Analysis of the decrease in attenuation achieved by properly bending plastic optical fibres

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Abstract: When light propagates along a plastic optical fibre that bends sharply, an important amount of power can be lost through radiation around the bend. An analysis is made of how to reduce the radiation loss by shaping the bend into a suitable curve. To calculate the total radiation loss around the bend, a geometric approach to describing propagation in multimode optical fibres is used. Radiation loss only occurs at reflection or turning points, and is described by the power-transmission coefficient T. For step-index optical fibres, T depends mainly on the angle of incidence at the interface and on the core and cladding refractive indices and, to a lesser degree, on the light wavelength. Different bend shapes can cause very different radiation losses around the bend. The purpose of this paper is to show which curves give rise to the lowest attenuations and identify the reasons.

1 Introduction

Plastic optical fibres can be bent so much that their bend radii begin to be comparable with the core diameter, in which case the radiation loss may be significant. Even maintaining the position of the two ends of the bent section and the direction of the optical fibre at both points, it is possible to find countless possible shapes for the bend. To calculate the total fraction of power radiated around each possible bend, we retain the ray paths of geometric optics and apply the corresponding transmission coefficients at the reflection points [1]. The sharper the bend, the higher the radiation loss is. On graded-index fibres, this is due to a displacement of the turning-point caustic towards the exterior of the bend, whereas on step-index fibres this behaviour is caused by an increase in the angle of incidence at the core–cladding interface. The curve that yields the least attenuation generally arises from a trade-off between improving the sharpest section and keeping up the good behaviour around the remainder of the bend. When dealing with graded-index plastic optical fibres, short sections having very small bend radii should be specially avoided, since graded profiles provoke higher attenuations than step ones when the optical fibre is bent [2]. From now on, we will centre our analysis on step profiles, which are far more common than graded ones in plastic optical fibres.

To understand the motivation for this work, it is necessary to comment on other work on bending losses. An important background knowledge of bending losses comes from the development of analytical expressions for losses in circularly curved waveguides. Different techniques have been used: analysing electric fields, dealing with fictitious currents, using perturbation methods, deriving equivalent straight waveguides or by means of a geometrical approach [3, 4]. The idea of seeking an optimum bend shape is not an original one, since the optimum bend shape of single-mode slabs, or planar waveguides, has been sought [5]. Nevertheless, we have detected a lack of specific studies on losses around arbitrarily curved plastic optical fibres. Moreover, our work helps to design electro-optical couplers based on liquid crystals and plastic optical fibres, because we clarify the dependence of bending losses on the bend shape and the refractive indices, which is essential to minimise the required coupling length between the two optical fibres [6–8].

This paper is in three parts. First, we explain the way of obtaining the total radiation loss mathematically. Then, we show the attenuation around bends shaped into different curves. Finally, we discuss the causes of the differences in attenuation when the shape of the bend changes, taking into account the effect of variations in the light wavelength and in the core and cladding refractive indices. The analysis serves to arrive at some conclusions about the best curve.

The objectives of our work are: (i) to show that power loss can be considerably improved by changing the shape of the bent section, even keeping the distance between its two ends, (ii) to clarify to what extent a curve with a very small bend radius along a very short distance could be a suitable choice to reduce the attenuation by improving the remainder of the curve, (iii) to analyse the influence of the core and cladding refractive indices on the total radiation loss, and (iv) to present an easy way of shaping the bent section which assures that the attenuation is going to be near enough to the absolute minimum.

2 Total radiation loss

In bent optical fibres, light power flows along the core within tubes of parallel rays of infinitesimal cross-section, which can undergo radiation at reflection or turn-

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ing points [9]. In the following, we are going to take the word rays to mean ray tubes. We will show that radiation loss around a bent optical fibre is mainly due to refraction of meridional rays, which are those rays that are contained in the plane defined by the core symmetry axis and the centre of curvature of the bend. Therefore, higher losses will result from meridional rays alone than from the set of meridional and skew rays together, although the qualitative behaviour of bend shapes will be similar [10]. First, we will centre our analysis on meridional rays, i.e. on the corresponding planar waveguides. Then we will compare the results with those obtained when skew rays are also taken into account. On a bent step-index optical fibre, meridional rays follow straight lines between reflections from the inner and outer interfaces, as shown in Fig. 1. As we have plotted, an LED illuminates a plastic optical fibre that bends sharply, R being the varying radius of curvature. We assume that the previous straight section is sufficiently large for the spatial steady state to be reached [11]. Therefore, the angle that each ray makes with the core symmetry axis is in the range between $-\theta_c$ and $+\theta_c$ at the entrance to the bend, θ_c being the complementary critical angle beyond which rays are refracting. At the end of the initial straight section, the light power distribution is given by the expression:

