# S28. Rings, Modules, Categories and Applications

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**Speakers:**

1. Pere Ara (Universitat Autònoma de Barcelona, Spain)
   *Towards a K-theoretic characterization of graded isomorphisms between Leavitt path algebras*
2. Lars Christensen (Texas Tech University, USA)
   *Injective modules and flat ring extensions*
3. Septimiu Crivei (Babes-Bolyai University, Cluj-Napoca, Romania))
   *Maximal exact structures on additive categories*
4. Juan Cuadra (Universidad de Almería, Spain)
   *Hopf orders and Kaplansky's sixth conjecture*
5. Sergio Estrada (Universidad de Murcia, Spain)
   *On covering ideals*
6. Ana Jeremías (Universidad de Santiago de Compostela, Spain)
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    *The center of a Leavitt path algebra of a row finite graph*

11. Agata Smoktunowicz (University of Edinburgh, Great Britain)
*On nil and Jacobson radicals in differential polynomial rings and tensor products*

12. Greg Stevenson (Universität Bielefeld, Germany)
*TBA*

13. Jan Stovicek (Charles University, Czech Republic)
*Tilting theory in the context of Grothendieck derivators*

14. Alberto Tonolo (Università di Padova, Italy)
*Filtrations induced by tilting modules*

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16. Alexander Zimmermann (Université de Picardie, France)
*Batalin-Vilkovsky structure on the Hochschild cohomology of Frobenius algebras*

# Towards a K-theoretic characterization of graded isomorphisms between Leavitt path algebras

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Hazrat gave in [2] a K-theoretic invariant for Leavitt path algebras as graded algebras. Hazrat conjectured that this invariant classifies Leavitt path algebras up to graded isomorphism, and proved the conjecture in some cases. In this talk, I will state Hazrat's Conjecture and I will report recent advances towards its resolution.

This talk is based on joint work with E. Pardo [1].

[1] Ara, P., Pardo, E., Towards a K-theoretic characterization of graded isomorphisms between Leavitt path algebras, *J. K-theory*, to appear.

[2] Hazrat, R., The graded Grothendieck group and the classification of Leavitt path algebras, *Math. Ann.* **355** (2013), 273–325.

# Injective modules and flat ring extensions

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Let $R \to S$ be a ring homomorphism. It is an elementary and well-known fact that given an injective $R$-module $E$, the $S$-module obtained by co-base change, $\mathrm{Hom}_R(S, E)$, is injective. The converse is not true, not even if $R$ is a commutative noetherian local domain and $S \neq R$ is its completion. Indeed, say $R$ is a 1-dimensional commutative regular local ring that is not complete, i.e. $R \neq \hat{R}$. (For example, $R$ could be a localization of a polynomial ring, $R = k[x]_{(x)}$, in which case $\hat{R}$ is the power series ring $k[\![x]\!]$.) As noticed in [1], one then has $\mathrm{Hom}_R(\hat{R}, R) = 0$, so the $\hat{R}$-module obtained by co-base change of $R$ is injective though $R$ is not injective as an $R$-module. In this setting, it follows from the results I will discuss that an $R$-module $E$ with $\mathrm{Ext}^1_R(\hat{R}, E) = 0$ has $\mathrm{Hom}_R(\hat{R}, E) \neq 0$, and if the $\hat{R}$-module $\mathrm{Hom}_R(\hat{R}, E)$ is injective, then $E$ is injective over $R$.

[1] Stephen T. Aldrich, Edgar E. Enochs, and Juan A. Lopez-Ramos, Derived functors of Hom relative to flat covers, *Math. Nachr.* **242** (2002), 17–26.

# Maximal exact structures on additive categories

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Quillen exact categories have recently reappeared into the mainstream due to some new applications to algebraic $K$-theory, model structures, approximation theory or functional analysis. They provide a suitable setting for developing homological algebra beyond abelian categories, as is often the case in the above settings.

Every additive category has an obvious smallest exact structure given by the split exact sequences. It is natural to wonder whether there exists, and then which is, the greatest exact structure on an arbitrary additive category. First we review the known results for preabelian [3], weakly idempotent complete [1] and additive [2] categories, the latter showing the existence of a maximal exact structure on any additive category. Then we make some steps towards its description.

