IKERLANAK

ZONING A CROSS-BORDER CITY

by

Juan Carlos Bárcena-Ruiz and F. Javier Casado-Izaga

2015

Working Paper Series: IL. 89/15

Departamento de Fundamentos del Análisis Económico I

Ekonomi Analisiaren Oinarriak I Saila

UPV EHU

University of the Basque Country

ZONING A CROSS-BORDER CITY

Juan Carlos Bárcena-Ruiz* **and F. Javier Casado-Izaga**†
Universidad del País Vasco, UPV/EHU.

Abstract

This paper investigates zoning in a cross-border linear city that consists of two bordering towns. In each town a local regulator has a say in the location of the local firm. The incentive to gain consumers from the other town, or not to lose local consumers, may push regulators to approve only locations for firms close enough to the frontier. When zoning is costly an asymmetric equilibrium may emerge: only one regulator resorts to zoning. In the case of towns of different sizes the regulator of the larger town is the only one that zones in an asymmetric equilibrium.

JEL Classification: L13, R32, R38.

Key words: Zoning, spatial competition, location choice.

*Corresponding author. Departamento de Fundamentos del Análisis Económico I. Facultad de Ciencias Económicas y Empresariales. Universidad del País Vasco, UPV/EHU. Av. Lehendakari Agirre 83, 48015, Bilbao, Spain. Phone: + 34 94 601 38 29. Fax: + 34 94 601 38 91. *E-mails*: juancarlos.barcena@ehu.es, franciscojavier.casado@ehu.es.

†Financial support from Ministerio de Ciencia y Tecnología (ECO2012-32299) and the University of the Basque Country (EHU14/05) is gratefully acknowledged.

1 INTRODUCTION

Modern cities exercise their zoning authority to regulate the use of land within their limits. Zoning is intended to preserve the health, safety, harmonic development and welfare of the community. Local authorities regulate the types of uses permitted in each zoning district as follows, on broad lines: residential, industrial, commercial, agricultural preservation and recreation areas. Zoning design is mostly concerned with the health and safety of residents. To avoid negative externalities heavy industrial activities are kept far enough from the areas reserved for residential use, but light industry may be located closer to residential areas. Some commercial activities that generate negative externalities, for example, discos or clubs, may be located far from residential or educational areas, while others with no negative externalities may be allowed within residential areas.

From an industrial organization point of view, when there are no negative externalities zoning may be used to determine future price competition between firms. When only close enough locations are allowed firms reduce prices to try to get more consumers, and when firms can only locate in locations far enough away price competition is mitigated. The location of firms also determines consumers' transportation costs. As a result, both consumers' and producers' surpluses crucially depend on the location of firms and a regulator may use zoning in order to raise welfare. Moreover, if neighboring towns are considered a strategic effect emerges that affects the regulation of land use. This paper thus seeks to shed light on a subject that, at least to our knowledge, has not been analyzed before by the literature on zoning: the strategic reasons that could lead the regulators of bordering towns to zone their urban areas, and the optimal zoning design.[1]

The seminal paper in analyzing the location of competing firms in a linear

[1] A related literature of spatial models studies tax competition assuming perfect competitive firms and local jurisdictions in metropolitan areas located on a line. See, for example, Braid (1993, 2000).

city is "Stability in competition" by Hotelling (1929). In his model two firms simultaneously locate within a linear city and once they are located both simultaneously decide their uniform prices. Consumers transport their purchase home at a linear transportation cost. A problem arises because there is no price equilibrium when firms are close enough, as pointed out by d'Aspremont et al. (1979). They show that under quadratic transportation costs the game has a price equilibrium for all locations of firms. Linked to these models, the literature on zoning has analyzed the effect on firms' profits and consumers' transportation costs, and then on social welfare. In this regard, Lai and Tsai (2004) study asymmetric zoning: all production activities are banned throughout an exclusively residential area that comprises an area close the left-hand border of the linear city. They show that the use of zoning as an industrial regulation device reduces firms' profits, which are transferred to consumers. Zoning may reduce the distortion in the total transportation costs of consumers and enhance social welfare. Chen and Lai (2008) study symmetric central zoning under spatial Cournot competition and show that social welfare may be improved by means of zoning regulation. Colombo (2012) analyzes a spatial non-discriminatory Cournot duopoly with a central zoning area that may be asymmetric. He concludes that the optimal size of the zoning zone is zero as the consumer surplus and the profits of firms decrease as the zoning area increases. Finally, Bárcena-Ruiz et al. (2015) study a duopoly where firms set mill prices. A regulator biased towards consumers allows the two firms to locate in a central area of the city while a regulator highly concerned about firms only allows them to locate outside the city boundaries.[2]

The papers cited above consider zoning in a single town. The model analyzed

[2] Bárcena-Ruiz and Casado-Izaga (2014) also analyze optimal zoning when two firms can price discriminate between consumers. Bárcena-Ruiz et al. (2014) study optimal zoning in a mixed duopoly framework. Matsumura and Matsushima (2012) show that restricting the locations of firms to the linear city reduces consumer welfare when firms sign strategic reward contracts with their managers. Hamoudi and Risueño (2012) study zoning in a circular city where firms and consumers are located on different sides.

in this paper studies a cross-border linear city composed of two bordering towns. The town to the left belongs to a country, state or region that is linked to another linear town from another country, state or region. Each town has its own regulator that attempts to maximize social welfare, considering only the surpluses of the consumers and the producer that reside in its town. For example, Irún in Spain, and Hendaye in France, are two bordering towns which make up a joint area that could be considered as a cross-border city. In each country the regulator takes into consideration the surpluses of local consumers and firms. Another example can be found in the twin cities of Niagara Falls, Ontario, and Niagara Falls, New York, on the border between Canada and the United States of America. The city composed by the two towns is a geographical space in which local and foreign firms compete.

