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Principle of functioning of a self-compensated fibre-optical displacement sensor based on diffraction-grating-ended POF

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Abstract
A self-compensated displacement sensor has been realized by placing a diffraction grating on the end face of a plastic optical fibre (POF), the method serving for any highly multimode optical fibre. By measuring the output powers at two diffraction angles corresponding to two different diffraction orders, a simple way to cancel out the fluctuations of the light source and other disturbances has been achieved.

Keywords: optical fibre sensor, plastic optical fibre, diffraction grating

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Fibre optical sensors can measure a great variety of physical parameters such as pressure, temperature or displacement [1, 2]. Sometimes changes in these parameters modulate the phase of the light carrier, in which case single-mode fibres and interferometric demodulation techniques are currently preferred. However, if intensity or amplitude modulation is used, then multimode fibres are commonly employed.

Multimode-fibre optical sensors are much simpler and more economical than single-mode-fibre ones. However, measurements carried out with them are based on the output optical intensity, so they are rather influenced by power fluctuations produced by variations in the light source emission intensity or by random fibre bending, among other disturbances. There are some recent studies that have been devoted to reducing this problem, particularly in the field of intensity-based displacement sensors. A possible solution was developed by Wang *et al* [3]. It consisted in employing two multimode fibres, one of them serving as a reference to measure fluctuations in the light source. More recently Das *et al* [4] developed a micro-displacement sensor using a thermal light source and a diffraction grating. Both works had in common that two wavelengths separated by means of a wavelength splitter were used and that distances of some hundreds of microns could be measured.

Lately, our team has been developing a method consisting in placing a diffraction grating on an obliquely polished end face of a plastic optical fibre (POF) [5]. A displacement sensor using this technique is presented in this paper. It is based on light intensity variations and it utilizes a diffraction grating placed on one end of a multimode optical fibre, which is tilted at an angle to the fibre cross section. This arrangement provides a self-compensation mechanism, eliminating the need for the wavelength splitter of the aforementioned sensors. The following sections give a discussion of the principles on which this sensor is based, the experimental results obtained and the possible applications of the sensor.

2. Theoretical study

The objective of the present study is to make a self-compensated displacement sensor by using the diffraction

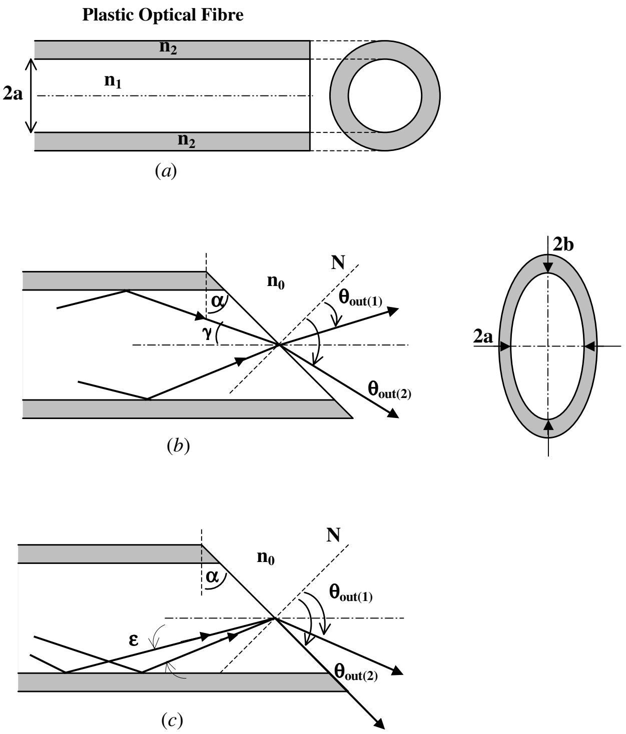

Figure 1. Geometry of the studied POF and propagation conditions through, (*a*) perpendicularly terminated fibre and circular end face, (*b*) fibre terminated at an angle and elliptical end face and (*c*) incidence and transmission of a reduced group of the modes. α is the light cone angle of the rays incident on the end facet that can leave the fibre. ε is the light cone angle of the rays incident on the end facet that can leave the fibre. N is the normal to the fibre end surface and finally a and b are the small and long axes, respectively, of the ellipse shaped fibre end.

properties of a grating when located at the extremity of a multimode optical fibre. The efficiency of a diffraction grating at a chosen wavelength depends on the amount of lines per unit length engraved on the grating and also on the width of the incident beam. Since the fibre employed is strongly multimode, the feasibility of the device requires a good adjustment of the diffraction angles by appropriately choosing the tilt angle of the output surface and the number of lines per millimetre in the grating. Next, we carry out this study.