1. Crivei, S., Maximal exact structures on additive categories revisited, *Math. Nachr.* **285** (2012), 440–446.
2. Rump, W., On the maximal exact structure of an additive category, *Fund. Math.* **214** (2011), 77–87.
3. Sieg, D., Wegner, S.-A., Maximal exact structures on additive categories, *Math. Nachr.* **284** (2011), 2093–2100.

# Hopf orders and Kaplansky's sixth conjecture

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2010 Mathematics Subject Classification. 16T05

A fundamental result in the Representation Theory of Finite Groups is Frobenius Theorem stating that the degree of any complex irreducible representation of a finite group $G$ divides the order of $G$. The proofs of this result use a specific property of the group algebra $\mathbb{C}G$: it may be defined over $\mathbb{Z}$ or, in other words, $\mathbb{Z}G$ is a Hopf order of $\mathbb{C}G$.

Kaplansky's sixth conjecture predicts that Frobenius Theorem holds for complex semisimple Hopf algebras. There are several partial results in the affirmative. Compared to the group case, the main difficulty to prove this conjecture is that it is not guaranteed that a complex semisimple Hopf algebra $H$ is defined over the integers or, more generally, over the ring of integers of a number field. Indeed, Larson proved that if $H$ admits a Hopf order over a number ring, then $H$ satisfies Kaplansky's sixth conjecture. The question whether a complex semisimple Hopf algebra can be defined over a number ring has been part of the folklore of Hopf Algebra Theory.

In this talk we will show that a complex semisimple Hopf algebra does not necessarily admit a Hopf order over a number ring. The family of examples that we will handle are Drinfel'd twists of certain group algebras. The twist contains a scalar fraction which makes impossible the definability of such Hopf algebras over number rings.

The results that will be presented are part of a joint work with Ehud Meir (University of Copenhagen), http://arxiv.org/abs/1307.3269.

## On covering ideals

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Ideal Approximation Theory [1] establishes an extension to ideals of morphisms in general exact categories of the usual theory of right and left (minimal) approximations by modules. This extension covers significant examples of covers and envelopes of modules which were not included in the original theory because they need to be stated in terms of morphisms instead of objects. This is the case of the existence of phantom covers of modules or the existence of almost split morphisms in the Abelian category $A$-mod of finitely presented modules over Artin algebras. In the talk we introduce a new criterion for the existence of covers of modules associated to ideals of morphisms, by using quiver representation techniques. As a consequence we get an alternative proof of the existence of phantom covers [2].

[1] Fu, X. H., Guil Asensio, P. A., Herzog, I., Torrecillas, B., Ideal approximation theory, *Adv. Math.* **244** (2013), 750–790.
[2] Herzog, I., The phantom cover of a module, *Adv. Math.* **215** (2007), 220–249.

# Geometric Applications of Hopf Algebroids

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It is well known the relationship between affine schemes and commutative rings. It is actually a perfect dictionary combining concepts from Algebra and Geometry. In this talk we will approach algebraic varieties and (separated) schemes as coequalizers of pairs of maps between affine schemes. This will lead us to see how can we associate a scheme (plus a covering) with a certain algebraic structure, a Hopf algebroid, restoring the dictionary algebra–geometry in this global context. In fact a Hopf algebroid is just the dual of an internal groupoid in the category of affine schemes. This formalism extends to the more general case framework of algebraic stacks. We'll discuss how to describe quasi-coherent sheaves in terms of comodules.

# Group actions on $sl_2$ and their polynomial identities

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2010 Mathematics Subject Classification. 16R50, 17B01, 16W20

All algebras will be over the field of the complex numbers. Assume that $G$ is a finite group that acts on the 3-dimensional simple Lie algebra $sl_2$. A result of F. Klein (1884) describes these groups: $C_n$, the cyclic group of order $n$, $D_n$, the dihedral group of order $2n$, $A_4$, $S_4$ and $A_5$. Recall that $S_n$ and $A_n$ stand for the symmetric and for the alternating groups on $n$ letters, respectively.

In this talk we describe the $G$-identities of $sl_2$, that is the identities for $sl_2$ with the corresponding $G$-action. It turns out that each of these groups embeds into $Aut(sl_2)$, and different embeddings of the same group result in the same $G$-identities.