Local authorities are able to regulate the use of land within their towns, but market prices in developed countries are not commonly regulated, and are only exceptionally set by the state. So in our model prices are freely decided by the two firms. Each regulator knows that even though prices are set by the firms they depend crucially on the locations of the rival firms. To maintain a structure close to the duopoly models analyzed in the seminal papers, we consider that only one firm operates in each town, though it can sell its product in both markets. Each regulator may use zoning policy to achieve the optimal location of the local firm. Given that consumers are uniformly distributed, zoning affects the local firm's location by limiting some urban area to residential use only.

We find that when zoning is not very costly both regulators zone their towns in order to place the local firm close to the border between the two towns. This result is also obtained when the sizes of the two towns are not very different.[3] Zoning regulation allows the regulators to achieve the optimal locations of firms. Thus, in both towns the areas further from the frontier between them are re-

[3] Given that consumers are uniformly distributed along the linear city when one town is larger than the other it has more consumers.

served for residential use only, while the rest of the areas can share residential and commercial uses. Each regulator pushes the local firm to locate closer to the frontier in order to try to gain consumers from the other town and to discourage local consumers from buying non local products. When the two towns are zoned and the same size, the locations that the regulators choose are closer to the frontier than those that minimize local consumers' transportation costs. Given these equilibrium locations it is not of interest to the regulator to locate the local firm closer to its rival because the increase in transportation costs for local consumers outweighs the foreign revenue that the local firm obtains.[4] When the two towns are zoned but of different sizes, the regulator of the larger town prefers the local firm to locate very close to its rival to reduce the loss of revenue from local consumers.[5]

But zoning costs may be meaningful: there are costs linked to studying regulation, to designing the maps that plot the different uses of different areas in the town, to uploading those maps and regulations to the web site that offers the information; and there are costs for the staff that informs about and watch for non-fulfilment of the norms, to mention just a few. In this case new equilibria may emerge: both regulators may refuse to use zoning when it is very costly, or the regulator of the smaller town may refuse zoning while the regulator of the larger town applies zoning to try to reduce the loss of local consumers who buy from the non local firm. This asymmetric equilibrium in which one town is zoned while the other is not may also emerge when the two towns are of the same size. However, when towns are of different sizes and there is an asymmetric equilibrium the regulator that resorts to zoning is the one from the larger town. This is because the larger town has more urban space and local revenues and when it is zoned the loss of revenue from local consumers supplied by the firm

[4] When the firms are located very close together the incentive to come close to the rival decreases as prices are lower and the revenues that the firm may obtain in the other town are lower.

[5] By locating closer to its rival, a firm reduces equilibrium prices and thus the loss of local consumers is not so important from a social welfare point of view.

located in the smaller town is small.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 focuses on the analysis of a cross-border city that consists of two identical towns. Section 4 studies the case of two asymmetric towns, and Section 5 draws conclusions.

2 THE MODEL

Consumers are distributed uniformly and with unitary density along a cross-border linear city that consists of two bordering towns. The town to the left extends along the interval $[0, \alpha]$, while the town to the right extends along the interval $[\alpha, 1]$. As a result, the two towns take the form of a cross-border linear city along the segment $[0, 1]$ with a frontier zone between them at α. Figure 1 summarizes the situation.

[INSERT FIGURE 1 AROUND HERE]

Each consumer buys only one unit of the good from the firm with the lowest delivered price, considered as the mill price plus transportation cost. Consumers transport their purchase home at a cost td^2, where t is a positive constant and d is the distance traveled from the firm's location to the consumer's home. Each consumer derives a surplus from consumption, gross of price and transportation costs, denoted by s. Thus, the gross consumer surplus in town 1 is αs and in town 2 is $(1 - \alpha)s$. We assume that s is large enough for all consumers to buy one unit of the product each.

There are two firms indexed by i $(i = 1, 2)$ competing in the linear city. Let x_i denote the location of firm i. Firm 1 is the local firm of town 1 and it is located within the limits of that town: $x_1 \in [0, \alpha]$. Firm 2 is the local firm of town 2 and it is located within the limits of town 2: $x_2 \in [\alpha, 1]$. Locations are a long term decision: once firms choose them they cannot be changed. Firms' production costs are normalized to zero.

The timing of the game is the following: in the first stage the regulators simultaneously decide whether to apply zoning or not. When there are zoning regulations the locations allowed for the local firm are announced. In the second stage both firms simultaneously decide their locations within their towns, taking into account any zoning constraints that may exist. In stage three, both firms simultaneously set uniform prices. We solve the game by backward induction to find the subgame perfect Nash equilibria.

Let p_i denote the price set by firm i $(i = 1, 2)$. The location of the consumer who is indifferent as regards buying from one firm or the other, $\overline{x}$, can be obtained from the following condition:

$$p_1 + t(\overline{x} - x_1)^2 = p_2 + t(\overline{x} - x_2)^2. \tag{1}$$

From (1) the following is obtained:

$$\overline{x} = \frac{p_2 - p_1}{2t(x_2 - x_1)} + \frac{x_2 + x_1}{2}. \tag{2}$$

Thus, the respective demands of firms 1 and 2 when they are located at different points (that is when they are not located at α), are given by q_1 and q_2:

$$q_1 = \begin{cases} \overline{x} & \text{if } 0 \leq \overline{x} \leq 1 \\ 1 & \text{if } \overline{x} > 1 \\ 0 & \text{if } \overline{x} < 0 \end{cases}, \quad q_2 = \begin{cases} 1 - \overline{x} & \text{if } 0 \leq 1 - \overline{x} \leq 1 \\ 1 & \text{if } 1 - \overline{x} > 1 \\ 0 & \text{if } 1 - \overline{x} < 0 \end{cases} \tag{3}$$

In order to get a subgame perfect Nash equilibrium, we first solve the third stage of the game to get equilibrium prices. In this stage firms simultaneously set their prices, and then their outputs are determined by expression (3). The objective function of firm i is:

$$\pi_i(p_i, p_j) = p_i q_i, i \neq j; i, j = 1, 2. \tag{4}$$

Substituting (2) and (3) in (4) and taking the first order condition with respect to prices for each firm we obtain the equilibrium prices when both firms sell the good:[6]

[6] The second order conditions of the problems that we analyze are always satisfied.

