TEMA 1.1: Hadrones

Operador paridad: $\Pi\psi(\vec r)=\psi(-\vec r)$ $\quad \Pi^2=1$

$\Pi Y_l^m(\hat n)=Y_l^m(-\hat n)=(-1)^l Y_l^m(\hat n)$

$\Pi_\pi=-1$ $\quad \Pi_n=\Pi_p=+1$

Espacio de carga: $p=\begin{pmatrix}1\\0\end{pmatrix}$ $\quad n=\begin{pmatrix}0\\1\end{pmatrix}$

Operador carga: $Q=\begin{pmatrix}1&0\\0&0\end{pmatrix}=\frac12(1+\tau_z)$,

$Q=\begin{pmatrix}1&0\\0&0\end{pmatrix}=\frac12(1+\tau_z)$,

$\tau_+=\begin{pmatrix}0&1\\0&0\end{pmatrix}=\tau_x+i\tau_y$ $\quad \tau_-=\begin{pmatrix}0&0\\1&0\end{pmatrix}=\tau_x-i\tau_y$

Isoespin: $\vec t=\frac12\sigma$ $\quad$ Carga isospin: $Z=\frac12+\tau_z$

$t_3 p=\frac12 p$ $\quad t_3 n=-\frac12 n$

$\pi^+\to t_z=1, \pi^0\to t_z=0, \pi^-\to t_z=-1$

$B_p=B_n=1$ $\quad B_\pi=0$ $\quad Z=t_z+\frac{B}{2}$

Isospin total: $\vec T=\sum_a \vec t_a$ $\quad \vec T^2=T(1+T)$ $\quad [\vec T,H_s]=0$

$Z=\sum_a\left(\frac12+t_{za}\right)=\frac{A}{2}+T_z$ $\quad T_z=\frac12(Z-N)$

Momento dipolar magnético: $\vec\mu=\frac{e}{2mc}\vec L$ $\quad \vec\mu_L=\frac{eg_L}{2mc}\vec L$

$p\to g_L=1$ $\quad n\to g_L=0$ $\quad g_s=2$ $\quad \vec\mu_s=\frac{eg_s}{2mc}\vec S$

$\mu_N=\frac{e\hbar}{2mc}$ $\quad \vec\mu_L=\mu_N g_L\frac{\vec L}{\hbar}$ $\quad \vec\mu_s=\mu_N g_s\frac{\vec S}{\hbar}$ $\quad \alpha=\frac{e^2}{\hbar c}$

Compton: $\lambda'=\lambda+\frac{h}{mc}(1-cos\theta)$

TEMA 1.2: Sistema de dos nucleones

$\nabla^2\psi+\frac{2M}{\hbar^2}(E-V)\psi=0$

Func. ond. d: $\psi=\Psi_{space}\chi_s^{ms}\phi_T^{T_z}$ $\quad \Psi_{space}=\frac{u_l(r)}{r}Y_l^m$

Paridades: $\Pi_S=(-1)^{S+1}$ $\quad \Pi_T=(-1)^{T+1}$ $\quad \Pi_L=(-1)^L$

Monopolo electrostático $l=0$

$M(E0,0)=\frac{1}{\sqrt{4\pi}}\sum_a e_a=\frac{1}{\sqrt{4\pi}}\int d^3rp(\vec r)=\frac{1}{\sqrt{4\pi}}Z_e$

Momento dipolar: $l=1$

$M(E1,m)=\sqrt{\frac{3}{4\pi}}\sum_a e_a r_a(n_a)_m=\sqrt{\frac{3}{4\pi}}d_m$

Momento cuadrupolar: $Q=\langle\psi|\hat Q|\psi\rangle$

$Q=\int\psi^*(r)(3z^2-r^2)\psi(r)d^3r$

Momento magnético: $\vec\mu=\sum_a\left(g^s\vec s+g^l\vec l\right)$

$\vec j=\vec l+\vec s$ $\quad \vec j^2=j(j+1)$ $\quad \vec l^2=l(l+1)$ $\quad \vec s^2=s(s+1)$