The analogous problem for the associative algebra of the $2 \times 2$ matrices was studied and solved by Berele (2004).

In each of the above cases for $G$ we give a basis of the $G$-identities of $sl_2$. The methods we use rely on the description of the graded identities of $sl_2$ as well as on Invariant theory.

## Descent and a construction of a generic form via symmetric monoidal categories


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Let $A$ be any algebra, perhaps with some extra structure, of finite dimension over a field $K$ of characteristic zero. (Examples for extra structure: a Hopf algebra, a comodule algebra.) Consider the following question: over what subfields of $K$ is $A$ defined, and in what ways? (or in other words: over which subfields of $K$ does $A$ admit a form, and what are these forms?). In this talk I will present a new approach to this problem. I will explain a construction of a symmetric monoidal category $\mathcal{C}_A$. The construction of this category will give rise to a subfield $K_0$ of $K$, such that if $A$ has a form over $K_1$, then $K_0$ is contained in $K_1$. I will explain how can one use Deligne's theory on symmetric monoidal categories to construct a "generic form" of $A$, that is- a form over a commutative ring $B$ (who is a $K_0$ algebra), such that every form of $A$ is given by a specialization of $B$. If time permits, I will also describe some applications in the theory of finite dimensional semisimple Hopf algebras.

## Localizations in derived categories of rings

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Given a compactly generated triangulated category $\mathcal{D}$, for example, the stable homotopy category of CW- spectra or the derived category of a ring, a smashing subcategory of $\mathcal{D}$ is a full triangulated subcategory, closed under small coproducts, such that the Verdier quotient $\mathcal{D}/\mathcal{X}$ is closed under small coproducts. $\mathcal{D}$ is said to have the "telescope property" if every smashing subcategory is compactly generated, meaning it is generated by a set of compact objects. In the context of derived categories of rings, the study of the telescope property, and of the compactly generated smashing subcategories in general, is of great interest and, in particular, it leads to the idea of "localization of rings at sets of compacts objects". If $R$ is a ring and $\Sigma$ is a set of compact objects of the derived category $\mathcal{D}(R)$, we say that a ring homomorphism $f : R \to S$ is a generalized universal localization of $R$ at $\Sigma$ (see [1]) if $S \otimes_R P = 0$ for every $P \in \Sigma$ and $f$ satisfies a universal property, namely for every ring homomorphism $g : R \to C$ such that $C \otimes_R P = 0$ for every $P \in \Sigma$, $g$ factors through $f$. If $\Sigma$ consists of two terms complexes, then $S$ is the universal localization in the sense of Cohn and Schofield (see [2]). It is known that, for a given set of compact objects, the generalized localization does not always exists. In this talk I will give sufficient conditions for its existence and I will introduce a further generalization, the differential graded localization (dg localization). Then I will prove a correspondence between equivalence classes of dg localizations and equivalence classes of compactly generated smashing subcategories of $\mathcal{D}(R)$. In the end I will show some connections between the dg localization and the generalized universal localization.

[1] Krause, H., Cohomological quotients and smashing localizations, *Amer. J. Math.* **127** (6) (2005), 1191–1246.

[2] Schofield, A. H., *Representations of rings over skew fields*, LMS Lecture Note Series **92**, Cambridge, 1985.

# The center of a Leavitt path algebra of a row finite graph

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In this talk we will speak about the new techniques we introduce in order to deepen into the structure of a Leavitt path algebra with the aim of giving a description of its center when the graph is row-finite. In particular, we will address our attention to the extreme cycles, which appear for the first time in the literature; they concentrate the purely infinite part of a Leavitt path algebra. Jointly with the line points and the vertices in cycles without exits, they are the key ingredients to determine the center. Our work will rely on our previous approach to the center of a prime Leavitt path algebra [1]. We will go further into the structure itself of the Leavitt path algebra. For example, the ideal $I(P_{ec} \cup P_c \cup P_l)$ generated by vertices in extreme cycles ($P_{ec}$), by vertices in cycles without exits ($P_c$) and by line points ($P_l$) will be a dense ideal in some cases, for instance in the finite one or, more generally, if every vertex connects to $P_l \cup P_c \cup P_{ec}$. Hence its structure will contain much of the information about the Leavitt path algebra. To deal with row-finite graphs, we will need to add a new hereditary set: the set of vertices whose tree has infinite bifurcations ($P_{b^\infty}$).

