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OUTPUT DYNAMICS IN AN ENDOGENOUS GROWTH MODEL

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Output dynamics in an endogenous growth model

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Abstract

The aim of this paper is to analyze whether endogenous growth RBC models are consistent with two stylized facts about U.S. output dynamics. First, GNP growth is positively correlated in the short run and it has a weak negative autocorrelation over longer horizons. Second, GNP appears to have an important trend-reverting component that has a hump-shaped MA representation. In particular, this article considers a stochastic version of Lucas' (1988) model in the absence of externalities in discrete time with two modifications: agents derive utility not only from consumption but also from leisure and labor adjustment costs are included. Results reveal that combining the endogenous character of the engine of growth with labor adjustment costs may help solve *the Cogley-Nason (1995) puzzle* since it provides a stronger propagation mechanism which, in turn, improves the model's ability to generate realistic output dynamics.

Keywords: Real business cycle models; endogenous growth; propagation mechanism.

JEL classification: E32; O41; C52.

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1 Introduction

Prescott (1986) points out that the business cycle phenomenon has three dimensions: the comovements of other variables with output, the relative volatilities of various series and the periodicity of output. Standard Real Business Cycle (RBC) models provide satisfactory results on both first and second dimensions but, as Cogley and Nason (1995) noted, they fail to reproduce two stylized facts about U.S. output dynamics: first, GNP growth is positively autocorrelated over short horizons and has a weak and possibly insignificant negative autocorrelation over longer horizons; and second, GNP appears to have an important trend-reverting component that has a hump-shaped MA representation. Furthermore, they conclude that standard RBC models have weak internal propagation mechanisms and, as a consequence, exogenous sources of dynamics are needed in order to replicate observed autocorrelation and impulse response functions. They also find that non-standard RBC models that incorporate lags or costs of adjusting labor input are only partially successful, since implausibly large transitory shocks are needed in order to match the transitory impulse response function found in data. As a result, they suggest that "RCB theorists ought to devote further attention to modeling internal sources of propagation".

Cogley and Nason (1995) follow the traditional approach in the RBC literature that assumes a strict exogenous engine of growth. An alternative approach is to consider a model that encompasses both cycles and endogenous growth[1]. This paper pursues this second line of research.

The contribution of different endogenous growth models to RBC literature has already been analyzed by King, Plosser and Rebelo (1988 b), Gomme (1993), Ozlu (1996), Einarsson and Marquis (1997) and Barañano (2001), among others. They show that the endogenous growth assumption may not be innocuous, since these models not only perform better than the standard exogenous growth model in explaining labor market fluctuations but also provide a stronger propagation mechanism[2]. Hercowitz and Sampson (1991), Perli and Sakellaris (1998), Jones, Manuelli and Siu (2000) and Matheron (2003) have also analyzed the cyclical properties of endogenous growth in dynamic stochastic general equilibrium models and show that doing so allows, among other things, to display the kinds of internal propagation mechanisms that are necessary to match the observed autocorrelation of output growth. However, they do not study their models' ability to reproduce the trend-reverting component found in U.S. GNP.

This paper is closely linked to Collard (1999). He also assesses the ability of endogenous growth models in solving *the Cogley-Nason (1995) puzzle*, but due to the specific set of functional forms used (he considers log-linear specifications for preferences and technology which allow him to obtain an analytic solution of output growth), his model fails in one respect, since it predicts that output, consumption and investment are perfectly correlated, which is obviously

[1]Endogenous growth models incorporate a mechanism that generates sustained growth without exogenous technological progress.

[2]Endogenous growth models have also been used to explain economic growth in the long-run. Lucas (1988), Rebelo (1991) and King and Rebelo (1990), among others, focus their papers on the ability of endogenous growth models to explain certain observed growth patterns which the standard exogenous growth model fails to account for. Furthermore, as noted by King and Rebelo (1993), the transitional dynamics of the basic exogenous growth model in order to explain sustained cross-country differences in growth rates induce extremely counterfactual implications. We follow Uzawa (1965) and Lucas (1988), who suggested human capital as the engine of growth.

counterfactual. In other words, Collard's (1999) model fails in explaining the first dimension of business cycles.

