, Marta Narváez-Clauss*, Martín Sombra

Departament d'Àlgebra i Geometria, Facultat de Matemàtiques, Universitat de Barcelona, Spain
marta.narvaez@ub.edu

The roots of the polynomial $x^n - 1$ lie on the unit circle and determine the vertices of a regular polygon of n sides. When n grows, these polygons tend to the circle. In other words, the n-roots of unity converge to the uniform distribution on S^1 when n tends to infinity. There are several results that let us extend this behavior more generally to the case of sequences of 'small' algebraic numbers on a curve. The size of an algebraic number is quantified by its Weil height. In this talk, I will explain the notion of height and state Bilus classical equidistribution theorem. If time permits, I will present an extension to higher dimension and expose a quantitative version for this result.

Iterated socles and Hilbert functions

Claudia Polini

Department of Mathematics, University of Notre Dame, U.S.A.
cpolini@nd.edu

In this talk we will survey recent results on iterated socles and applications to Hilbert functions.

Monomial Pommaret bases

Eduardo Sáenz-de-Cabezón*, Werner M. Seiler

Departamento de Matemáticas y Computación, Universidad de La Rioja, Spain
eduardo.saenz-de-cabezon@unirioja.es

A Pommaret basis of a polynomial submodule $\mathcal{M}$ is one kind of involutive basis that has certain features which help the description of the structure of $\mathcal{M}$. Any polynomial submodule has, after an appropriate change of coordinates when necessary, such a basis. If $\mathcal{M}$ is a monomial ideal I then a change of coordinates may change its monomiality. This property leads to the fact that only ceratin monomial ideals have a finite Pommaret basis. We call these ideals *quasi-stable*. They can be seen as a generalization of stable ideals. In [2] Seiler constructs a (nonminimal) free resolution for quasi-stable ideals based on Pommaret bases. That resolution can be seen as a generalization of the famous explicit minimal resolution that Eliahou and Kervaire constructed in [1] for stable monomial ideals. We will explore in this talk the Pommaret bases of quasistable ideals and their free resolutions. We will focus on understanding these ideals as a generalization of stable ideals in a deeper sense.

[1] Eliahou, S., Kervaire, M., Minimal resolutions of some monomial ideals, *J. Algebra* **129** (1990), 1–25.
[2] Seiler, W. M., A combinatorial approach to involtion and δ-regularity II: Structure analysis of polynomial modules with Pommaret bases, *Appl. Algebra Engrg. Comm. Comput.* **20** (2009), 261-338.

Generalized Koszul properties of commutative local rings

Liana M. Sega

Department of Mathematics and Statistics, University of Missouri, U.S.A.
segal@umkc.edu

We study several properties of commutative local rings that generalize the notion of Koszul algebra. The properties are expressed in terms of the Ext algebra of the ring, or in terms of homological properties of powers of the maximal ideal of the ring. We analyze relationships between these properties and we identify large classes of rings that satisfy them. In particular, we prove that the Ext algebra of a compressed Gorenstein local ring of even socle degree is generated in degrees 1 and 2.

This is joint work with Justin Hoffmeier.

Associated primes of powers of edge ideals

N.V. Trung

Institute of Mathematics, Hanoi, Vietnam
nvtrung53@gmail.com

We present a complete combinatorial classification of the associated primes of every fixed power of the edge ideal of a graph. This will be done by using matching theory. It turns out that these associated primes are characterized by certain kind of subgraphs of the given graph.

On the dual graph of a Cohen-Macaulay algebra

Matteo Varbaro

Dipartimento di Matematica, Università di Genova, Italy
`varbaro@dima.unige.it`

Given an algebraic set X in the projective n-space, its dual graph $G(X)$ is the graph whose vertices are the irreducible components of X and whose edges connect components that intersect in codimension one. Hartshorne's connectedness theorem says that if X is arithmetically Cohen-Macaulay, then $G(X)$ is connected. In the talk I will discuss the following question: If we know more, like the degree of X or that X is arithmetically Gorenstein, can we say something more on the connectivity properties of $G(X)$? In particular, Hartshorne's theorem tells us that, if X is arithmetically Cohen-Macaulay, given two vertices of $G(X)$ there is at least one path connecting them. Can we say that there are more such paths, under some further assumption? Can we say what is the minimal length of such paths? I will present two results in both directions, obtained in a recent collaboration with Bruno Benedetti.

On the divisors of a module, their Rees algebras and blow up

Santiago Zarzuela

Departament d'Àlgebra i Geometria, Universitat de Barcelona, Spain
szarzuela@ub.edu

Paraphrasing W. V. Vasconcelos, the divisors of a finitely generated module E over a ring R are ideals of R that carry important information about the structure and properties of E. In this talk I shall review some classical examples as the Fitting ideals or the determinant and relate them to some more recently introduced divisors like the norm of E as defined by O. Villamayor and the (generic) Bourbaki ideal defined by A. Simis, B. Ulrich and W. V. Vasconcelos, attached to the Rees algebra of E. The founded relationships among these divisors allow to describe in some cases universal properties with respect to E of their blow ups.

This is a joint work with Ana L. Branco Correia.