2.1. Transmission through a multimode optical fibre cut at an angle to the fibre symmetry axis

In most of the applications of optical fibres, the end of the fibre is cut at a right angle to the fibre symmetry axis, as shown in figure 1(*a*), which corresponds to a tilt angle α of 0°. In this case, when neither mode coupling effects nor fibre length are very large, the output light rays lie inside a revolution cone whose semi-angle θ of aperture is characterized by the fibre numerical aperture (NA $= \sin\theta$) and, therefore, by the core and cladding refractive indices (n_1 and n_2 respectively). When the tilt angle α of the output end face is different from zero, i.e. when this is elliptical with semi-axes a and b instead of circular of radius a (figure 1(*b*)), then the directions of the transmitted rays into the air are modified. Moreover, the maximum inclination angles for rays not to undergo total internal reflection at the fibre end face have also changed.

On the other hand, typical POF of 0.98 mm core diameter allows the propagation of more than a million modes in the visible spectrum. For example, at a wavelength of 650 nm, the number of modes N in a 1 mm step-index POF with a numerical aperture NA $= 0.5$ is

$$N = \frac{V^2}{2} = \left(\frac{2\pi a}{\lambda}\right)^2 \frac{n_1^2 - n_2^2}{2} = 2.8 \times 10^6, \qquad (1)$$

where a is the core radius and λ is the wavelength. Light rays can be considered to be guided if the angle with the normal at each reflection point is not greater than the critical angle given by $\theta_c = \sin^{-1}(n_2/n_1)$. Now let us focus our attention on meridional rays contained in the plane along the long axis b. If all of them are guided, these can reach the output end propagating at an angle γ to the fibre symmetry axis between $+\cos^{-1}(n_2/n_1)$ and $-\cos^{-1}(n_2/n_1)$. This is shown in figure 1(*b*). The tilt angle α of the end face can limit the amount of light transmitted to the outside of the fibre.

Considering that all the modes within the above-mentioned range are propagating along the POF, and that this has been cut at an angle α, it is easy to check that the directions of the transmitted rays with respect to the normal to the end face are between $\theta_{out(1)}$ and $\theta_{out(2)}$, given by

$$\theta_{out(1)} > \sin^{-1}\left\{\frac{n_1}{n_o}\cos[\alpha + \sin^{-1}(n_2/n_1)]\right\}, \qquad (2a)$$

$$\theta_{out(2)} < \sin^{-1}\left\{\frac{n_1}{n_o}\cos[\alpha - \sin^{-1}(n_2/n_1)]\right\}, \qquad (2b)$$

where n_o is the refractive index of the outer medium (1 in the case of the air).

The condition for a ray to be transmitted towards the exterior of the fibre is that the incident angle should be smaller than the critical angle given by $\sin^{-1}(n_o/n_1)$. When the tilt angle α increases, some incident rays may not fulfil this condition, in which case those rays that do not satisfy the inequality will undergo total internal reflection, and they will be radiated through the core–cladding interface. The maximum value for $\theta_{out(2)}$ is 90° (figure 1(*c*)).

The fibre used, both in our calculations and in our experiments, is a 2 m long 1 mm diameter step-index poly(methyl methacrylate) (PMMA) POF. Specifically, our fibre is an ESKA SH-4001™, which has an attenuation of 165 dB km^{-1} at a wavelength of 650 nm [6]. Other characteristics are $n_1 = 1.492$ and NA $= 0.5$. The large diameter of step-index POF makes it possible to terminate the fibre at a large tilt-angle, which significantly reduces the range of angles for rays to be transmitted. For example, for an angle of inclination $\alpha = 60°$ and a maximum angle to the fibre symmetry axis γ, only a small range of incident angles, differing in ε degrees at most, will be transmitted (figure 1(*c*)). For $\alpha = 60°$ instead of $\alpha = 0°$, the NA diminishes from 0.5 to 0.11.