The aim of this paper is to analyze the contribution of RBC models with endogenous growth in characterizing the above mentioned stylized facts about U.S. GNP dynamics without constraining the specific set of functional forms used to obtain a closed-form solution. In particular, this article considers a stochastic version of Lucas' (1988) human capital accumulation model in discrete time with two modifications: agents derive utility not only from consumption but also from leisure and labor adjustment costs are included[3]. Our results show that combining the endogenous character of the engine of growth with labor adjustment costs may help solve *the Cogley-Nason (1995) puzzle* since it provides a stronger propagation mechanism which, in turn, improves the model's ability to replicate the above mentioned observations.

The rest of the paper is as follows: Section 2 describes the endogenous growth model considered and the calibration procedure used. In Section 3, the solution method used is briefly described and the quantitative results obtained are shown. Finally, Section 4 concludes.

2 The Model

This paper considers a stochastic discrete time version of Lucas' (1988) model in the absence of externalities with two modifications. On the one hand, agents derive utility not only from consumption but also from leisure. On the other hand, as suggested by Shapiro (1986) and Cogley and Nason (1995), labor adjustment costs are included.

The economy consists of a large number of productive families which own both the production factors and the technology used in two production activities: the production of the final good (market sector) and the production of new human capital (human capital sector)[4]. The population size is assumed to be constant. At any point in time, individuals must decide what fraction of their time they devote to each of these activities, and how much time they set aside for leisure. The time endowment is standardized to one, so that l_t denotes the fraction of time given over to leisure and n_t the fraction of time devoted to the production of the consumption good.

The technology of the consumption good is described by a production function with constant returns to scale with respect to physical capital and efficient labor. As already mentioned, we also consider labor adjustment costs. In particular, the production function is modified so that there are quadratic adjustment costs in labor input. The specification adopted implies that the marginal cost of adjusting the labor input increases in the rate of employment. Formally, the production technology is made (log) linear in the cost of adjustment,

[3]Comovements of other variables with output and the relative volatilities of various series for this endogenous growth model without labor adjustment costs are studied in Barañano (2001) and in this respect results are successful.

[4]The introduction of a home production sector competing with the market sector has already been used in RBC literature. See for example Benhabib *et al.* (1991), Greenwood and Hercowitz (1991), Gomme (1993) and Ozlu (1996).

$$\begin{aligned}\ln\ y_t &= \ln[F^m(k_t, Z_t, n_t h_t)] - \frac{\eta}{2}\left[\frac{\Delta(n_t h_t)}{n_{t-1}h_{t-1}}\right]^2 \\ &= \ln[A_m Z_t k_t^{\alpha}(n_t h_t)^{1-\alpha}] - \frac{\eta}{2}\left[\frac{\Delta(n_t h_t)}{n_{t-1}h_{t-1}}\right]^2,\ with\ 0 < \alpha < 1\end{aligned}$$

where $n_t h_t$ represents the qualified labor units, the term in brackets shows the percentage change in efficient labor, A_m is the parameter which measures the productivity of this sector, k_t and h_t are the stocks of physical capital and human capital in per-capita terms, respectively, η is the labor adjustment cost parameter, and finally Z_t is a technology shock characterized by the following autoregressive process:

$$\ln(Z_t) = \rho_2\ \ln(Z_{t-1}) + (1-\rho_1)\ln(\bar{Z}) + \varepsilon_t,$$

where $\ln(\bar{Z})$ is the mean of $\ln(Z_t)$ and ε_t is a serially independent innovation with standard deviation σ_Z.

The law of motion for physical capital is

$$k_{t+1} = y_t - c_t + (1-\delta_k)k_t,$$

where c_t denotes consumption and δ_k represents the depreciation rate of physical capital, which is assumed to be constant.

New human capital is assumed to evolve according to the following equation:

$$h_{t+1} = A_h\theta_t(1 - l_t - n_t)h_t + (1-\delta_h)h_t,$$

where A_h measures the productivity of this sector, δ_h denotes the depreciation rate of human capital and θ_t is a shock which follows a first order autoregressive process given by:

$$\ln(\theta_t) = \rho_2\ \ln(\theta_{t-1}) + (1-\rho_2)\ln(\bar{\theta}) + \epsilon_t,$$

where $\ln(\bar{\theta})$ is the mean of $\ln(\theta_t)$ and ϵ_t follows a white noise process with standard deviation σ_θ. It is further assumed that θ_t is uncorrelated to shock Z_t.