$$p_1 = \frac{t}{3}(x_2 - x_1)(2 + x_1 + x_2), \ p_2 = \frac{t}{3}(x_2 - x_1)(4 - x_1 - x_2). \quad (5)$$

The demands of firms 1 and 2 can now be obtained using (2) and (3):

$$q_1 = \frac{2 + x_1 + x_2}{6}, \ q_2 = \frac{4 - x_1 - x_2}{6}. \quad (6)$$

From expressions (4) to (6) the profits of firms 1 and 2 are:

$$\pi_1 = \frac{t}{18}(x_2 - x_1)(2 + x_1 + x_2)^2, \ \pi_2 = \frac{t}{18}(x_2 - x_1)(4 - x_1 - x_2)^2. \quad (7)$$

The results of the third stage of the game do not depend on the relative sizes of the towns, but only on zoning regulations as they may condition the values of x_1 and x_2. Expressions for equilibrium prices (5), firms' demands (6) and firms' profits (7) can be used throughout the different scenarios analyzed in this paper.

When a regulator decides to zone its town the local firm must locate within the area set by the regulator. Given that consumers are distributed along the city, each town may have two different zones: an exclusively residential area and another that can be shared by consumers and the local firm.

Each regulator's objective function W_i is the sum of the local consumer surplus and the profits of the local firm π_i:

$$W_i = CS_i + \pi_i, \ i = 1, 2. \quad (8)$$

To obtain the objective function for each regulator, first consider that the locations and prices set by the two firms are such that $\overline{x} > \alpha$. In that case, town 1's social welfare (W_1) comprises the gross surplus of local consumers αs, minus the transportation costs of local consumers (who all buy from firm 1), plus the profits that firm 1 captures from its sales in town 2. The sale incomes of firm 1 in its town are only a monetary transfer from domestic consumers to the local firm. When $\overline{x} = \alpha$ each market is covered by the local firm. Finally, when

$\overline{x} < \alpha$, town 1's social welfare is the gross surplus αs, minus the transportation costs of local consumers (those who live between 0 and $\overline{x}$ buy from firm 1 so they transport their purchase from x_1, and the rest buy from firm 2 and transport their purchase from x_2), minus the profits that firm 2 captures from sales in town 1. Following a similar reasoning W_2 can be written. Equations (9) and (10) show these expressions:

$$W_1 = \begin{cases} \alpha s - \int_0^{\alpha} t(x-x_1)^2 dx + p_1(\overline{x}-\alpha) & \text{if } \overline{x} > \alpha \\ \alpha s - \int_0^{\alpha} t(x-x_1)^2 dx & \text{if } \overline{x} = \alpha \\ \alpha s - \int_0^{\overline{x}} t(x-x_1)^2 dx - \int_{\overline{x}}^{\alpha} t(x-x_2)^2 dx - p_2(\alpha-\overline{x}) & \text{if } \overline{x} < \alpha \end{cases} \quad (9)$$

$$W_2 = \begin{cases} (1-\alpha)s - \int_{\alpha}^{\overline{x}} t(x-x_1)^2 dx - \int_{\overline{x}}^{1} t(x-x_2)^2 dx - p_1(\overline{x}-\alpha) & \text{if } \overline{x} > \alpha \\ (1-\alpha)s - \int_{\alpha}^{1} t(x-x_2)^2 dx & \text{if } \overline{x} = \alpha \\ (1-\alpha)s - \int_{\alpha}^{1} t(x-x_2)^2 dx + p_2(\alpha-\overline{x}) & \text{if } \overline{x} < \alpha \end{cases} \quad (10)$$

Figure 2 depicts W_1 when $\overline{x} > \alpha$. The lined area above $p_1 + TC_1$ (from $x = 0$ to $x = \alpha$) measures the consumer surplus in town 1. The lined area below p_1 (from $x = 0$ to $x = \overline{x}$) measures the profits of firm 1: local profits ($p_1\alpha$) and profits from its sales to residents in town 2 ($p_1(\overline{x} - \alpha)$).

[INSERT FIGURE 2 AROUND HERE]

Figure 3 depicts W_2 when $\overline{x} > \alpha$. The lined areas above $p_1 + TC_1$ (from $x = \alpha$ to $x = \overline{x}$) and above $p_2 + TC_2$ (from $\overline{x}$ to 1) measure the consumer surplus in town 2, and the lined area below p_2 (from $x = \overline{x}$ to $x = 1$) measures the profits of firm 2.

[INSERT FIGURE 3 AROUND HERE]

Since each regulator may be interested in capturing consumer revenues from the other town, or in not losing some revenues from local consumers, we study optimal locations and welfare from the regulator's point of view. We describe the zoning mechanism that achieves the optimal locations depending on the behavior of the other regulator. Finally, we study the regulator's incentives to zone a town when zoning is costly considering that the two towns may be of the same or different sizes.

3 ANALYSIS OF THE SYMMETRIC CITY: BOTH TOWNS ARE OF THE SAME SIZE

In this section we show the results for the symmetric case, that is when $\alpha = 1/2$. When there are no restrictions on the location of the firms, given that $\partial\pi_1/\partial x_1 < 0$ and $\partial\pi_2/\partial x_2 > 0$, both firms locate at the end points of the linear city (i.e. $x_1 = 0$ and $x_2 = 1$) as shown by d'Aspremont et al. (1979). Let superscript NN denote that towns 1 and 2 are not zoned. In that case firms share the market equally and obtain the same profits. It is easy to see that the profits of each firm, the price set by each one, and social welfare in each town are given by:

$$\pi_i^{NN} = \frac{t}{2},\ p_i^{NN} = t,\ W_i^{NN} = \frac{s}{2} - \frac{t}{24},\ i = 1, 2.$$

Substituting equilibrium prices (5) in (6) gives $q_1 \gtreqless q_2$ and $p_1 \gtreqless p_2 \Leftrightarrow x_1 \gtreqless 1 - x_2$. That is, in equilibrium the firm located closer to the middle of the market has the greater market share. This firm sets a higher price than its rival but the location advantage guarantees it the highest demand. Thus, when the two towns are of the same size the firm located closer to the middle of the market captures some consumers from the other town.