2.2. Fibre end face tilted diffraction-grating-ended multimode optical fibre

On the output end of a multimode optical fibre polished obliquely at an angle with the fibre cross section lies a

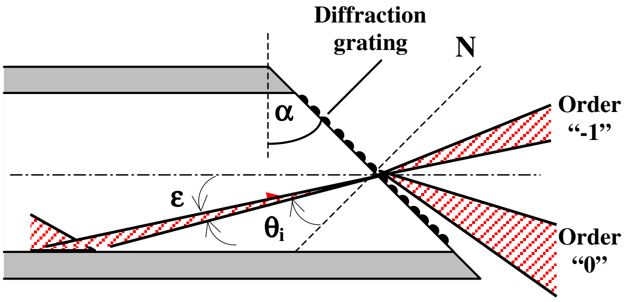

Figure 2. Diffraction caused by a diffraction grating located in the extremity of a tilted fibre. Order (−1) and order 0 mean diffraction orders −1 and 0.

thin-film diffraction grating, as shown in figure 2. The tilt angle α and the period of the diffraction grating have been tuned to obtain only two of the diffraction orders among all the possible ray output angles. Transmitted rays out of the fibre and the diffraction grating will be those for which the incidence angle is smaller than $\sin^{-1}(n_o/n_1)$. Let θ_i be the ray incidence angle with respect to the normal to the fibre-grating interface. Light diffracted by the diffraction grating, in the case of a monochromatic wave, will follow the equation [7]:

$$\sin\beta_m = n_1 \sin\theta_i \pm \frac{\lambda}{d} m, \tag{3}$$

where β_m is the angle between the diffracted ray and the normal, m is an integer that stands for the diffraction order ($0, \pm1, \pm2, \ldots$), λ is the light wavelength and d is the period of the diffraction grating. Incident rays corresponding to all propagating modes will be between $\theta_{imin} = \alpha - \gamma$ and $\theta_{imax} = \alpha + \gamma$. If only diffraction orders of one sign are looked for (say, negative), it is clear that we only need to consider θ_{imin}. If only a group of rays differing in an angle smaller than or equal to ε is considered, then the incidence angle, as a function of the angles α, γ, ε and the outer refractive index, satisfies:

$$\theta_i = \alpha - \gamma + \varepsilon < \sin^{-1}(n_o/n_1), \tag{4}$$

ε is the light cone angle of the rays incident on the end facet that can leave the fibre. Incident rays outside this range are radiated towards the cladding. This study only focuses on transmitted rays. In view of the application we are interested in, we will only consider the orders 0 and –1. The diffraction angles corresponding to these orders have the same sign as the incident angle. The angular separation between two consecutive orders is, from (3), given by the equation:

$$\Delta\sin(\beta) = \sin\beta_m - \sin\beta_{m+1} = \frac{\lambda}{d}. \tag{5}$$

It can be seen that $\Delta\sin(\beta)$ depends both on λ and d through the quotient λ/d.

The application of equation (4) to the case of a PMMA POF, with $n_1 = 1.492$ and NA $= 0.5$, yields an angle γ_c of 20°, which means that for $n_o = 1$ (index of air), the angle θ for transmission to occur should be smaller than 42°. For a tilt angle α of 60°, we have $\varepsilon = 2°$. As we have shown before, POF because of their large diameter (980 μm) can guide more than two million modes in the visible spectrum. In an angular range of 2°, approximately 10% of the total guided modes are contained. It is these modes that contribute to the transmission output of the fibre, and, therefore, to the diffraction by the grating.