$$P(\theta) = P_0(\cos\theta)^N \tag{1}$$

P_0 being a constant. We will assume $N = 0.3$, since that is the typical value of N when the light source is an LED [12]. On the bend, rays will be refracting at either of the interfaces if $\theta > \theta_c$. Otherwise, they will be tunnelling at the outer interface and bound at the inner one [13]. We will name the core and cladding refractive indices n_{co} and n_{cl}, respectively. For PMMA optical fibres, $n_{co} = 1.491$ and, if the numerical aperture is 0.5, it follows that $n_{cl} = 1.40$, so $n_{co} = 1.06 \times n_{cl}$.

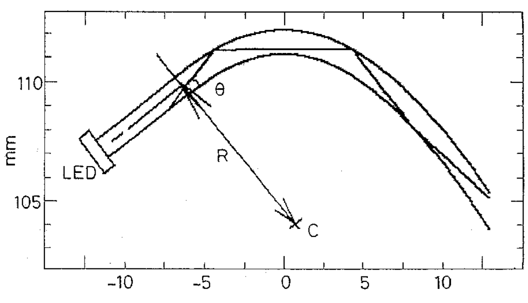

Fig. 1 *LED illuminates a plastic optical fibre which bends sharply after a straight section*
C = centre of curvature

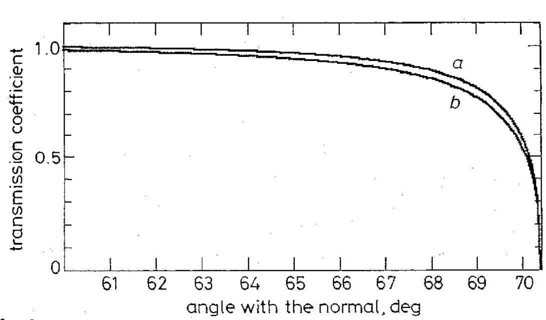

Fig. 2 *Transmission coefficient for electric fields parallel and perpendicular to the interface*
Scalar approximation for $n_{co} = 1.06\, n_{cl}$
a E perpendicular
b E parallel

The well known classical Fresnel transmission coefficient for refracting rays is independent of polarisation phenomena when $n_{co} \simeq n_{cl}$ (scalar approximation). Fig. 2 shows the transmission coefficient corresponding to PMMA fibres with the above refractive indices. This Figure serves to compare two polarisation directions at an angle of 90° to each other. The curves have been calculated for electric fields (E) parallel and perpendicular to the interface, respectively.

As the error made in using the scalar or weak-guidance approximation is still small, eqn. 2 can be employed to obtain the transmission coefficient for refracting rays [14]:

$$T = 1 - \frac{\text{power in the reflected ray}}{\text{power in the incident ray}} = \frac{4\sin\theta(\sin^2\theta - \sin^2\theta_c)^{1/2}}{\{\sin\theta + (\sin^2\theta - \sin^2\theta_c)^{1/2}\}^2} \tag{2}$$

This equation holds for all rays with $\theta_c \le \theta \le \pi/2$, neglecting that the cladding has an outer interface, which is a good approximation since most of the light energy propagates in the core.

We will designate the core radius ρ and the free-space wavelength λ. In our calculations, ρ is assumed to be 0.5mm. Let r_{rad} be the radius of the radiation caustic, where tunnelling rays appear in the cladding. The transmission coefficient for tunnelling rays [15] on a weakly guiding, step-profile fibre is given by

$$T = |T_f|\exp\left(-2k\rho L_b[\ln\{p + (p^2-1)^{1/2}\} - (p^2-1)^{1/2}/p]\right) \tag{3}$$

where $k = 2\pi/\lambda$, T_f is the analytic continuation of the classical Fresnel transmission coefficient, L_b is an invariant associated with each path around the bend, and the symbol p stands for the quotient $r_{rad}/(R + \rho)$. At the outer interface, $L_b = n_{co}\cos\theta$, while at the inner interface we have

$$L_b = [(R-\rho)/(R+\rho)]n_{co}\cos\theta$$

$|T_f|$ and r_{rad} are given by:

$$|T_f| = \{4/(n_{co}^2 - n_{cl}^2)\}(n_{co}^2 - L_b^2)^{1/2}(L_b^2 - n_{cl}^2)^{1/2}$$
$$r_{rad} = (R+\rho)(n_{co}/n_{cl})\cos\theta$$