Next, we study the case where both towns are zoned, and then we solve the case where only one town is zoned. Finally, we provide the optimal zoning mechanism for each case and show the regulators' incentives to zone their cities.

Firms Locations with regulatory Restrictions in both Towns

From the preceding analysis it is easy to obtain the main results. Note that $\partial\pi_1/\partial x_1 < 0$ and $\partial\pi_2/\partial x_2 > 0$, so the location that maximizes the profits of a firm for a given location of its rival when there are zoning constraints is the location that maximizes the distance between the two firms, taking into account that each firm must locate at the locations permitted in its town. Firm 1 will try to locate as close as allowed to the left border of the city ($x_1 = 0$). Similarly, firm 2 will try to locate as close to the right border of the city ($x_2 = 1$) as the zoning constraint allows.

Now assume that there are regulatory constraints within the two towns. As a benchmark we consider the locations of the firms that maximize the joint welfare of the two towns. In that case it is well known that $x_1 = 1/4$ and $x_2 = 3/4$.[7] However, if one firm adopts such a location the regulator of the other town may increase its welfare by pushing its firm towards the middle of the market, thus gaining market share at the expense of the rival firm. Taking into account local welfare, regulators will encourage their firms to adopt locations in order to sell to consumers of the rival town. This gives rise to an effect that pushes both firms towards the middle of the market. Proposition 1 shows equilibrium locations and describes the zoning mechanism.

PROPOSITION 1: *When* $\alpha = 1/2$ *the equilibrium locations are:* $x_1^* = 5/16$ *and* $x_2^* = 11/16$. *The zoning regulations forbid firm 1 to locate within the interval* $[0, 5/16)$ *and firm 2 to locate within the interval* $(11/16, 1]$.

Proof: Assume that firm 1 is located closer to the middle of the market and may thus capture some consumers that live in town 2. In that case, $W_1 = s/2 - \int_0^{1/2} t(x - x_1)^2 dx + p_1(\overline{x} - 1/2)$ and the regulator's reaction function is: $x_1(x_2) = (-20 - 2x_2 + \sqrt{478 + 80x_2 + 16{x_2}^2})/6$. Solving similarly for the regulator of town 2 gives $W_2 = s/2 - \int_{1/2}^{\overline{x}} t(x - x_1)^2 dx - \int_{\overline{x}}^{1} t(x - x_2)^2 dx - p_1(\overline{x} - 1/2)$, so the reaction function is: $x_2(x_1) = (10 - x_1 - 2\sqrt{16 - 5x_1 + {x_1}^2})/3$. Operating gives $x_1 = 5/16$ and $x_2 = 11/16$. This means that firms 1 and 2 adopt a symmetric location and obtain the same market share. Due to the symmetry of the model,

[7] Consumers' transportation costs are minimized when each firm is located in the middle of its town and then joint social welfare is maximized.

the same is obtained when it is assumed that it is firm 2 which may capture consumers from town 1.

Proof. Firms locate optimally when the zoning regulations forbid firm 1 to locate within the interval $[0, 5/16)$ and firm 2 to locate within the interval $(11/16, 1]$. To show that the zoning regulation works note that given the location of firm 2, firm 1 locates as far as possible from its rival to mitigate price competition and thus maximize profits. With the suggested zoning regulation firm 1 chooses the closest location to $x = 0$ that is allowed, in this case $x_1^* = 5/16$. From firm 2's point of view the analysis is symmetrical and thus $x_2^* = 11/16$. ∎

Proposition 1 shows that firms locate symmetrically with regard to the common frontier between the two towns. Moreover, the left zone of town 1, $x \in [0, 5/16)$, and the right zone of town 2, $x \in (11/16, 1]$, are for residential use only. The remaining zones of the towns (i.e. the central area of the linear cross-border city) may be shared by firms and consumers. In equilibrium, given that firms are located symmetrically, both firms sell the same quantities and obtain the same profits. The locations that the regulators choose for the two firms are such that both are closer to the border between the two towns than the locations that minimize the transportation costs of local consumers. From these equilibrium locations it is not in the interest of the regulator to locate the local firm closer to the rival. This is because the increase in local consumers' transportation costs and the reduction in the prices paid by non local consumers outweigh the additional market share that the firm obtains. Note that when firms are located very close together the incentive of the regulators to locate the local firm closer to the rival are reduced, as both the prices and the revenues that the firm may obtain in the other town are lower.

Let superscript ZZ denote that both towns are zoned. Assume that zoning has a fixed cost f that must be paid by the regulator, which for the sake of simplicity is the same in both countries. It is easy to see that the profits of each firm, the price set by each one, and social welfare in each town are given by:

$$\pi_i^{ZZ} = \frac{3t}{16},\ p_i^{ZZ} = \frac{3t}{8},\ W_i^{ZZ} = \frac{s}{2} - \frac{19t}{1536} - f,\ i = 1, 2.$$

Firms Locations with regulatory Restrictions in one Town Only

Now assume that there are location constraints in one town only. Given that both towns are of the same size, it can assumed with no loss of generality that only the regulator of town 1 restricts the location of its firm. The results when only firm 2 is zoned are symmetric. Proposition 2 shows equilibrium locations.