This is a joint work with María Guadalupe Corrales García, Dolores Martín Barquero, Cándido Martín González and José Félix Solanilla Hernández.

[1] M. G. Corrales García, D. Martín Barquero, C. Martín González, M. Siles Molina, J. F. Solanilla Hernández, Centers of path algebras, Cohn and Leavitt path algebras; preprint.

# On nil and Jacobson radicals in differential polynomial rings and tensor products

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2010 Mathematics Subject Classification. 16N20, 16N40, 16W25, 16S99, 16T05, 16L99

The first part of the talk will describe a construction of a finitely generated, infinitely dimensional algebra $A$ such that algebras $A \otimes A$ and $A \otimes A^{op}$ are nil. This answers a question of Puczyłowski from 1993 and also recent questions of Chebotar, Ke, Lee and Puczyłowski.

In the second part of this talk it will be shown that differential polynomial rings over locally nilpotent rings need not be Jacobson radical, answering a question of Shestakov.

The second part of this talk is a joint work with Michał Ziembowski.

[1] A. Smoktunowicz, Infinitely dimensional nil algebras $A \otimes A^{op}$ and $A \otimes A$ exist, http://arxiv.org/abs/1403.2557.
[2] A. Smoktunowicz, M. Ziembowski, Differential polynomial rings over locally nilpotent rings need not be Jacobson radical, preprint; http://arxiv.org/abs/1311.3571.

# Tilting theory in the context of Grothendieck derivators

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This is a report on joint work with Moritz Groth [2, 3]. Representation theory was classically concerned with studying representations of quivers in the category of vector spaces, while the concept of Grothendieck derivator [1] axiomatizes homotopy theory of representations of small categories in Quillen model categories. It turns out that it is possible to generalize standard techniques (reflection functors, the Nakayama functor, Auslander-Platzeck-Reiten tilting modules) to the context of derivators, extending their applicability to *any* stable homotopy theory. As every stable Grothendieck derivator is enriched over topological spectra, some of these constructions take a universal form in the derivator of spectra.

In the talk I will discuss the above mentioned techniques and a recent progress in the area.

[1] Groth, M., Derivators, pointed derivators, and stable derivators, *Algebr. Geom. Topol.* **13** (2013), 313–374.
[2] Groth, M., Stovicek, J., Tilting theory via stable homotopy theory, preprint; http://arxiv.org/abs/1401.6451.
[3] Groth, M., Stovicek, J., Tilting theory for trees via stable homotopy theory, preprint; http://arxiv.org/abs/1402.6984.

# Filtrations induced by tilting modules

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In [1] Miyashita introduced tilting modules of finite projective dimension over an arbitrary associative ring $R$. Following his definition, a left $R$-module ${}_RT$ is said to be a classical tilting module of projective dimension $\leq r$ if it satisfies the following three conditions:

1. ${}_RT$ has a projective resolution $0 \to P_r \to ... \to P_0 \to T \to 0$ where the $P_i$'s are finitely generated projective left $R$-modules;

2. $\mathrm{Ext}^i_R(T,T) = 0$, if $1 \leq i \leq r$;

3. there exists an exact sequence $0 \to R \to T_0 \to ... \to T_r \to 0$ where the $T_i$'s are summands of a finite direct sum of copies of ${}_RT$.

The class $T^\perp := \bigcap\limits_{i \geq 1} \mathrm{KerExt}^i_R(T,-)$ is called the $n$-tilting class induced by $T$. If ${}_RT$ is a classical 1-tilting module, any module $M$ admits a filtration

$$M = M_1 \geq M_0 \geq 0$$

with $M_1/M_0$ in $\mathrm{KerHom}(T,-)$ and $M_0 \in T^\perp$. In this talk we try to understand how this construction can be generalized for $r \geq 2$.

[1] Miyashita, Y., Tilting modules of finite projective dimension, *Math. Z.* **193** (1) (1986), 113–146.