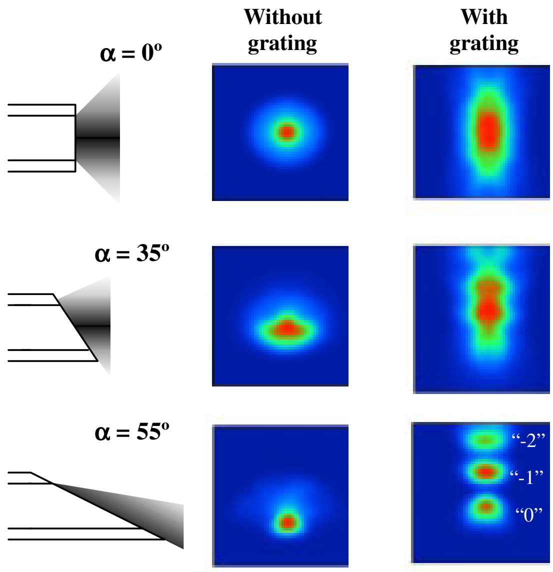

Figure 3. Intensity patterns of the light irradiated by an end face fibre tilted for $\lambda = 660$ nm.

Table 1. Diffracted angles by a grating in a fibre.

	Diffraction angle $\alpha = 60°, \gamma = 20, \varepsilon = 2°, d = 1.8\ \mu$m		
Order	$\lambda = 470$ nm	$\lambda = 530$ nm	$\lambda = 660$ nm
0	70.0°–90°	73.8°–88.6°	72.9°–84.3°
−1	46.7°–50°	41.7°–44.8°	36.1°–38.9°
−2	29.2°–31.9°	21.8°–24.3°	12.9°–15.2°

Table 1 shows the calculation of the diffracted rays as a function of the wavelength, for a grating period d of 1.8 μm. The values of the refractive indices as a function of the wavelength have been obtained from [8]. In table 1, we include the range of angles for the diffracted rays corresponding to the orders 0, −1 and −2, for three different wavelengths. If only two of the orders are required instead of three, we can decrease the period of the diffraction grating until the order −2 is eliminated.

Figure 3 shows the calculated intensity distribution irradiated by the fibre SH-4001 with different angles of inclination α and the effect on the end face with a diffraction grating of period 1.8 μm. The grooves of the diffraction grating are along the small axis of the elliptical output end face of the POF. The intensity patterns were deduced by means of ray tracing. It is observed that as the inclination angle α is increased the diffraction orders are more resolved. For $\alpha = 55°$, we observe three orders corresponding to $m = 0$, −1 and −2. For smaller angles α the diffractive orders are overlapped due to the angular spread ε considered in the simulation, according to the inequality (4).

3. Application to the displacement sensor

The new displacement sensor is based on two diffraction orders of the grating: 0 and −1. By using the values obtained in the previous section for POF and LEDs of different wavelengths,

