Extended abstract:
A coalitional theory of oligopoly

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Revised: February 13, 2009

"The difficult problem that arises from the relations of a very small number of competing firms has been much studied in recent years, but there has not yet developed any very close agreement on the solution."
— John R Hicks (1935, p.12)

A new theory of oligopoly based on well established tools from coalitional game theory is presented and studied. The new theory sheds new light on the issue, and addresses limitations of current oligopoly literature.

Mainstream oligopoly theory assumes:
- Prices exist in and out of equilibrium.
- Oligopolists have a priori market power.
- Oligopolists have very limited trading possibilities with each other, and cannot create explicit cartels or mergers.

These three assumptions already appear in Cournot’s (1838) work on monopoly and oligopoly, and continue ever since up to modern game theoretic analysis.

This latter analysis models the oligopolists as players that make strategic choices, such as prices, quantities, products’ characteristics and location, while the consumers are assumed to be passive price takers. The strategic choices, even when they do not constitute an equilibrium, always lead to prices, which in turn determine the payoffs of the economic agents, oligopolists and consumers alike.

The application of coalitional games to oligopoly is not new. Shitovitz (1973) and Gabszewicz and Mertens (1971) extended Aumann’s (1964) work on continuum economies to oligopoly and established conditions that preserve the equivalence of the core with the set of competitive price equilibria. These works have the limitation that by construction only Pareto efficient outcomes are possible in an economy, making market failure impossible.

My theory of oligopoly allows for market failure, and does not concentrate on finding equivalence between existing oligopoly equilibria and coalitional equilibria. On the contrary, it concentrates on establishing different predictions than the existing oligopoly literature, providing new insights, and exposing limitations of the strategic models.

My theory is logically divided into two parts. In the first part I present a model that explains why linear pricing may occur in an oligopoly framework, in and out of equilibrium. In the second part, I study models in which prices always exist in and out of equilibrium, by assumption. While the two models are logically independent, the first may be viewed as a reasoning of the assumptions of the second model.

Now, I discuss the two parts in more detail.

**Part 1: Linear pricing in monopoly and oligopoly**

This article establishes conditions in which the outcome of the interaction between oligopolists and consumers will be a consumer-wise competitive allocation. In these allocations, every consumer maximizes his utility within a competitive budget set. The results makes use of a core equivalence theorem in perfectly competitive economies.

Consider economies with a continuum of small consumers, and a finite set of large oligopolists. Further assume that consumers can participate directly only in finite coalitions and can only handle transactions that are negligible relative to the market. To enable trade between the oligopolists and consumers, assume that each oligopolist offers a menu of transactions to each consumer, and each consumer can choose repeatedly from the menus offered to him. In addition, consumers can re-trade among themselves in finite coalitions.

The main result is that under these conditions only consumer wise competitive allocations can be obtained.

On the other hand, when consumers can be involved in transactions with many participants and large quantities of goods, as in Shitovitz (1973) non-linear pricing may occur, indicating that the presence of...
friction is required for linear pricing.

The result is quite general when compared to the underlying economic structures in oligopoly
literature. I assume that there is a numéraire commodity that consumers may have any positive or
negative quantity of it, and that preferences are continuous and strictly monotonic with respect to this
commodity.

Further, there is a possibly infinite set of potential commodities. However, only a finite subset of it is
realized. These potential commodities may represent different locations or qualities, while oligopolists
are assumed to eventually produce only a finite number of different commodities.

When we consider consumer-wise competitive allocations, only realized commodities and the
numéraire have prices. In a consumer's budget set potential commodities that are not realized have
always zero quantity, thus cannot be traded.

The formal description of the assumptions on trading possibilities is modeled with the core with
respect to finite coalitions (the f-core) of atomless economies.

For every system of menus that the oligopolists offer to the consumers, an economy consisting only of
consumers is defined. The menus appear in the economy as individual production sets of the consumers.
As this economy is atomless, the f-core may be applied. The result is obtained by using an f-core
equivalence theorem by Hammond (1999). Hammond (1999) generalizes the well known result of
Hammond, Kaneko, and Wooders (1989), who restricted their attention to exchange economies.
Allocations in the economy that their restriction to the consumers' sector is in the f-core of the above
described game are called "small coalition stable" allocations. Thus, the result is:

**Theorem 1.1** An allocation is small coalition stable if and only if it is a consumer-wise competitive
allocation.

### Part 2: Oligopoly as a coalitional game

The consumer-wise competitive coalitional game (C3-game) models an oligopoly as a coalitional game
allowing coalitions to consider only consumer-wise competitive allocations.

As opposed to strategic models, in this model market power is not a priori assigned, as both
oligopolists and consumers play a similar active role in the game. In addition, no limitation on
cooperation among oligopolists is imposed. The model, therefore, may predict the extent of market
power of firms, and when collusion among oligopolists is sustainable in equilibrium.

A C3-game may be applied to essentially every well known one-period oligopoly model studied earlier
as a strategic game. At this stage, my work concentrates on the core of these games, although studying
other solution concepts, and in particular the Shapley value, seems interesting as well.

I begin by reviewing general results, and then turn to results that are more specific to particular
models.

#### The worse than JPM outcome paradox

Consider an economy with an oligopoly, further assume that all oligopolist derive utility only from the
numéraire commodity. A joint profit maximization (JPM) outcome, is a consumer-wise competitive
allocation that maximizes the joint profit of the oligopolists.