PROPOSITION 2: *When $\alpha = 1/2$ the equilibrium locations of the two firms are: $x_1^* = (-220 + \sqrt{574})/6$ and $x_2^* = 1$. Zoning regulation in town 1 does not allow firm 1 to locate within the interval $\left[0, (-220 + \sqrt{574})/6\right)$.*

Proof: The preceding analysis shows that in equilibrium firm 2 is located at $x_2 = 1$ since it is not subject to regulations. Note that although in stage 2 firms decide their locations simultaneously, the regulator of town 1 chooses the location restrictions before, in stage 1. Given that firm 2 always locates at $x_2 = 1$ the result of the game is the same when the social welfare of town 1 is maximized subject to $x_2 = 1$, as when the reaction function of the regulator of town 1 is calculated for $x_2 = 1$. For the sake of simplicity, we use the latter approach.[8]

Proof. As firm 1 maximizes welfare it always locates closer to the middle of the market than firm 2. Thus, the regulator of town 1 chooses to obtain consumers from town 2. The preceding analysis shows that $x_1(x_2) = (-20 - 2x_2 + \sqrt{478 + 80x_2 + 16x_2{}^2})/6$. Thus, solving the first order condition for the regulator of firm 1 when $x_2 = 1$ gives this solution: $x_1^* = (-220 + \sqrt{574})/6 \simeq 0.3264$ and $x_2^* = 1$. The zoning regulation in town 1 guarantees that firm 1 locates at $x_1^* \simeq 0.3264$ because it is not allowed to locate within the interval

[8] This approach is valid as long as $x_2 = 1$. When x_2 depends on x_1 this way of finding the solution is not valid as one has to proceed backwards.

$\big[0, (-220+\sqrt{574})/6\big)$ and it thus locates as far as allowed from firm 2 at $x_1^* \simeq 0.3264$. ∎

Given that only town 1 is zoned, firm 1 locates closer to the border between the two towns than when the two firms are regulated. Thus, firm 1 gains market share at the expense of its rival and obtains greater profits. When only one town is zoned the rival locates at the opposite end. The equilibrium location chosen by the regulator of the firm in the zoned town is such that locating the firm closer to its rival would produce a positive effect, a gain of market share. However, there are two negative effects: the increase in the transportation costs of local consumers and another negative effect that emerges because the firm must reduce the prices paid by all the consumers of the other town and not only by new ones. In the equilibrium location for the zoned firm these two negative effects balance the gain of more non local consumers.

Superscript ZN (or NZ) denotes that town 1 is zoned and town 2 is not (or town 1 is not zoned and town 2 is). It is easy to compute the profits, outputs and prices set by the two firms, and the social welfare of each town for the ZN case:

$$\begin{aligned}
\pi_1^{ZN} &= \frac{(10556-407\sqrt{574})t}{1944},\ \pi_2^{ZN} = \frac{(53396-2207\sqrt{574})t}{1944}, \\
q_1^{ZN} &= \frac{\sqrt{574}-4}{36},\ q_2^{ZN} = \frac{40-\sqrt{574}}{36}, \\
p_1^{ZN} &= \frac{(16\sqrt{574}-343)t}{54},\ p_2^{ZN} = \frac{(847-34\sqrt{574})t}{54}, \\
W_1^{ZN} &= \frac{s}{2}+\frac{7(82\sqrt{574}-1957)t}{1944}-f,\ W_2^{ZN} = \frac{s}{2}-\frac{(1445\sqrt{574}-34526)t}{1296}.
\end{aligned}$$

The town that zones obtains a greater welfare if the cost of zoning is not high (i.e. $f < 0.0996t$). The case when only town 2 is zoned (NZ) is defined similarly.

To Zone or not to Zone: The Regulators' Choice

Having solved the third and second stages of the game taking into account the optimal location restrictions, we now study whether or not the regulators have incentives to zone their towns. From the results obtained in the four different subgames we get the following pay-off matrix (results are approximated to make the analysis more intuitive). This matrix shows the social welfare obtained by the two towns depending on whether or not their regulators decide to zone.

[INSERT TABLE 1 AROUND HERE]

Proposition 3 shows the equilibria of the first stage of the game.

PROPOSITION 3: *When the fixed cost is such that* $f \leq 0.06t$ *both towns are zoned. When* $0.06t \leq f \leq 0.069t$ *one regulator zones its town and the other does not. When the fixed cost is very high* $f \geq 0.069t$ *both regulators decide not to zone.*

Proof: $W_1^{ZZ} - W_1^{NZ} = 5(-221069 + 9248\sqrt{574})t/41472 - f > 0$ if and only if $f < 5(-221069 + 9248\sqrt{574})t/4147 \backsimeq 0.06t$. $W_1^{ZN} - W_1^{NN} = (-6809 + 287\sqrt{574})t/972 - f > 0$ if and only if $f < (-6809 + 287\sqrt{574})t/972 \backsimeq 0.069t$. For town 2 the results are symmetric. As a result, when $f < 0.06t$ zoning is a dominant strategy for both firms, and so (ZZ) is the only equilibrium. When $0.06t < f < 0.069t$, for both firms the best response to zoning is not to zone and the best response to not zoning is to zone; in this case there are two equilibria: (ZN) and (NZ). When $f > 0.069t$ not zoning is a dominant strategy for both firms, so (NN) is the only equilibrium. It is straightforward to compute that along the lines that limit each zone, $f = 0.06t$ and $f = 0.069t$, the equilibria at the areas located on both sides of each line can be obtained.

Because zoning is a dominant strategy when it is free, the only equilibrium is when both regulators zone their towns: (ZZ). This is because the zoning mechanism permits the regulators to locate the firms optimally, obtaining the greatest welfare. However, when $f > 0$ other equilibrium configurations may appear since the increase in welfare obtained by zoning may be offset by the fixed cost of zoning. When the fixed cost is low enough, $f < 0.06t$, zoning is a dominant strategy and (ZZ) is still the only equilibrium. In this case, the increase in welfare by zoning still offsets its cost for both towns. When $0.06t < f < 0.069t$ two asymmetric equilibria emerge, and one regulator zones but the other does not: (ZN) and (NZ).[9] In this case, when the other town is not zoned the gain from zoning is positive as the fixed zoning cost is not very high. But once that one town is zoned, welfare gains from zoning the other town are negative because the high fixed cost cannot be covered given that the rival firm is zoned and will then behave very aggressively. Finally, when the fixed cost is very high, $f > 0.069t$, in equilibrium both regulators decline to zone: (NN). In this last case the cost of zoning is so high that it offsets the increase in welfare in both towns.