$\vec j\cdot\vec l=\frac{j(j+1)+l(l+1)-s(s+1)}{2}$ $\quad \vec j\cdot\vec s=\frac{j(j+1)+s(s+1)-l(l+1)}{2}$

$\vec s\cdot\vec l=\frac{\vec j^2-\vec s^2-\vec l^2}{2}$

Op. mom. mag.: $\vec\mu=g^s\vec s+g^l\vec l=g(j,l,s)\vec j$

Landé: $g(j,l,s)=g^l\frac{\langle\vec j\cdot\vec l\rangle}{j(j+1)}+g^s\frac{\langle\vec j\cdot\vec s\rangle}{j(j+1)=}$

$=\frac{1}{2j(j+1)}\{(g^l+g^s)j(j+1)+(g^l-g^s)[l(l+1)-s(s+1)]\}$

$g_p^s=2\mu_p=5.58$ $\quad g_n^s=-3.82$ $\quad g_p^l=1$ $\quad g_n^l=0$

$u_0=Asin(Kr)$ $\quad r<R$ $\quad Be^{-\kappa r}$ $\quad r>R$

K=$\frac1\hbar\sqrt{2M(V_0-E)}$ $\quad \kappa=\frac1\hbar\sqrt{2ME_B}$

TEMA 1.3: Interacción nucleón-nucleón:

Mom. mag. deuterón: $\mu_d\simeq\mu_p+\mu_n=0.8797\mu_N$

Pot. local: $V(1,2)=V_c+V_s(\vec\sigma_1\cdot\vec\sigma_2)+V_TS_{12}(\vec r)+V_T'S_{12}(\vec p)+V_{LS}\vec L\cdot\vec S+V_Q(\vec L\cdot\vec S)^2$

$S_{12}=3\left(\vec\sigma_1\cdot\frac{\vec r}{r}\right)\cdot\left(\vec\sigma_2\cdot\frac{\vec r}{r}\right)-(\vec\sigma_1-\vec\sigma_2)$ $\quad (\vec r\cdot\vec p)^2=r^2p^2-L^2$

$\vec S_i^{\,2}=\frac34\hbar^2$ $\quad \vec S_1\cdot\vec S_2=\frac14\vec\sigma_1\cdot\vec\sigma_2$ $\quad \vec S_1\cdot\vec S_2=\frac12(\vec S^2-\vec S_1^{\,2}-\vec S_2^{\,2})$

Pot. Paris: $S=1$ $V(1,2)=V_{c1}(r)+V_T(r)S_{12}(\vec r)+V_{so}(r)\vec L\cdot(\vec\sigma_1+\vec\sigma_2)$

$S=0$ $V(1,2)=V_{co}(r)$

Reid soft: $V=V_c(\mu r)+V_{12}(\mu r)S_{12}+V_{LS}(\mu r)\vec L\cdot\vec S$

Yukawa: $V_g^2\frac{e^{-\mu r}}{r}$

Lennard Jones: $V_{LJ}(r)=4E_{PO}\left[\left(\frac\sigma r\right)^{12}-\left(\frac\sigma r\right)^6\right]$

TEMA 2.1: Propiedades generales de los núcleos

$A\simeq\frac{\frac43\pi R^3}{\frac43\pi r^3}$ $\quad R\simeq r_o\cdot A^{1/3}$ $\quad r_0>r\sim 1fm$

Densidad de carga: $\rho(r)=\frac{\rho_0}{1+e^{\frac{r-R_{1/2}}{a}}}$ $\quad A>20$ $\quad t=4.4a$

