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WILEY-VCH

Phys. Status Solidi A **208**, No. 8, 1845–1848 (2011) / **DOI** 10.1002/pssa.201084111

physica status solidi a
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# Computational analysis of the amplification features of active plastic optical fibers

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Received 3 October 2010, revised 14 March 2011, accepted 20 June 2011
Published online 15 July 2011

**Keywords** ASE threshold, dye lasers, fiber lasers, numerical aperture, polymer optical fibers, rhodamine 6G

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Polymer optical fibers doped with organic dyes can be used as efficient optical amplifiers and lasers in the visible region. We computationally study the spectral evolution with distance of the pulses propagating along such fibers. The main goal is to analyze the lasing threshold and the slope efficiency as functions of the fraction of spontaneous emission that contributes to laser emission. The discussion focuses on rhodamine-6G doped fibers.



**1 Introduction** Plastic optical fibers (POFs) have raised a great interest in applications such as local area networks, sensors and, more recently, also in the field of fiber lasers and amplifiers in the visible region when they are doped [1, 2]. The most frequently employed solid host is poly (methyl methacrylate) (PMMA) and many works have been carried out to develop PMMA fiber lasers and amplifiers [1, 3–9]. In this paper, rhodamine 6G is used as organic dopant. It has large emission and absorption cross sections, which allows developing fiber lasers and amplifiers in short fiber lengths [5]. Moreover, such devices can be designed to work in a wide range of wavelengths, because many materials with different emission and absorption cross sections can be chosen [10].

Lasers typically consist of a gain material between two reflecting mirrors. This set-up is designed to produce stimulated amplification of internal photons [5, 6, 11]. However, similar light emission is obtained when a mirrorless doped fiber or a liquid solution is optically pumped [7, 12–16]. Such light presents many of the properties of laser radiation (narrow linewidth, existence of a threshold power, and directionality) but these are less developed [17]. This kind of emission is called amplified spontaneous emission (ASE) or mirrorless lasing.

We have developed a mathematical model with which we have analyzed the evolution of the output light as a function of several parameters, including the fraction of the spontaneous emission lying in guided directions ($\beta$). As far as we know, this parameter has not been thoroughly studied in dye-doped POFs yet. All these analyses allow developing POF lasers with a good threshold-to-efficiency relation.

**2 Theoretical model** The behavior of the system is modelled by a set of partial differential equations, initial conditions, and boundary conditions. The unknown functions to be determined are the resulting light power ($P$) and the molecule population density in the excited state ($N_2$). The independent variables are usually the time $t$ and the position $z$ along the fiber. Since one of the goals of this work is to carry out a spectral analysis of $P$, we have also included the wavelength $\lambda$ as one of the independent variables, i.e., $P(t, z, \lambda)$ [18, 19].

We have considered that the doped POF is pumped from one end ($z = 0$) at wavelength $\lambda_p$. The evolutions of $P$ and of the population density in the excited state $N_2(t, z)$ can be described by three partial differential equations [19]. Let us now focus our attention on one of these rate equations, namely:

$$\begin{aligned}\frac{\partial P}{\partial z} &= \sigma_{\rm emi}(\lambda_{\rm k}) N_2 P - \sigma_{\rm abs}(\lambda_{\rm k}) N_1 P \\ &\quad - \frac{1}{c/n_1}\frac{\partial P}{\partial t} + \frac{N_2}{\tau}\frac{hc}{\lambda_{\rm k}}\sigma^{\rm sp}_{\rm emi}(\lambda_{\rm k})\beta A_{\rm c}.\end{aligned} \tag{1}$$

The first term in Eq. (1) is related to the stimulated emission. The second and third ones are respectively the attenuation term due to material absorption and the propagation term of the power inside the fiber. The refractive index of the fiber core has a spectral dependence that is well approximated by an empirical formula [20]. We also take into account that $N_1 = N - N_2$, where $N_1$ is the population density in the non-excited state and $N$ is the total population density, which depends on the dopant concentration. In the fourth term, which accounts for the spontaneous emission, $\sigma_{\mathrm{emi}}^{\mathrm{sp}}(\lambda_{\mathrm{k}})$ represents the probability of spontaneous emissions. It has the same qualitative shape as the stimulated-emission cross section and it can be easily determined experimentally. The parameters $A_{\mathrm{c}}$ and $h$ are the cross section of the fiber core and Planck's constant, respectively. $\tau$ is the fluorescence lifetime of the dyes, which is 4.8 ns in the case of rhodamine 6G in PMMA [1]. The direction of spontaneous photons is isotropically random, and only a fraction $\beta$ will contribute to laser emission. The value of $\beta$ is equal to the fraction of spontaneous emissions lying in guided directions in the fiber, assuming that all of them will eventually contribute to stimulated emissions. For a fiber whose core refractive index is $n_1$ and whose cladding refractive index is $n_2$, we can estimate $\beta$ as the following quotient, in which $I_0$ is the light intensity (in W/sr) emitted spontaneously by an atom:

$$\beta = \frac{\int_0^{\theta_{\mathrm{c}}} I_0 2\pi \sin\theta \mathrm{d}\theta}{\int_0^{\pi} I_0 2\pi \sin\theta \mathrm{d}\theta} = \frac{1-\cos\theta_{\mathrm{c}}}{2} = \frac{n_1 - n_2}{2n_1} \qquad (2)$$

where $\theta_{\mathrm{c}}$ is the complementary critical angle [2]. For a typical PMMA fiber, $n_1 = 1.492$ and $n_2 = 1.405$, so $\beta = 0.029$ typically. The lowest possible value for $n_2$ is 1, which would correspond to $\beta = 0.165$.

In this paper, we show the influence of $\beta$ on the emission properties and on the lasing threshold. Some researchers define the threshold as the pump energy necessary for the full width at half maximum (FWHM) of the emitted intensity to drop to half the value observed when the fiber is pumped with low energy [21]. Others point out that the output energy above threshold depends linearly on the pump. Specifically, the slope of the straight line (see Fig. 1) is defined as the slope efficiency, and the lasing threshold is defined by its intersection with the axis of abscissas. We have chosen the latter definition in this paper.

To solve the aforementioned equations, we will consider that $P$ and $N_2$ are 0 at $t = 0$ and also that a Gaussian pulse of pump power is launched into the fiber at $z = 0$. The temporal FWHM of the pumped Gaussian pulse will be 5 ns in all cases to facilitate comparisons. The output powers used for the spectral graphs will represent the time-integrated powers.

The numerical scheme employed is based on finite differences. The discretization of $z$ and $t$ is uniform, i.e., with constant steps $\delta z$ and $\delta t$ such that $z_i = i\,\delta z$, $t_j = j\,\delta t$, where $i$ and $j$ are integers. For $\lambda$, variable step sizes $\delta\lambda$ can be used to allow for a finer discretization in the spectrum regions of interest. The judicious choice of these parameters is critical

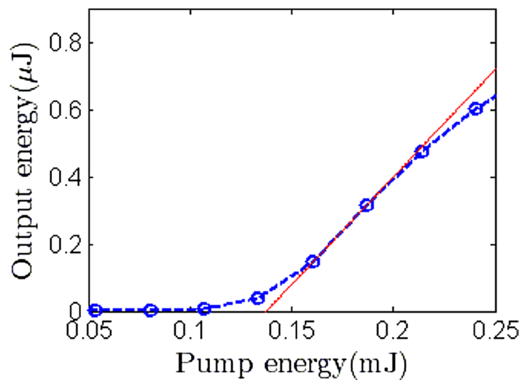


**Figure 1** (online color at: www.pss-a.com) Output energy (at 547 nm) exciting a 1 cm rhodamine-6G-doped POF with a concentration of $3 \times 10^{22}$ molecules/m$^3$.

in order to achieve numerical convergence with the desired precision and computational cost.

An array $N_2(i, j)$ is used to store the discrete values of $N_2(t, z)$, and an array $P(i, j, k)$ is used to store the light power distribution $P(t, z, \lambda)$. Finite differences, centred whenever possible, are used to approximate all the terms of the rate equations. Both arrays ($P$ and $N_2$) are filled column by column, i.e., a full column is calculated before going on to the next one.

## 3 Computational results and discussion

Firstly, let us analyze the evolution of the emitted light when a fiber of 1 cm is pumped with different energies. The pump wavelength $\lambda_{\mathrm{p}}$ is 532 nm. The origin of times has been chosen in such a way that $P_{\mathrm{p}}$ peaks at $t = 10$ ns in all cases, its value being very small at $t = 0$. The core diameter is 1 mm. The concentration of rhodamine 6G considered is $3 \times 10^{22}$ molecules/m$^3$, which is a value that can be dissolved in PMMA.