# A point-free approach to surjunctivity and direct finiteness

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We will describe a "point-free strategy" to partially solve some classical problems about the representations of a given group $G$. The problems we are interested in are the following:

(Linear) Surjunctivity Conjecture. A map is *surjunctive* if it is non-injective or surjective. Given a set $A$ endow $A^G$ with the product of the discrete topologies. The *Surjunctivity Conjecture* (due to Gottschalk [1]) states that any continuous and $G$-equivariant map $\phi : A^G \to A^G$ is surjunctive, provided $A$ is finite. Similarly, given a field $\mathbb{K}$ and supposing that $A$ is a $\mathbb{K}$-vector space, the *L-Surjunctivity Conjecture* states that any continuous, linear and $G$-equivariant map $\phi : A^G \to A^G$ is surjunctive, provided $A$ is finite dimensional.

Stable Finiteness Conjecture. A ring $R$ is *directly finite* if $xy = 1$ implies $yx = 1$ for all $x, y \in R$. Furthermore, $R$ is *stably finite* if $\mathrm{Mat}_k(R)$ is directly finite for all $k \in \mathbb{N}_+$. The *Stable Finiteness Conjecture* (due to Kaplansky [2]) states that the group ring $\mathbb{K}[G]$ is stably finite for any field $\mathbb{K}$. Notice that, $\mathrm{Mat}_k(\mathbb{K}[G]) \cong \mathrm{End}_{\mathbb{K}[G]}(\mathbb{K}[G]^k)$, so an equivalent way to state the Stable Finiteness Conjecture is to say that any surjective endomorphism of a free right (or left) $\mathbb{K}[G]$-module of finite rank is injective.

We will define the category of *quasi frames* (or *qframes*), that is a suitable subcategory of the category of semilattices. Both the L-Surjunctivity and the Stable Finiteness Conjecture can be stated in terms of qframes. We will show that these conjectures are true in case $G$ is sofic and, in this case, their validity follows from a general theorem proved in the category of qframes.

[1] Walter Gottschalk, Some general dynamical notions, *Recent advances in topological dynamics (Proc. Conf. Topological Dynamics, Yale Univ., New Haven, Conn., 1972; in honor of Gustav Arnold Hedlund)*, 120–125. Lecture Notes in Math., vol. 318, Springer, Berlin, 1973.

[2] Irving Kaplansky, *Fields and rings*, reprint of the second (1972) edition, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, IL, 1995.

# Batalin-Vilkovisky structure on the Hochschild cohomology of Frobenius algebras

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The Hochschild cohomology ring over a finite dimensional algebra $A$ over a field $k$ carries a lot of structure. Besides the usual ring structure the Gerstenhaber Lie algebra structure obtained a lot of attraction. It is often quite complicate to obtain an explicit description of this Lie bracket even in small examples. All this is part of an even bigger structure, called a differential calculus by Gelfan'd, Daletskii, Tamarkin and Tsygan. Lambre generalised this notion to a differential calculus with duality, and he proved that the structure of a differential calculus with duality implies the structure of a Batalin-Vilkovisky algebra. In a Batalin-Vilkovisky algebra the Lie bracket has a very nice and explicit form. Ginzburg showed that the Hochschild cohomology algebra of a Calabi-Yau algebra has a Batalin-Vilkovisky structure and Tradler showed that the Hochschild cohomology algebra of a finite dimensional symmetric algebra has a Batalin-Vilkovisky structure. Kowalzig and Krähmer extend the result of Ginzburg to some twisted version. We use their ideas and combine them with the work of Lambre to extend Tradler's result to Frobenius algebras with diagonalisable Nakayama automorphism. We give an explicit criterion when our hypotheses are satisfied and apply them to certain classes of algebras of tame representation type.

[1] V. Ginzburg, Calabi-Yau algebras, http://arxiv.org/abs/math/0612139.
[2] N. Kowalzig and U. Krähmer, Batalin-Vilkovsky structures on Ext and Tor, *J. Reine Angew. Math.*, to appear.
[3] T. Lambre, Dualité de Van den Bergh et Structure de Batalin-Vilkovisky sur les algèbres de Calabi-Yau, *J. Noncommut. Geom.* **4** (3) (2010), 441–457.
[4] T. Tradler, The Batalin–Vilkovisky algebra on Hochschild cohomology induced by infinity inner products, *Ann. Inst. Fourier* **58** (7) (2008), 2351–2379.