As stated above, it is assumed that consumers derive their utility from the consumption of the final good and from leisure. Future utility is discounted at a rate β and preferences are described by the following utility function[5]:

$$U(c_t, l_t) = \lambda \ln c_t + (1-\lambda)\ln l_t, \qquad 0 \leq \lambda \leq 1.$$

[5]Note that this function satisfies the conditions needed to ensure the existence of a balanced growth path. For more details on this issue, see King, Plosser and Rebelo (1988a, pp. 201-202).

As is well known, in the absence of external effects, public goods and distortionary taxation, the solution to the planner's problem is the competitive equilibrium allocation.

The problem faced by the central planner is to choose sequences for consumption, hours worked, leisure, physical capital and human capital that maximize the expectation of the discounted stream of utility given by:

$$
\begin{gathered}
\max_{n_t, c_t, l_t, k_{t+1}, h_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t), \\
s.t. \quad c_t + k_{t+1} = \frac{A_m Z_t k_t^{\alpha} (n_t h_t)^{1-\alpha}}{\exp\left\{ \frac{\eta}{2} \left[\frac{\Delta(n_t h_t)}{n_{t-1} h_{t-1}} \right]^2 \right\}} + (1-\delta_k) k_t, \\
\ln(Z_t) = \rho_1 \ \ln(Z_{t-1}) + (1-\rho_1) \ln(\bar{Z}) + \varepsilon_t, \\
h_{t+1} = A_h \theta_t (1 - l_t - n_t) h_t + (1-\delta_h) h_t, \\
\ln(\theta_t) = \rho_2 \ \ln(\theta_{t-1}) + (1-\rho_2) \ln(\bar{\theta}) + \epsilon_t, \\
c_t \geq 0, \ k_{t+1} \geq 0, \ h_{t+1} \geq 0, \\
0 \leq l_t \leq 1, \ 0 \leq 1 - l_t - n_t \leq 1,
\end{gathered}
$$

where $Z_0, \theta_{0,} k_0, h_0$ and n_0 are exogenously given.

The first-order conditions for this problem are:

$$
\begin{aligned}
U_2(c_t, l_t) &= U_1(c_t, l_t) \left[\tfrac{1-\alpha}{n_t} - \eta \tfrac{\Delta(n_t h_t) h_t}{(n_{t-1} h_{t-1})^2} \right] y_t \\
&\quad + \beta E_t \left\{ U_1(c_{t+1}, l_{t+1}) \, \eta \tfrac{\Delta(n_{t+1} h_{t+1}) n_{t+1} h_{t+1} h_t}{(n_t h_t)^3} \, y_{t+1} \right\}, \qquad (1) \\
U_1(c_t, l_t) &= \beta E_t \left\{ U_1(c_{t+1}, l_{t+1}) \left[\frac{\alpha}{k_{t+1}} y_{t+1} + 1 - \delta_k \right] \right\}, \qquad (2) \\
\frac{U_2(c_t, l_t)}{A_h \theta_t h_t} &= \beta E_t \left\{ \frac{U_2(c_{t+1}, l_{t+1})}{A_h \theta_{t+1} h_{t+1}} \left[A_h \theta_{t+1} (1 - l_{t+1}) + 1 - \delta_h \right] \right\}, \qquad (3) \\
h_{t+1} &= A_h \theta_t (1 - l_t - n_t) h_t + (1-\delta_h) h_t, \qquad (4) \\
k_{t+1} + c_t &= \frac{A_m Z_t k_t^{\alpha} (n_t h_t)^{1-\alpha}}{\exp\left\{ \frac{\eta}{2} \left[\frac{\Delta(n_t h_t)}{n_{t-1} h_{t-1}} \right]^2 \right\}} + (1-\delta_k) k_t, \\
\lim_{t \to \infty} E_t \beta^t U_1 k_{t+1} &= 0, \\
\lim_{t \to \infty} E_t \beta^t \frac{U_2}{A_h \theta_t h_t} h_{t+1} &= 0,
\end{aligned}
$$

where E_t is an operator whose expectations are conditional on the information available up to period t.