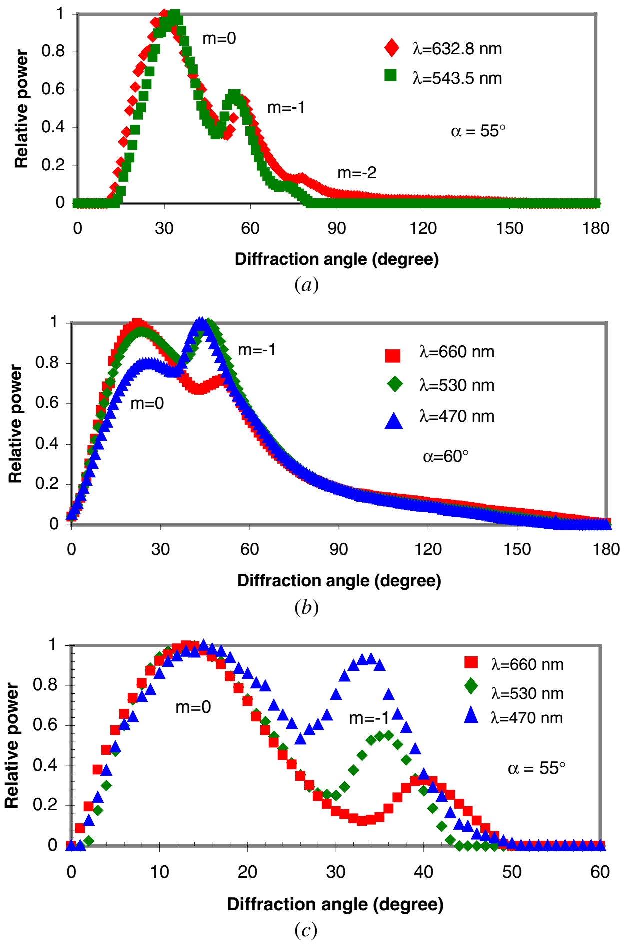

Figure 4. Experimental measure using a diffraction grating of period 1.8 μm in a tilted POF. (*a*) Distribution across the far field radiation pattern in end fibre at $\alpha = 55^\circ$ with grating and different source diode LEDs, (*b*) transmission of the diffracted light corresponding to order 0 and -1 in end fibre at $\alpha = 60^\circ$ for three different wavelength LEDs and (*c*) distribution across the far field radiation pattern in end fibre at $\alpha = 55^\circ$ with grating and different source lasers.

the angles for each diffraction order have been experimentally corroborated. Results are shown in figure 4. It can be noted that the efficiency of the diffraction grating is higher for smaller wavelengths, although the angular separation decreases. In this figure the diffraction angle is taken as the complement of β_m.

From the data corresponding to $\alpha = 55^\circ$ (figure 4(*a*)) and $\alpha = 60^\circ$ (figure 4(*b*)) we observed clearly an improvement of the diffraction efficiency for $m = -1$, mainly due to a smaller range for the value of the angular spread ε.

We also observed that the separation between peaks corresponding to different diffraction orders is larger for larger wavelengths, as predicted by equation (5) and that their peak values are in the range given in table 1. The efficiency is also better for shorter wavelengths as should be expected from basic physics.

In addition, we can see that the order -2 does not appear, due to the characteristics of the light source. The LED is an incoherent source and the diffraction efficiency for such a source is weak enough not to be observed. This can be appreciated in figure 4(*c*) where the light sources are

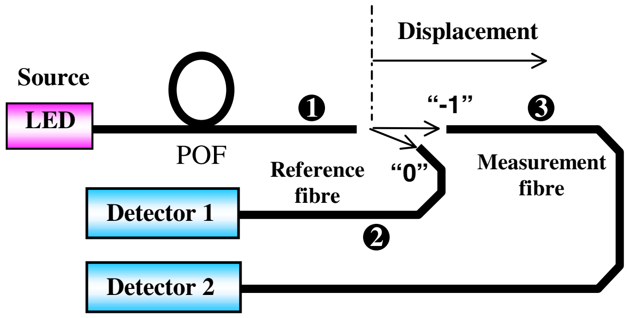

Figure 5. Scheme of the proposed displacement sensor. This consists of an emitting fibre, with its diffraction grating, and two receiving fibres: one for the order 0 (fibre 2), which serves as a reference, and the other for the order –1 (fibre 3), which measures the displacement.

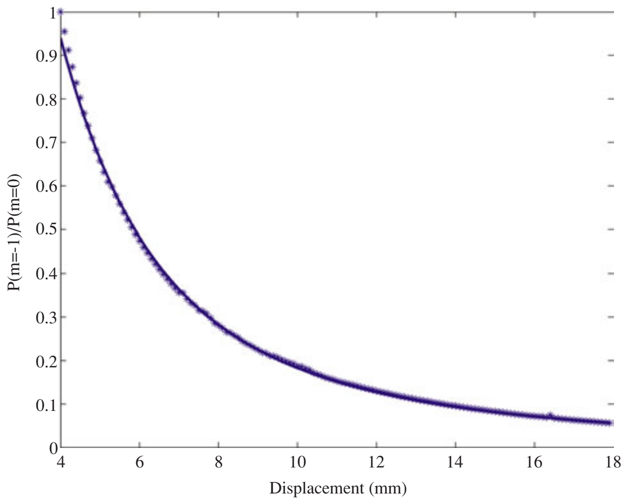

Figure 6. Relative power of the order $m = -1$ with respect to the order $m = 0$ against displacement for a wavelength of 660 nm. The experimental data are plotted with asterisks and the best fitting curve with a continuous line.

two He–Ne lasers at 632.8 and 543 nm. In this case we observed the diffracted beam corresponding to order 2 is very weak when compared with the other two diffracted beams $m = 0, -1$.