Figure 1 illustrates the expected relationship between input and output energies [7, 15]. The output energy corresponds to $\lambda = 547$ nm. When the pump energy is low, the output energy grows with a small slope, because the output is mainly due to spontaneous emission. Above threshold, population inversion occurs and spontaneously emitted photons are amplified in a single pass through the fiber. Besides, the population in the non-excited state is reduced, so the absorption coefficient $[\sigma_{\mathrm{abs}}(\lambda_{\mathrm{p}})N_1]$ decreases. Saturation effects are observed well above threshold, which reduce the slope of the curve. The pump energy for which the threshold value is attained is $E_{\mathrm{p}} = 0.1382$ mJ, as can be seen in Fig. 1.

When the pump energy launched into the fiber is increased up to the threshold value, the FWHM of the emitted spectrum narrows [Fig. 2(top)]. This fact can be explained because the number of modes decreases due to energy transfer from low power modes to those situated near the emission peak [7, 16]. In Fig. 2 (bottom), we can observe shifts of both peak and average wavelengths towards the maximum of the emitted cross section [18] when the fiber is

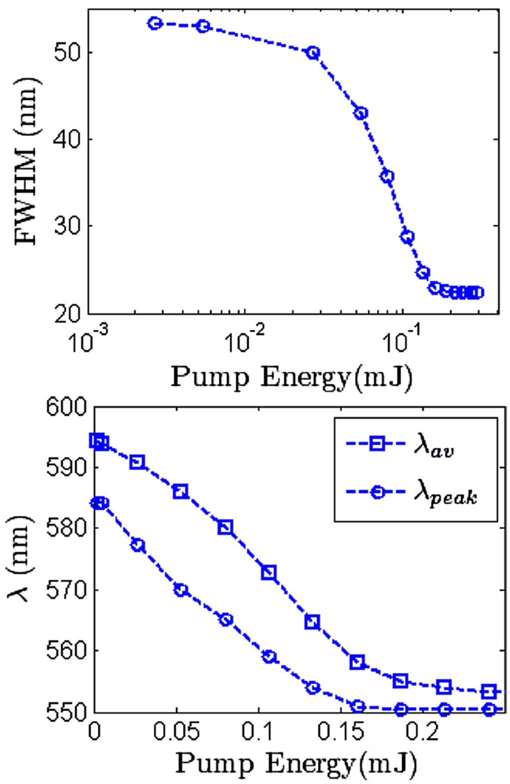


**Figure 2** (online color at: www.pss-a.com) Top: Evolution of the spectral width as a function of the pump energy. Bottom: average and peak wavelengths in the same conditions as in Fig. 1.

pumped above threshold. The average wavelength has been calculated as follows:

$$\lambda_{av} = \frac{\int_{-\infty}^{\infty} \lambda P(\lambda)\mathrm{d}\lambda}{\int_{-\infty}^{\infty} P(\lambda)\mathrm{d}\lambda} \qquad (3)$$

Let us now analyze the evolution of the emitted light as a function of $\beta$. Figure 3 shows the evolution of the output energy for different pump energies and values of $\beta$. This parameter is a function of the fiber numerical aperture (NA).

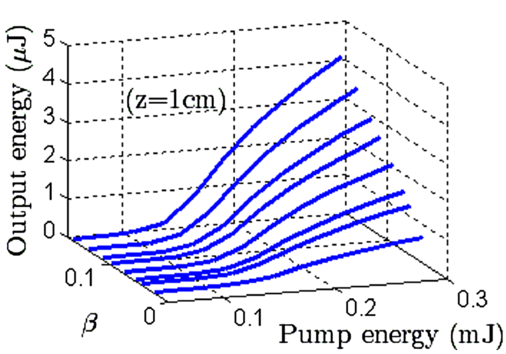


**Figure 3** (online color at: www.pss-a.com) Evolution of the output energy (at 547 nm) for different values of $\beta$ when the fiber is pumped with several pump energies, for the same concentration as in Fig. 1.

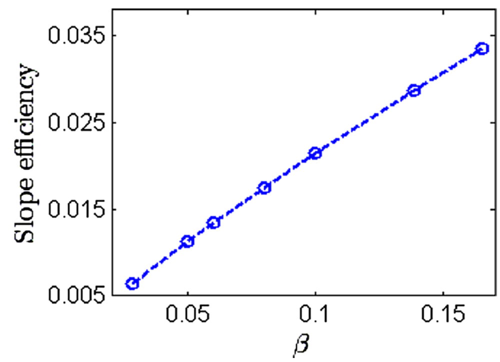


**Figure 4** (online color at: www.pss-a.com) Evolution of the slope efficiency as a function of $\beta$ (at 547 nm) in the case of a 1 cm fiber with the same concentration as in Fig. 1.