4 ROBUSTNESS ANALYSIS: TOWNS ARE OF DIFFERENT SIZES

The preceding section studies the symmetric case, but many cross-border towns are of different sizes. For example, in the aforementioned case of Irún and Hendaye the population of Hendaye is around 23% of that of Irún. In the case of the twin cities of Niagara Falls the population on the US side is around 60% of that on the Canadian side. To model this asymmetric situation we study the case when $\alpha = 1/3$, that is, the town on the left is half the size of the town on the right. To complete this analysis, at the end of Section 4 we show the results when the two towns are very different in size, specifically $\alpha = 1/10$. This enables us to study whether or not the results obtained when the two towns are of same size are robust to changes in their relative sizes.

Firms' locations when neither firm is constrained are the same as before: $x_1 = 0$ and $x_2 = 1$. We consider now that both towns are zoned. This setting is easy to solve because substituting (5) in (2) gives $\overline{x} = (2 + x_1 + x_2)/6$. As a consequence, in equilibrium the firm located on the left (in the small town) always captures some consumers from the large town: $\overline{x} > \alpha = 1/3$. Given that

[9] As seen in Section 3.2, the town that zones obtains greater social welfare and thus both towns want to be the one that zones. If the zoning cost is sufficiently higher in one than in the other, the town with the lower cost zones and the other does not. Thus there is only one asymmetric equilibrium.

firm 1 captures some consumers that live in town 2, social welfare in town 1 is: $W_1 = s/3 - \int_0^{1/3} t(x - x_1)^2 dx + p_1(\overline{x} - 1/3)$, and the regulator's reaction function is: $x_1(x_2) = (-8 - x_2 + \sqrt{70 + 16x_2 + 4x_2{}^2})/3$. Solving similarly for regulator 2, taking into account that this regulator chooses the value of x_2 that maximizes $W_2 = 2s/3 - \int_{1/3}^{\overline{x}} t(x - x_1)^2 dx - \int_{\overline{x}}^{1} t(x - x_2)^2 dx - p_1(\overline{x} - 1/3)$, it is obtained that $x_2(x_1) = (32 - 3x_1 - 2\sqrt{184 - 48x_1 + 9x_1{}^2})/9$. The equilibrium locations are determined where the two reaction functions cross: $x_1^* \simeq 0.1315$ and $x_2^* \simeq 0.5482$.

To achieve equilibrium locations zoning regulations must forbid firm 1 to locate within the interval $[0, 0.1315)$ and firm 2 to locate within the interval $(0.5482, 1]$. Zoning regulation works because given the location of firm 2, firm 1 maximizes its profits by locating as far as possible from its rival. Thus, firm 1 chooses the closest location allowed to $x_1 = 0$, that is: $x_1^* \simeq 0.1315$. For firm 2 the analysis is similar. The location chosen by the regulator of the large town is very close to the rival firm so as to reduce the loss of local consumers; locating closer to the rival means that equilibrium prices are lower and the loss of revenue from local consumers is not so substantial for social welfare.

Consider now that only one town is zoned. Towns are different in size and zoning restrictions are different depending on whether locations are constrained in the small town or the large one. Now assume that only town 1 is zoned. From the preceding analysis it is known that in equilibrium when firm 2 is not constrained it locates at $x_2 = 1$, so the game can be solved as in Proposition 2. The regulator of town 1 knows that firm 1 gets some consumers from market 2, so evaluating $x_1(x_2) = (-8 - x_2 + \sqrt{70 + 16x_2 + 4x_2{}^2})/3$ for $x_2 = 1$ gives the solution $x_1^* \simeq 0.1623$ and $x_2^* = 1$. The zoning regulation guarantees that firm 1 locates at $x_1^* \simeq 0.1623$ because this firm is not allowed to locate in the interval $[0, 0.1623)$. Comparing this result with that obtained without zoning, the firm from the small town locates closer to the frontier between the two towns in order to gain market share. The firm from the large town is located very far away because it is not regulated, so prices are very high. So the regulator of the small town finds it of interest to make the local firm move towards the frontier.

The results when only town 2 is zoned are straightforward taking into account that $x_1 = 0$,[10] so $x_2(x_1) = (32 - 3x_1 - 2\sqrt{184 - 48x_1 + 9{x_1}^2})/9 \simeq 0.5412$. When the optimal location of firm 2 is decided by a regulator while firm 1 decides its own location, the locations chosen are $x_1^* = 0$ and $x_2^* \simeq 0.5412$. Zoning regulation does not allow firm 2 to locate within the interval $(0.5412, 1]$. Comparing this result with that obtained without zoning, the local firm from the large town locates closer to the border between the two towns to reduce the loss of revenue from local consumers. This is achieved by reducing both the loss of consumers and the prices paid by local consumers that buy from the foreign firm.

Consider now the regulators' incentives to zone their towns. For the sake of simplicity, assume that zoning has a fixed cost f which is the same in both towns. Taking into account that $\overline{x} > \alpha$, expressions (9) and (10) give the solutions for each pair of locations. The matrix below summarizes the social welfare obtained by the two towns in each subgame.

[INSERT TABLE 2 AROUND HERE]

Proposition 4 shows the equilibria in the first stage of the game.

PROPOSITION 4: *When the fixed cost is such that* $f \leq 0.0097t$ *both towns are zoned. When* $0.0097t \leq f \leq 0.1506t$ *the regulator of the large town zones it and the regulator of the small town does not. When the fixed cost is very high* $f \geq 0.1506t$ *both regulators decline to zone.*

Proof: It is straightforward to compute that from the viewpoint of regulator 1 $W_1^{ZZ} \gtreqless W_1^{NZ} \Leftrightarrow f \lesseqgtr 0.0097t$, and $W_1^{ZN} \gtreqless W_1^{NN} \Leftrightarrow f \lesseqgtr 0.0137t$. From the viewpoint of regulator 2 $W_2^{ZZ} \gtreqless W_2^{ZN} \Leftrightarrow f \lesseqgtr 0.1399t$, and $W_2^{NZ} \gtreqless W_2^{NN}$

[10] This approach is valid since $x_1 = 0$ is always the case when town 1 is not zoned. When x_1 depends on x_2 it is necessary to proceed backwards.