Equation (1) shows the optimal way of determining the fraction of time devoted to the production of goods. The marginal utility from an additional labor unit has to be equal to its marginal disutility. Labor adjustment costs not only affect current marginal utility but also the expected utility via future output. Hence, due to the presence of labor adjustment costs, firms do not adjust labor input completely in the current quarter. Their optimal response is to defer a part to the subsequent quarter.

Equation (2) governs the accumulation of physical capital and establishes that on the margin, the expected return to acquiring an additional unit of physical capital must equal the cost it causes in utility terms today.

Equation (3) governs the accumulation of human capital. Given that $1 - l_t$ denotes the fraction of time not allocated to leisure, this equation establishes that, at the margin, the expected return in current period utility from an additional unit of human capital must equal its cost.

In the steady state, the variables k_t, y_t and c_t grow at a constant rate which is equal to the human capital growth rate, while n_t and l_t remain constant. Therefore, non-stationary time series are obtained from the first order conditions characterizing the social planner problem. For the sake of simplicity in computations, the first-order conditions can be rewritten as:

$$U_1(\hat{c}_t, l_t) = \beta \left(\frac{h_{t+1}}{h_t}\right)^{-1} E_t \left\{ U_1(\hat{c}_{t+1}, l_{t+1}) \left[\frac{\alpha \hat{y}_{t+1}}{\hat{k}_{t+1}} + 1 - \delta_k\right]\right\}, \tag{5}$$

$$\begin{aligned} U_2(\hat{c}_t, l_t) = {} & U_1(\hat{c}_t, l_t) \left[\frac{1-\alpha}{n_t} - \eta \frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}} \frac{h_t}{n_{t-1}h_{t-1}}\right] \hat{y}_t \\ & + \beta E_t \left\{ U_1(\hat{c}_{t+1}, l_{t+1}) \, \eta \frac{n_{t+1} - n_t \frac{h_t}{h_{t+1}}}{n_t \frac{h_t}{h_{t+1}}} \frac{n_{t+1}h_{t+1}}{n_t h_t} \frac{\hat{y}_{t+1}}{n_t} \right\}, \end{aligned} \tag{6}$$

$$\frac{U_2(\hat{c}_t, l_t)}{A_h \theta_t} = \beta \left(\frac{h_{t+1}}{h_t}\right)^{-1} E_t \left\{ \frac{U_2(\hat{c}_{t+1}, l_{t+1})}{A_h \theta_{t+1}} \left[A_h \theta_{t+1}(1 - l_{t+1}) + 1 - \delta_h\right]\right\}, \tag{7}$$

$$\frac{h_{t+1}}{h_t} = A_h \theta_t (1 - l_t - n_t) + 1 - \delta_h, \tag{8}$$

$$\hat{c}_t + \hat{k}_{t+1}\frac{h_{t+1}}{h_t} = \frac{A_m Z_t \hat{k}_t^{\alpha} n_t^{1-\alpha}}{\exp\left\{\frac{\eta}{2}\left[\frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}}\right]^2\right\}} + (1-\delta_k)\hat{k}_t, \tag{9}$$

where $\hat{c}_t = \dfrac{c_t}{h_t}$ and $\hat{k}_t = \dfrac{k_t}{h_t}$.

2.1 Calibration

In order to solve and simulate the model, we must assign values to the following parameters:

Technology parameters: A_m, α, δ_k, ρ_1, σ_Z, η
Preferences: β, λ
Human capital production: A_h, δ_h, ρ_2, σ_θ

We follow the calibration procedure suggested by Kydland and Prescott (1982). The values for structural parameters and some steady state variables are displayed in Table I and, except for η, they are explained in detail in Barañano (2001).

Parameter α, which measures the capital's average share of per-capita GNP, was chosen to be equal to 0.36. The physical capital depreciation rate, δ_k, was set equal to 0.025 (which is equivalent to 10% per annum), based on the study carried out by Kydland and Prescott (1982). Parameter A_m was normalized to unity.

Parameters of the autoregressive process which characterizes the technology shock dynamics (Z_t) are usually chosen on the basis of calibration studies well known in this literature. At Prescott's (1986) suggestion, the value assigned to ρ_1 is 0.95, given the persistency of the residuals. The value for σ_Z was chosen in order to replicate the volatility of per-capita GNP observed in U.S. data[6].