To test the displacement sensor, the wavelength of 660 nm has been chosen. In this case, diffracted power in the order 0 is greater than that in the order -1. The set-up has been plotted in figure 5. This consists of an emitting fibre, with its diffraction grating, and two receiving fibres: one for the order 0 (fibre 2), which serves as a reference, and the other for the order -1 (fibre 3), which measures the displacement. No lenses were used to focus light into the receiving fibres. Each receiving fibre is connected to a photodiode. The intensity collected by fibre 3 is a function of the displacement. To cancel out power fluctuations in fibre 1 we use the quotient between the powers collected by fibres 3 and 2. Optical power guided by fibre 1 can fluctuate due to lack of stability of the source or to bends along the fibre, but the quotient of the powers of the two orders is not affected.

For LEDs of wavelength $\lambda = 660$ nm, we have measured the optical power from fibre 3 as a function of distance. In figure 6, the upper curve shows the measurement with a LED of a certain power P, and the lower one corresponds to half the power of the same LED. Therefore, the LED power is not a limiting factor in the design of the displacement sensor. Power variations of the LED are self-compensated by the design.

The dependence of relative power on distance follows a curve like:

$$\frac{P_{m=-1}}{P_{m=0}} = \frac{1}{a + bx + cx^2}, \tag{6}$$

where a, b and c are constants and x is the displacement from the zero position. This behaviour should be expected from the inverse square power dependence on the distance. The inclusion of the linear term is to allow for an arbitrary origin. By means of a least square fitting we obtained the best estimates of these parameters which give the smaller standard deviation σ. The best estimation for these parameters was $a = 0.424\,387$, $b = -0.069\,565$ and $c = 0.057\,355$. With these values the dispersion is $\sigma = 0.008$.

In figure 6, both the experimental data with asterisk symbols as well as the best fitting curve with a continuous line are plotted. The curve fits the experimental points quite well over the whole measuring range.

In summary, the proposed sensor works fine in the range of 4000–14 000 μm. Smaller distances cannot be measured due to the impossibility of resolving the diffraction orders spatially. Fortunately this is not a problem because we can arbitrarily fix the zero displacement position.

4. Conclusion

We have presented a new displacement sensor based on a diffraction grating placed on the output end face of a POF polished at a tilted angle. By measuring two diffraction orders transmitted through the diffraction grating, one as reference and the other as a function of the displacement, we have obtained a way to compensate power fluctuations. By using a red LED as light source, we have achieved a measuring range from 4000 to 14 000 μm with a resolution of 20 μm. A wider range can be obtained if a laser source is employed.

This method can also be utilized to measure other physical parameters such as temperature, pressure or strain. In addition, by using several wavelengths we could measure different physical parameters simultaneously.

Acknowledgments

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References

[1] Lopez-Higuera J M 2002 *Handbook of Optical Fibre Sensing Technology* (New York: Wiley)
[2] Zubia J and Arrue J 2001 Plastic optical fibres: an introduction to their technological processes and applications *Opt. Fibre Technol.* **7** 101–40
[3] Wang A *et al* 1995 Self-referenced fibre optic sensor for microdisplacement measurement *Opt. Eng.* **34** 240–2
[4] Das K *et al* 2002 Fibre-optic microdisplacement sensor based on diffraction grating *Microwave Opt. Technol. Lett.* **32** 446–9
[5] Lomer M, Zubía J and López-Higuera J M 2002 Wavelength demultiplexing using an optical diffraction grating on POF end fibre *POF 2002: 11th Polymer Optical Fibre Conf. (Tokyo, Sept. 2002)* pp 73–6
[6] www.pofeska.com
[7] Loewen E C and Popov E 1997 *Diffraction Grating and Applications* (New York: Dekker)
[8] Ishigure T *et al* 1996 Optimum refractive index profile of the graded-index polymer optical fibre, toward gigabit databit data link *Appl. Opt.* **35** 2048–53