$\Leftrightarrow f \lesseqgtr 0.1506t$. As a result, when zoning is not very costly, $f < 0.0097t$, as zoning is a dominant strategy the two regulators zone their towns: (ZZ). When $0.0097t < f < 0.1506t$ an asymmetric equilibrium emerges: the regulator of the small town (town 1) does not zone its town but the other does: (NZ). Finally, when the fixed cost is very high, $f > 0.1506t$, the only equilibrium is that in which both regulators decline to zone: (NN). Along the lines that limit each zone, $f = 0.0097t$ and $f = 0.1506t$, the equilibria in the areas on both sides of the line are obtained.

When zoning entails a cost but that cost is not too high, $f \in (0.0097t, 0.1506t)$, there is an asymmetric equilibrium in which the regulator of the large town zones it but the other does not. This is because the large town has more urban space and local revenues, so the incentive to zone the town in order to reduce the loss of revenue from local consumers is very high. Once the large town is zoned the welfare gains from zoning the small town are negative when $f > 0.0097t$ because of the high fixed cost. Therefore, compared with the case when the two towns are identical, the asymmetry in the size of the towns permits one asymmetric equilibrium to be selected: only the large town is zoned.

The case analyzed of two towns with different sizes when $\alpha = 1/3$ reflects a situation such that there is still an interior solution for the location of firm 1 when the small town is zoned. Our findings show that when the two towns are very different in size (for example when $\alpha = 1/10$), the firm located in the small town is pushed to locate as far as possible from its rival when the small town is zoned. As a consequence, its regulator does not need to zone. With regard to the regulator of the large town, by zoning town 2 it could obtain the optimal location of firm 2 when the fixed cost f is not very high; that location is: $x_2 = 0.4$. Thus, when the fixed zoning cost is positive (and then the regulator of town 1 decides not to zone the small town) but not very high, the regulator of the large town decides to zone it. Obviously when the fixed zoning cost is very high both regulators decline to zone their towns. Our conclusion is that when the two towns are extremely different in size there is no equilibrium in which both towns are zoned because zoning is not useful in the small town.

5 CONCLUSIONS

Zoning regulations are a useful device available to local authorities to regulate not only the use of land to preserve health or safety, but also to achieve social welfare goals. Regulators may find it of interest to zone their towns in order to reduce local consumers' transportation costs, to obtain revenue from consumers located in a neighboring town, or to reduce the number of local consumers that buy products from other towns. In the absence of zoning regulation firms try to locate far from their rivals to mitigate price competition as long as they do not take into account the nationality of the consumers, because they only consider their total profits. From a regulator's point of view the local or foreign nature of consumers is important: local consumers that are served by a local firm transfer their money to a local producer with no loss of welfare because consumer demand is inelastic. When local consumers are served by a foreign firm, the amount that they pay reduces local social welfare.

We consider a linear city with two cross-border towns and find that when the two towns are of the same size the regulators use zoning to push firms to capture consumers from the foreign town and to keep local consumers from buying foreign products. Regulators may find it of interest to zone their towns when it is not very costly. Under zoning regulations firms are located closer than they would be in the absence of such regulations. When the two towns are different in size this effect is also present and the regulator of the larger town has a greater incentive to zone its town. This is because it has more urban space and local revenue and by this means the regulator reduces the loss of revenue from local consumers who do not buy local products and also reduces the transportation costs of local consumers. It is impossible to find an equilibrium in which only the small town is zoned. Finally, when the two towns are very different in size and zoning is costly it is not useful in the small town, so there is no equilibrium in which both towns are zoned. But in this setting zoning is still a useful device for the larger town.

REFERENCES

Bárcena-Ruiz, Juan Carlos, and F. Javier Casado-Izaga. 2014. "Zoning under Spatial Price Discrimination." *Economic Inquiry*, 52, 659-665.

Bárcena-Ruiz, Juan Carlos, F. Javier Casado-Izaga, and Hamid Hamoudi. 2014. "Optimal Zoning of a Mixed Duopoly." *Annals of Regional Science,* 52, 141-153.

Bárcena-Ruiz, Juan Carlos, F. Javier Casado-Izaga, Hamid Hamoudi, and Isabel Rodríguez. 2015. "Optimal Zoning in the Unconstrained Hotelling Game." *Papers in Regional Science,* forthcoming. DOI: 10.1111/pirs.12132.

Braid, Ralph M. 1993. "A Spatial Model of Tax Competition with Multiple Tax Instruments." *Journal of Urban Economics,* 47, 88-114.

Braid, Ralph M. 2000. "Spatial Competition between Jurisdictions which Tax Perfectly Competitive Retail (or Production) Centers." *Journal of Urban Economics,* 34, 75-95.

Chen, Chin-Sheng, and Fu-Chuan Lai. 2008. "Location Choice and Optimal Zoning under Cournot Competition." *Regional Science and Urban Economics*, 38, 119-126.

Colombo, Stefano. 2012. "On Optimal Zoning in a Linear Town with Cournot Competitors." *Letters in Spatial and Resource Sciences*, 5, 113-118.

d'Aspremont, Claude, J. Jaskold Gabszewicz, and Jacques-François Thisse. 1979. "On Hotelling's Stability in Competition." *Econometrica*, 47, 1145-1150.

Hamoudi, Hamid, and Marta Risueño. 2012. "The Effects of Zoning in Spatial Competition." *Journal of Regional Science*, 52, 361–74.

Hotelling, Harold. 1929. "Stability in Competition." *Economic Journal*, 39, 41-57.

Lai,Fu-Chuan, and Tsai Jyh-Fa. 2004. "Duopoly Locations and Optimal Zoning in a Small Open City." *Journal of Urban Economics*, 55, 614-626.

Matsumura, Toshihiro, and Noriaki Matsushima. 2012. "Locating Outside a Linear City Can Benefit Consumers." *Journal of Regional Science*, 52, 420-432.