The minimum value of $\beta$ used is 0.029, which corresponds to NA = 0.5, and the maximum one is 0.165 (which happens when the refractive index of the cladding is $n_2 = 1$).

For a 1 cm fiber, Fig. 3 shows that the output energy reaches higher values when $\beta$ is higher. For high values of $\beta$, there are more photons that can propagate along the fiber and interact with excited molecules, generating a higher number of laser emissions. In consequence there is more output energy.

The slope of the curves increases strongly as $\beta$ approaches its maximum. This means that the slope efficiency has a strong dependence on the fraction of the spontaneous emission that contributes to laser emission. The efficiency when $\beta$ is the highest, in comparison with the case in which $\beta$ is small, is almost an order of magnitude higher, and the evolution of the slope efficiency with $\beta$ is almost linear (see Fig. 4). The pump energy needed to achieve the threshold is reduced when $\beta$ is higher (see Fig. 5), for the reason already explained. The same behavior has been observed, for example, in random lasers [22].

In Fig. 6, the spectral width of the emitted light has been represented as a function of $\beta$ above threshold for a fiber length of 1 cm. We can see that the spectral FWHM is reduced by 0.5 nm when the value of $\beta$ used is increased from

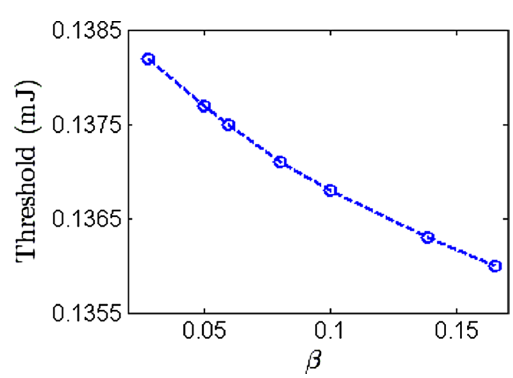


**Figure 5** (online color at: www.pss-a.com) Evolution of the threshold pump energy as a function of $\beta$ (at 547 nm) in the case of a 1 cm fiber with the same concentration as in Fig. 1.

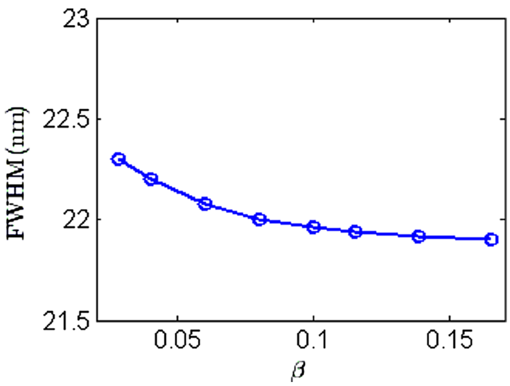


**Figure 6** (online color at: www.pss-a.com) Evolution of the FWHM as a function of $\beta$ in the same conditions as in Fig. 1.

0.029 to 0.165. We have also observed that there are no changes in the location of the spectral peak of the fiber emission when $\beta$ is increased.

**4 Conclusions** In this work, we have computationally analyzed the spectral features of the emission in dye-doped POF lasers. For this purpose, we have employed *ad-hoc* numerical algorithms. We have reproduced computationally the spectral narrowing of the output energy when the fiber is pumped above threshold. We have also calculated the evolution of the slope efficiency and of the threshold as a function of $\beta$. Lower threshold energies can be achieved when the fraction of spontaneous emission contributing to lasing ($\beta$) is higher. Spectral behaviors obtained experimentally are reproduced properly by our computational results. This work can be very useful to develop POF-based optical amplifiers and lasers.

**Acknowledgements** This work was supported by Ministerio de Ciencia e Innovación, University of the Basque Country UPV/EHU, Gobierno Vasco/Eusko Jaurlaritza, and the European Commission's 7th Framework Programme (FP7), under projects TEC2009-14718-C03-01, GIU05/03 and UE08-16, AIRHEM, FIBERSENS and S-PE09CA03, and AISHAII, respectively. The research leading to these results has also received funding from the European Commission's Seventh Framework Programme [FP7/2007-2013] under grant agreement no. 212912. One author has a research fellowship from Vicerrectorado de Euskara y Plurilingüismo, UPV/EHU, while working on his PhD.

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