The labor adjustment parameter, η, has been calibrated from estimates in Shapiro (1986). We follow Cogley and Nason (1995) and take $\eta = 0.36$ as the baseline value and we check whether the results are sensitive to changes in η. They point out that this value probably overstates the size of aggregate labor adjustment costs. However, when human capital is included, labor is measured in efficiency units; consequently, not only the hours worked but also human capital are subject to adjustment costs. Hence, the same baseline value seems to be more suitable when human capital is included than in an exogenous growth model.

The discount factor is chosen such that, in the steady state, a real interest rate of 1% per quarter is earned on physical capital. This value is derived from the fulfillment of the following first-order condition in the deterministic steady state:

$$\beta \left(\frac{h_t}{h_{t-1}} \right)^{-1} 1.01 = 1,$$

given the homogeneity properties of the utility function[7].

The value of parameter λ, which governs the importance of consumption relative to leisure in the utility function, is established to guarantee that the fraction of time allocated to producing goods is 0.24 in the steady state, which is the fraction of time spent working by the U.S. working-age population (Gomme (1993) and Greenwood and Hercowitz (1991)).

[6] Similar exercises are performed by Gomme (1993), Hansen (1985) and Einarsson and Marquis (1997).

[7] This condition is obtained by combining equations (2) and (4).

Parameter A_h has been chosen to ensure that the growth rate of output in the steady state matches the 1.4% observed annual growth rate for output. Estimates for human capital depreciation rate, δ_h, range from approximately 0.6% to 13.3% per year (see Heckman (1976) and Rosen (1976)). The value adopted is $\delta_h = 0.005$ (i.e. 2% per annum). There is not much evidence on the parameters of the stochastic processes for the human capital accumulation. Coefficient ρ_2 has been assigned the same value that ρ_1 in the technology shock (0.95). The value for σ_θ was set equal to 0.004. It is assumed that both shocks are uncorrelated[8].

3 Results

The resolution method used in this paper is Uhlig's (1999) Log-Linear Method (LLM). This procedure can easily be summarized in the following steps:

Step 1: Log-linearize the first-order conditions which characterize the equilibrium of the model in order to make all the equations approximately linear in the log-deviations from the steady state.
Step 2: Solve for the recursive equilibrium law of motion by using the method of undetermined coefficients suggested by Uhlig (1999), which is simple and of general applicability[9].

There is an extensive literature on testing for unit roots in GNP[10]. RBC theorists usually assume that technology shocks have a unit root or near unit root and, due to this assumed specification of technology shocks, standard RBC models are able to replicate the persistence found in U.S. output. However, endogenous growth models would replicate this stylized fact regardless of the specification of the shocks since, as pointed out by Mc Callum (1989), endogenous growth models that show constant returns to scale with regard to the factors that are accumulated exhibit the unit root property, which implies that transitory shocks affect the level of growing variables in the long-run[11]. Hence, relaxing the presumption underlying standard RBC models that growth components are determined by different factors from those causing business cycles improves the ability of the model to endogenously mimic this stylized fact.

The purpose of this paper is to analyze whether endogenous growth RBC models are consistent with two stylized facts about the time series properties of U.S. aggregate output[12]. First, GNP growth is positively correlated in the short run and it has a weak negative autocorrelation over longer horizons. Second, GNP appears to have an important trend-reverting component that has a hump-shaped MA representation. The procedure can be summarized as follows. We generate artificial time series for output by simulating various RBC models. Autocorrelation and

[8]Non-zero cross correlations between ϵ_t and ε_t would affect individuals' incentive to move resources across sectors as discussed in Benhabib *et al.* (1991). Lower values of this correlation entail more frequent divergence between Z_t and θ_t and, hence, more frequent incentives to switch between sectors over time.

[9]For more details, a computational appendix is available from the authors upon request.

[10]Whether macroeconomic aggregates contain a unit root is not a matter of consensus. See Murray and Nelson (2000) for a survey on this issue.

[11]A technical appendix is available from the authors upon request.

[12]See, for example, Nelson and Plosser (1982), Watson (1986), Campbell and Mankiw (1987), Cochrane (1988) and Blanchard and Quah (1989).

impulse response functions were estimated for each artificial sample (each model was simulated 1000 times) and then we compared our results with U.S. data from 1955:3 to 1984:1. Results were collected into empirical probability distributions used to calculate the probability of observing the statistics estimated from U.S. data under the hypothesis that the data were generated by a particular RBC model. Formally, this procedure can be regarded as a specification test of a particular RBC model which can also be used as an informal guide to model reformulation.