The total radiation loss around the bend is found by integrating over all ray directions from $-\theta_c$ to $+\theta_c$ and over the core cross-section. eqn. 3 is valid provided that θ is not too close to θ_c. If rays are close enough to the critical angle to lose the majority of their power at each reflection, the transmission coefficient given by the equation may not correspond with reality. Nevertheless, when $\theta_c - \theta$ is bigger than 0.5°, T is low or, in other words, the reflection coefficient R is high, and thus necessarily accurate, so the error made in the integration over all ray directions is negligible.

3 Behaviour of rays around sections bent in different shapes

Tunnelling rays, except for those very close to θ_c, undergo very little attenuation compared with that of refracting rays [16]. Nevertheless, the attenuation is by no means negligible. The results obtained for angles of incidence close to θ_c, for PMMA and PS fibres, for which θ_c is 19.59° and 27.94°, respectively, are shown in Table 1.

To calculate the total radiation loss, we have chosen 2000 rays, which enter the bend through 20 points

Table 1: Transmission coefficient at angles of incidence close to θ_c, for PMMA and PS slabs

PMMA (θ_c = 19.59°) (n_{co} = 1.49, n_{cl} = 1.40)		PS (θ_c = 27.94°) (n_{co} = 1.59, n_{cl} = 1.40)	
θ	T	θ	T
17°	0.001	27.1°	0.001
18°	0.027	27.5°	0.100
19°	0.249	27.7°	0.509
19.2°	0.453	27.84°	0.957
19.49°	0.585	28.04°	0.285
19.69°	0.642	30°	0.761
21°	0.8163	31°	0.823

spaced out the same distance apart and distributed over the whole width of the core. At each of these points, rays under consideration make 100 different angles with the core-symmetry axis, from $-\theta_c$ to $+\theta_c$ with equal increments. Ray powers vary with θ in the manner described by eqn. 1. If we add the powers of the 2000 rays at both ends of the bend, the total radiation loss is the difference between the two sums. An increase in the number of rays gives rise to the same results, which indicates that the approximation is good. The fraction of power still present after the conic sections plotted in Fig. 3 is shown in Table 2 for PMMA and PS fibres. For PMMA optical fibres, two different light wavelengths have been considered, so as to illustrate that the total output power decreases as the wavelength increases. The reason is that the attenuation of tunnelling rays increases with wavelength. In Table 3 we show the results that we have obtained when both meridional and skew rays are considered, i.e. when ray tracing is done in a three-dimensional space, taken into account the fact that the waveguides are circular instead of planar.

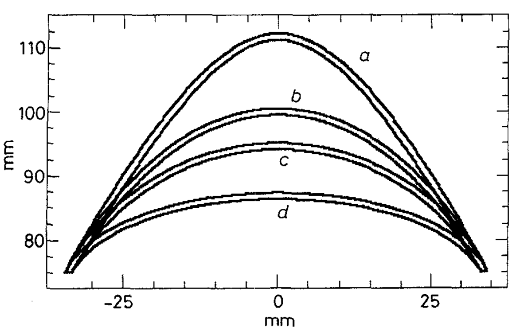

Fig. 3 *Conic curves depicting the meridional sections of four bent plastic waveguides*
a Hyperbola
b Ellipse 1
c Circumference
d Ellipse 2

Table 2: Percentage of output power relative to the input power, for PMMA and PS slabs

	PMMA (θ_c=19.59°)	PS (θ_c=27.94°)	PMMA (θ_c=19.59°)
	λ = 650nm	λ = 650nm	λ = 560nm
Hyperbola	50.2%	82.0%	50.7%
Ellipse 1	83.5%	95.2%	83.9%
Circumference	83.8%	92.5%	84.2%
Ellipse 2	61.6%	81.2%	62.1%

Table 3: Percentage of output power relative to the input power, for circular waveguides

	PMMA (θ_c=19.59°)	PS (θ_c=27.94°)
	λ = 650nm	λ = 650nm
Hyperbola	83.9%	90.9%
Ellipse 1	98.9%	100%
Circumference	98.7%	98.8%
Ellipse 2	80.3%	88.2%

4 Discussion

We can see that the output powers obtained for the four circular waveguides are higher than those obtained for the four planar waveguides. This is because only those rays close to the meridional plane undergo high radiation losses. Even so the qualitative behaviour of the four bends is the same for planar and for circular waveguides, so we will concentrate on the two-dimensional simplification in the following explanations.