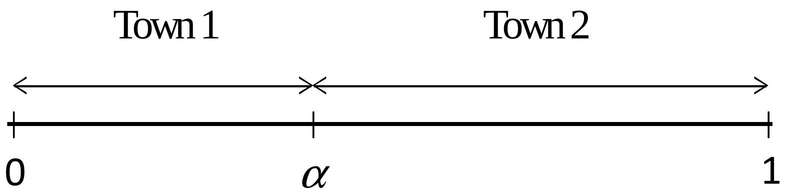

FIGURE 1: The cross-border linear city.

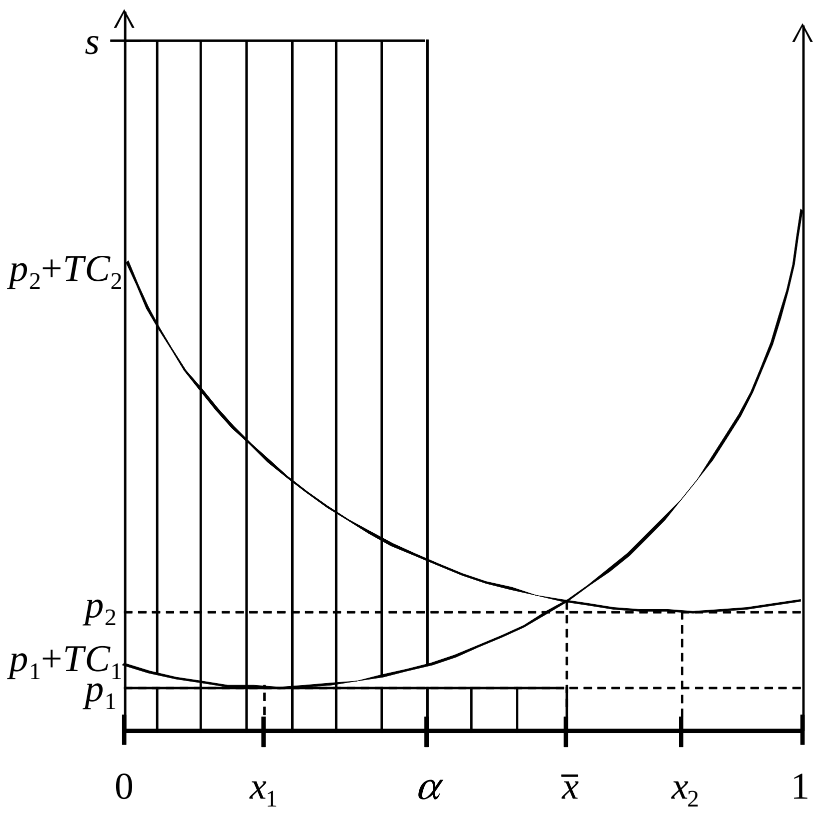

FIGURE 2: Town 1's social welfare when $\overline{x} > \alpha$.

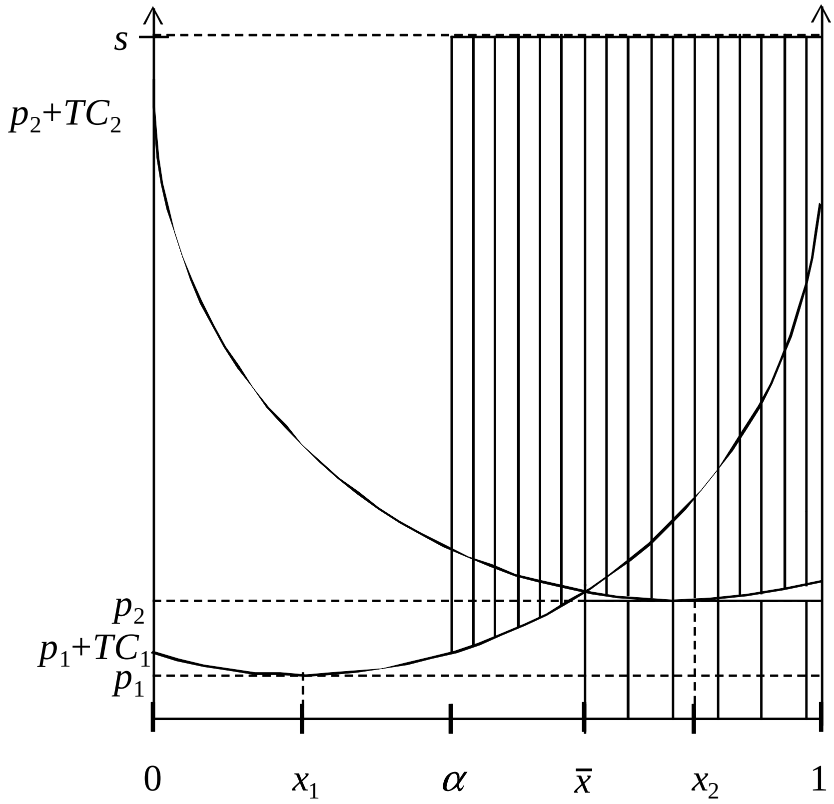

FIGURE 3: Town 2's social welfare when $\overline{x} > \alpha$.

TABLE 1: Payoff matrix in the first stage of the game

Town 1\2	Z	N
Z	$\frac{s}{2} - 0.0124t - f$ $\frac{s}{2} - 0.0124t - f$	$\frac{s}{2} + 0.0273t - f$ $\frac{s}{2} - 0.0723t$
N	$\frac{s}{2} - 0.0723t$ $\frac{s}{2} + 0.0273t - f$	$\frac{s}{2} - 0.0417t$ $\frac{s}{2} - 0.0417t$

TABLE 2. Payoff matrix in the first stage of the game

Town 1\2	Z	N
Z	$\frac{s}{3} + 0.0387t - f$ $\frac{2s}{3} - 0.0809t - f$	$\frac{s}{3} + 0.168t - f$ $\frac{2s}{3} - 0.2208t$
N	$\frac{s}{3} + 0.029t$ $\frac{2s}{3} - 0.0871t - f$	$\frac{s}{3} + 0.1543t$ $\frac{2s}{3} - 0.2377t$