Four RBC models are considered: a standard exogenous growth model without labor adjustment costs, a standard exogenous growth model with labor adjustment costs, Lucas' (1988) model without labor adjustment costs and Lucas' (1988) model with labor adjustment costs.

3.1 Autocorrelation Functions

We analyze whether the above mentioned models replicate the sample autocorrelation function (ACF) for output growth. We compute generalized statistics to test the match between actual and theoretical ACF's:

$$Q_{acf} = (\widehat{c} - c)' \widehat{V}_c^{-1} (\widehat{c} - c),$$

where $\widehat{c}$ stands for actual ACF and c is the model-generated one, which was estimated by averaging the ACF's across the ensemble of artificial series,

$$c = \frac{1}{N} \sum_{i=1}^{N} c_i,$$

where c_i is the ACF on replication i, and N=1000 is the number of replications. Matrix $\widehat{V}_c$ denotes the covariance matrix and is estimated by averaging the ensemble outer product of the ACF's for simulated data:

$$\widehat{V}_c = \frac{1}{N} \sum_{i=1}^{N} [c_i - c] [c_i - c]' .$$

Generalized Q statistics are approximately $\chi^2(p)$, where p is the number of lags in c. Following Cogley and Nason (1995), we report the results for $p = 8$.

The first column of Table II reports Q_{acf} statistics for each model. Probability values are in parentheses. A large value of Q_{acf} indicates that the theoretical ACF is a poor match for the actual ACF. Our results show that the adjustment cost endogenous growth model is the only one that passes the autocorrelation test. Nonetheless, this result is not robust to changes in the value of η since the model is rejected when $\eta = \frac{0.36}{4}$ at conventional significance levels (see the first column of Table III).

Figure 1 illustrates the results for each model. The solid line shows the actual ACF and dotted lines show artificial ACF's. Note that labor adjustment costs are crucial for generating serial correlation in both exogenous and endogenous growth models. Results are sensitive to changes in the value of η. Once this specification is considered, the endogeneity of the technological progress enhances the model's ability to replicate the observed ACF. The propagation mechanism embodied in the model provides some intuition for these results.

3.2 Impulse Response Functions

We also analyze whether those models replicate observed impulse response functions (IRF's). The IRF's are obtained by using the structural VAR technique developed by Blanchard and Quah (1989). For the implementation of this technique, a second-order VAR was estimated for per-capita output growth and hours. We compute the following statistic to test the match between actual and theoretical IRF's :

$$Q_{irf} = (\widehat{r} - r)'\widehat{V}_r^{-1}(\widehat{r} - r),$$

where $\widehat{r}$ is the actual IRF and r is the model-generated one, which was estimated by averaging across the ensemble of artificial series,

$$r = \frac{1}{N}\sum_{i=1}^{N} r_i,$$

where r_i is the IRF on replication i, and N=1000 is the number of replications. Matrix $\widehat{V_r}$ denotes covariance matrix and is estimated by averaging the ensemble outer product of the IRF's for simulated data:

$$\widehat{V_r} = \frac{1}{N}\sum_{i=1}^{N} [r_i - r]\,[r_i - r]'.$$

We truncate again at lag 8. The second and third columns of Table II report Q_{irf} statistics for each model. Monte Carlo probability values are in parentheses. (Exogenous growth models are driven by a single shock, so their bivariate VAR's have stochastic singularities). We infer from this table that results improve when the adjustment cost endogenous growth model is considered. This model has some success at matching not only the permanent IRF but also the transitory IRF. In the former dimension, results are sensitive to changes in the value of η, but, in the latter dimension, the model passes the test even when $\eta = \frac{0.36}{4}$ at the 5-percent level.

Figure 2 illustrates the IRF's for Lucas' (1988) model. The solid lines show the actual IRF's and the dotted lines show artificial IRF's. The adjustment cost endogenous growth model generates a hump in the transitory IRF as shown in the data. Results are sensitive to the choice of η. As shown in Figure 2, the higher the labor adjustment cost parameter, the higher the hump displayed.