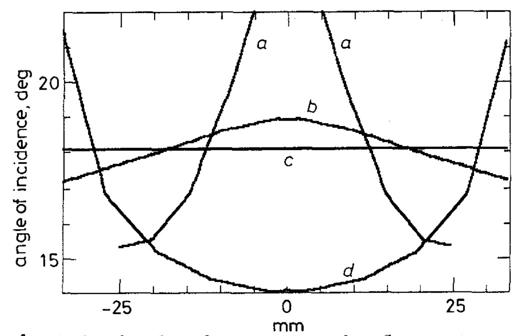

Fig. 4 *Angles of incidence for a given ray at the reflection points, around the four curves*
a Hyperbola
b Ellipse 1
c Circumference
d Ellipse 2

In Fig. 4 we show, joined by straight lines, the successive angles that a given ray makes with the outer interface at the reflection points around the four bent sections. The origin on the horizontal axis is located just halfway between the ends of the straight sections. Let us assume that the core and cladding refractive indices are those of the above PMMA optical fibre. In that case, our ray will lose very little power around the circumference, since the reflection coefficient is practically unity. In the same way, the total radiation loss will be very high around the hyperbola and the second ellipse, as the ray undergoes refraction at several points, but much lower around the first ellipse, because there is only the tunnelling effect. In general, curves for which the majority of the rays are not refracting at any point give rise to low attenuations.

The transmission coefficients corresponding to our ray around the four curves have been plotted in Fig. 5, which illustrates how high they can be when refraction occurs. In general, a circular arc yields a relatively very low radiation loss as long as θ is sufficiently smaller than θ_c for the majority of the reflections. If not, the other conic curves would be even worse. In addition, the bigger θ_c, the less the total attenuation is increased by short sharp sections, as can be checked by examining the power obtained for the hyperbola and the second ellipse in Table 2.

There are more complicated shapes for which the

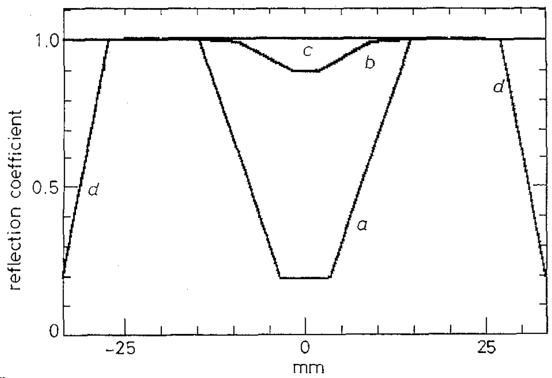

Fig. 5 *Reflection coefficients for the ray analysed in Fig. 4, around the four curves*
a Hyperbola
b Ellipse 1
c Circumference
d Ellipse 2

bend radius is always greater than that presented by the corresponding circular arc, as is the case of the oval shown in Fig. 6. Another possible option, also using a fourth-order mathematical curve, is the cardiod, which has the property that the second derivative of the bend radius with respect to arc length is constant. Although it is the curve preferred in road design, it practically coincides with a circular arc (see Fig. 6) and its minimum radius of curvature is a little smaller. In general, the local curvature has to change sufficiently slowly to reduce radiation loss.

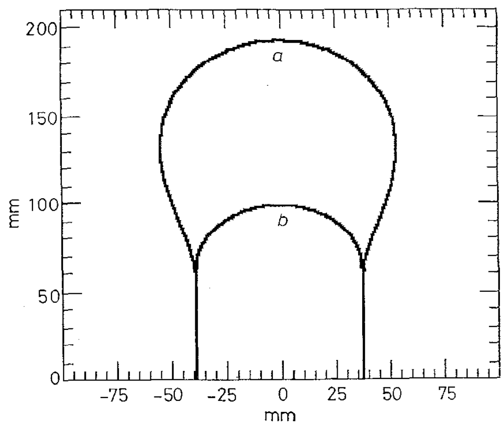

Fig. 6 *Two fourth-order curves into which the optical fibre could be shaped*
a Oval
b Cardiod

4 Conclusions

The influence of the shape of bends on the total radiation loss along two of the most common optical fibres has been analysed, as well as the effect produced by variations in the light wavelength. A circumference can cause slightly higher attenuation than an ellipse close to it, depending on the refractive indices, but curves with short very sharp sections are always bad. Even so, the easiest curve to achieve, yet one of the best, is the circumference.

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