**Theorem 2.1** The core of a C3-game never contains an allocation that is Pareto dominated by a JPM
outcome.

The proof follows directly from the definition of the core. If an allocation is Pareto dominated by
another, the dominated allocation is not in the core as it may be improved upon by the grand coalition
using the dominating allocation.

Although mathematically trivial, the result is important. A common phenomenon in strategic oligopoly
models is that the equilibrium outcome may be worse for all parties, including the consumers, than the
unique JPM outcome of the economy. For example, this happens in either price or quantity competition
where the two goods produced by the duopolists are complements in the consumers' demand function.

#### Competitive price equilibria

**Theorem 2.2** Any competitive equilibrium allocation is a member of the core of the C3-game.

Again this result is not surprising, and it follows from the facts that a competitive equilibrium
allocation is consumer-wise competitive, and the well know result that it is in the core (of the regular game) of the economy.

While this result may look negative at a first glance, in my view it conveys an important insight. The C3-game and its core do not imply that market power necessarily exists. If a competitive equilibrium exists in the economy, it may also prevail in an oligopoly situation. On the other hand, the competitive outcome is by no means the only possible outcome. The other outcomes may reflect different degrees of actual or equilibrium market power.

It should also pointed out that in models with increasing returns to scale competitive equilibrium usually does not exist, but the core of the C3-game is often non-empty.

The multiplicity of core outcomes is not a drawback of the C3-game, but a general property of the core of coalitional games. A possible way to avoid this is to consider another solution concept, such as the (NTU) Shapley value. The value may provide a more accurate prediction on the extent of market power, than the core. While this conclusion is similar to Aumann (1973), it follows from a completely different line of reasoning.

**The Shapley-Shubik (1969) oligopoly**

I now turn to study a particular model of oligopoly that was previously studied as various strategic games. This allows to compare the new theory to older well known results. The first version of this model was studied by Shapley and Shubik (1969) and further studied by many including Singh and Vives (1984), H äckner (2000), Amir and Jin (2001), and Hsu and Wang (2005). It is also presented in Shy's (1996) textbook.

There is a continuum of identical consumers. There is a numéraire commodity, and k perfectly divisible commodities that may be produced by the oligopolists.

Each consumer has a quasi-linear utility function, linear in the numéraire commodity. The non-linear part is quadratic, and symmetric in all k goods. A single parameter of the utility function determines the extent of substitution among the different goods. The consumer's demand function is linear in prices.

There is a finite set of oligopolists. Every oligopolist can produce some of the k goods. If an oligopolist may produce a certain good, production is costless. Oligopolists utility functions are linear in the numéraire, and they do not derive any utility from other goods. All players in the economy have a zero endowment. (This is equivalent to saying that only the numéraire commodity is present in the endowments.)

Evidently, in every such economy, competitive equilibrium is characterized by all k commodities having the price zero. The linear demand function implies that consumers purchase a finite quantity of each good. This also implies that the C3-game associated with such an economy, has a non-empty core.

**Monopoly**

In a monopoly setting, a single monopolist may produce the k goods. In every core allocation, all goods have identical prices. However, this identical price may belong to a large range. The competitive price being the lowest, and the textbook monopoly price being the highest.

**Symmetric oligopoly**

Now consider a symmetric oligopoly. There are k oligopolists each producing a different commodity. Here again, the prices of all commodities are identical, but the price range becomes smaller, while the minimum competitive price remains possible, the maximum price may become lower.

This is highly intuitive. In every coalitional game, merging several players into one player makes the core bigger as it reduces coalitional opportunities.

However the result differs from both strategic price or quantity competition. First, the model is not exposed to the worse than JPM outcome paradox. Second, when the k goods are complements or poor substitutes in the consumers’ preferences the model predicts endogenous merger or cartelization of the oligopolists. This is reflected in the result that the price range stays as large as in the monopoly situation. It is only when the goods are sufficiently substitutes, that the maximum price is lower than in monopoly.

**Asymmetric duopoly**

Consider a simple asymmetric duopoly. k=2. One oligopolist can produce both goods, and the other only one good. Here, the similarity to strategic models reduces further.

While in our model prices of all commodities remain identical, in strategic models they become
different. The price of the commodity produced by both oligopolists is lower. The reason for that is that
different prices are a mechanism to transfer surplus among the oligopolists. This points out an
important reason for a creation of a dead weight loss, often neglected by the literature.
In my model, prices are a mechanism to transfer surplus from the consumers to the oligopolists as
other type of surplus transfers are impossible. On the other hand my model allows a division of profits
among oligopolists independently from production. Indeed, in all core outcomes of the C3-game of this
economy, the oligopolist that can produce a single good makes zero profit.
The prices of the two goods in my model are always identical, and the maximum price, which is
positive, gives the consumers the same utility that they could get by consuming only one good with a zero
price.

Conclusions
The models presented here present new ideas about how oligopoly may be studied. The fact that they
use existing game theoretic tools, rather than inventing new ones, allows to concentrate on economic
issues in oligopoly theory.
As pointed earlier, essentially every one-period oligopoly model can be re-modeled as a C3-game. This
opens the door to many studies. Moreover, in my view, the insights illuminated by the core of the
C3-game of simple versions of the Shapley-Shubik oligopoly indicate that pursuing studying oligopoly
within this approach may be fruitful.
In particular, oligopolies where firms choose a finite number of locations or qualities of their products
have not yet been studied as coalitional games. Another topic that requires more attention, is that of
mergers, both horizontal and vertical.

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