Hence, it follows from the results that incorporating labor adjustment costs into the model improves its ability to reproduce the observed ACF, but an endogenous propagation mechanism is also needed in order to obtain realistic output dynamics of GNP.

3.3 Propagation Mechanism

Cogley and Nason (1995) argue that standard RBC models cannot generate the right pattern of output dynamics via their internal structure due to the weakness of the propagation mechanism embodied in them. Indeed, they must rely on external sources of dynamics to replicate both stylized facts[13]. Non-standard exogenous growth models that incorporate lags or costs of adjusting labor input although endogenously generate positive autocorrelation in output growth and a small hump in the transitory IRF, they need implausibly large transitory shocks in order to match the transitory IRF found in data. As a result they suggest that RBC theorists ought to devote further attention to understanding how shocks are magnified and propagated over time. In this section we analyze the internal propagation mechanism in the four models considered.

In order to asses the importance of the propagation mechanism embodied in these four models, we analyze the dynamic response functions of hours and output to shocks Z_t and θ_t. Note that, although the specification adopted for both shocks is an AR(1) process, the former can be interpreted as a transitory shock and the latter as a permanent shock, since technology shocks, unlike human capital shocks, do not affect the growth rate.

In comparison with the standard exogenous growth model (EGM), the Lucas' (1988) model provides a stronger internal propagation mechanism due to the fact that individuals not only substitute between market activity at different dates but also between market and human capital accumulation activities at a point in time. In particular, they respond to favorable technology shocks by devoting more time to work, while in periods of recession they respond by accumulating more human capital, maintaining a smooth path for leisure. Figure 3 reports the response of hours to a 1% technology shock in both models. This figure shows that hours fluctuate much more when this second sector is considered[14].

The ACF for output growth depends on the effects caused by both transitory and permanent shocks. The graphs of figure 4 illustrate the response of output to both transitory, Z_t, and permanent, θ_t, shocks, respectively. In response to a favorable transitory shock, labor flows into the market and out of the human capital sector, causing output to increase at impact. Due to the presence of labor adjustment costs, firms do not adjust labor input completely in the current quarter. Their optimal response is to defer a part to the subsequent quarter. Hence, output rises again in the subsequent period. Eventually, the income effect of a technology shock begins to offset the substitution effect, causing hours and output to decline from their peak. This generates a hump-shaped IRF of output to technology shocks (see transitory IRF in figure 2). Thus, a favorable technology shock generates positive autocorrelation in the transitory component of output growth. As shown in figure 4, not only the impact effect of a technology shock but also the lagged effects are larger than in the EGM. This generates a stronger serial correlation in output growth.

Apart from this effect we must take into account the resulting from a permanent shock. In response to a favorable permanent shock, individuals decide to devote more resources to accumulate human capital. As shown in figure 4, in this case, output falls at impact and is followed

[13]They find that standard exogenous growth models strongly damp transitory shocks, so most of the variation in output growth is due to permanent movements, which means that output dynamics are nearly the same as impulse dynamics.

[14]By internal propagation mechanism we mean those forces or properties of the model that amplify the effect of the technology shock and cause the deviation from the steady state to persist. As Barañano (2001) pointed out, when a single shock is considered, Lucas' (1988) model needs a lower technology shock in order to reproduce the volatility of U.S. output. We also infer from this result that Lucas' (1988) model provides a stronger propagation mechanism.

by further small declines. Subsequently, as human capital productivity declines, output rises back toward its initial trend. Thus, a positive human capital shock also generates positive autocorrelation in the permanent component of output growth. Hence, this second effect reinforces the serial correlation in output growth. Notice that this second effect only takes place when this second sector is included.

To sum up, the introduction of labor adjustment costs in Lucas' (1988) endogenous growth model allows for some improvements in explaining two stylized facts about output dynamics in the U.S.: a positive autocorrelation of output growth and an important trend-reverting component in GNP that has a hump-shaped transitory impulse response function. But it must be noticed that autocorrelation results depend on the size of the labor adjustment costs considered. We infer from this result that, as suggested by Cogley and Nason (1995), RBC theorists should devote further attention to the internal propagation mechanisms embodied in RBC models.

4 Conclusions

As Cogley and Nason (1995) noted, standard Real Business Cycle (RBC) models fail to reproduce two stylized facts about U.S. output dynamics: GNP growth is positively autocorrelated in the short run and has a weak negative autocorrelation over longer horizons, and GNP appears to have an important trend-reverting component that has a hump-shaped impulse-response function. Furthermore, these authors find that standard RBC models must rely on external sources of dynamics to replicate both stylized facts due to the weakness of their internal propagation mechanisms. Non-standard RBC models that rely on lags or costs of adjusting labor input are only partially successful, since they also need implausibly large transitory shocks in order to match the transitory IRF found in data. As a result, they suggest that RCB theorists ought to devote further attention to understand how shocks are magnified and propagated over time.

The aim of this paper is to analyze whether RBC models with endogenous growth are consistent with the above mentioned empirical regularities about U.S. output dynamics without constraining the specific set of functional forms used to obtain a closed-form solution. In particular, this article considers a stochastic version of Lucas' (1988) model in the absence of externalities with two modifications: agents derive utility not only from consumption but also from leisure, and labor adjustment costs are included. We show that introducing endogenous growth overcomes some of the main shortcomings of RBC models. Our results show that combining the endogenous character of the engine of growth with labor adjustment costs may help solve *the Cogley-Nason (1995) puzzle,* since it provides a stronger propagation mechanism which, in turn, enhances the model's ability to reproduce the above mentioned observations.

5 References

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Table I. Parameter and steady state values[a].

Parameter	Value	Interpretation
β	0.9936	Subjective discount factor
α	0.36	Share of physical capital in the final good technology
δ_k	0.025	Depreciation rate of physical capital
δ_h	0.005	Depreciation rate of human capital
A_m	1	Scale parameter in the final good technology
A_h	0.0266666	Scale parameter in the human capital production function
λ	0.3769	Consumption weight in utility function
η	0.36	Size of labor adjustment costs
σ_Z	0.007	Standard deviation of ε_t
σ_θ	0.004	Standard deviation of ϵ_t
ρ_2	0.95	Persistence of θ_t
ρ_1	0.95	Persistence of Z_t
v	0.0036	Growth rate
$\bar{n}$	0.24	Hours worked

[a] For parameters with a time dimension, the unit of time is a quarter of a year.

Table II. Test statistics for the autocorrelation and impulse response functions

Model	Q_{acf}	$Q_{irf}\ (Y_P)$	$Q_{irf}\ (Y_T)$
Exogenous growth model	31.82	–	–
	(0.0001)		
Exogenous growth model with adjustment costs	16.60	–	–
	(0.034)		
Endogenous growth model	28.66	37.90	39.18
	(0.0003)	(0.025)	(0.025)
Endogenous growth model with adjustment costs	10.17	19.84	12.11
	(0.25)	(0.07)	(0.09)

Table III. Sensitivity analysis

Endogenous growth model with adjustment costs	Q_{acf}	$Q_{irf}\ (Y_P)$	$Q_{irf}\ (Y_T)$
$\eta = 0.36$	10.17	19.84	12.11
	(0.25)	(0.07)	(0.09)
$\eta = \frac{0.36}{2}$	12.54	22.79	16.58
	(0.128)	(0.058)	(0.064)
$\eta = \frac{0.36}{4}$	16.03	27.22	22.85
	(0.041)	(0.042)	(0.051)

Figure 1: ACF for output growth

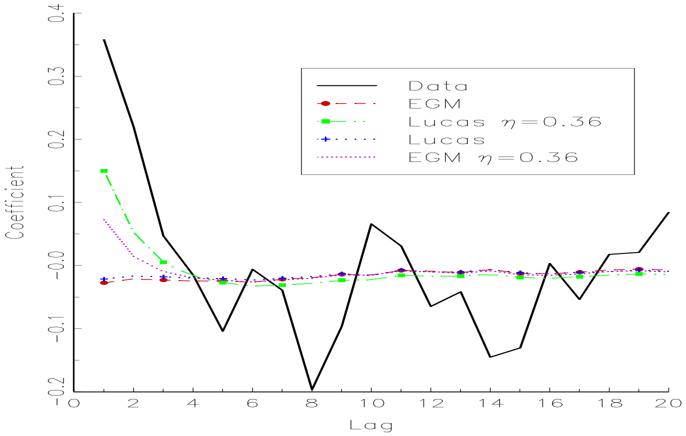

Figure 2: Impulse-response function from the Blanchard-Quah technique