$R_s=1.128A^{1/3}$ $\quad R_{1/2}=R_s-0.89A^{-1/3}$

$\rho_0=0.17\frac{Z}{A}fm^{-3}$ $\quad a=0.54fm$

Energia enlace: $B(Z,N)=[Zm_p+Nm_n-m(Z,N)]c^2$

Energia separación: $S_m(Z,N)=[m(Z,N-1)+m_n-m(Z,N)]c^2=B(Z,N)-B(Z,N-1)$

E.pairing: $\delta_n(Z,N)=S_n(Z,N)-S_n(Z,N-1)$ $\quad$ N par

Números mágicos: $2,8,20,50,82,(126,184)^*$ * n

$$l=\begin{cases} n\hbar & impar-impar\\ (n+1\frac12)\hbar & nucleos-impares\\ 0 & par-par\end{cases}$$

$V=\int\frac{\rho(\vec r')}{|\vec r-\vec r'|}d\vec r'$ $\quad V\simeq\frac qr+\frac{\vec p\cdot\vec x}{r^3}+\frac{Q_{ij}x_ix_j}{r^5}+\ldots$ $\quad \vec A(\vec r)\simeq\frac{\vec\mu\times\vec r}{r^3}+\ldots$

Mom. mag. nuclear: $\mu=g^lj+(g^s-g^l)<s_z>$

$\mu=g^l(j-\frac12)+\frac12g^s$ $\quad j=l+\frac12$

$\mu=g^l\frac{j(j+\frac32)}{j+1}-\frac{jg^s}{2(j+1)}$ $\quad j=l-\frac12$

Tensor 4pol: $Q_{ij}=\int\left(3x_i'x_j'-r'^2\delta_{ij}\right)\psi^*(\mathbf r')\psi(\mathbf r')d\mathbf r'$

$V_Q=Ze\frac{z^2}{2r^5}Q-Ze\frac{x^2+y^2}{4r^5}Q,$

$Q=\int(3z'^2-r'^2)\psi^*(\mathbf r')\psi(\mathbf r')d\mathbf r'$

Funciones spin angular:

$\mathcal Y_\ell^{j=\ell\pm1/2,m}=\frac{1}{\sqrt{2\ell+1}}\left[\pm\sqrt{\ell\pm m+\frac12}\,Y_\ell^{m-1/2}(\theta,\phi)\chi_{\text{spin}+}+\sqrt{\ell\mp m+\frac12}\,Y_\ell^{m+1/2}(\theta,\phi)\chi_{\text{spin}-}\right]$

$\chi_{\text{spin}+}=\begin{pmatrix}1\\0\end{pmatrix}$ $\quad \chi_{\text{spin}-}=\begin{pmatrix}0\\1\end{pmatrix}$

$Q=-\langle r^2\rangle\frac{2j-1}{2j+2}$ $\quad \langle r^2\rangle=\int_0^\infty u_\ell^2(r)r^2dr$ $\quad \langle r^2\rangle=R^2$

$Q=Q_{sp}\left(1-2\frac{n-1}{2j-1}\right)$ $\quad Q_{sp}=-\langle r^2\rangle\frac{2j-1}{2j+2}$

β^- decay: ${}^A_ZX_N\to{}^A_{Z+1}Y_{N-1}+e^-+\bar\nu$

β^+ decay: ${}^A_ZX_N\to{}^A_{Z-1}Y_{N+1}+e^++\nu$

Ex de m: $\Delta(Z,A)=M(Z,A)-Am_u=(M(Z,A)(u)-A)u$

Masa nuclear: $m(Z,A)=M(Z,A)-Zm_e+\frac{1}{c^2}\sum_{i=1}^Z B_i$

Energía enlace: $B/c^2=Zm_p+Nm_n-m(Z,A)$

$=Zm_p+Nm_n-[M(Z,A)-Zm_e]$

$=Zm(^1H)+Nm_n-M(Z,A)$

TEMA 2.2: Modelos nucleares

Modelo de gota liquida:

Energía de volumen: $B_1=a_vA$

Energía de superficie: $B_2=-a_sA^{2/3}$ $\quad a_s=4\pi R^2$

Energía de Coulomb: $B_3=-a_cZ^2A^{-1/3}$ $\quad a_c=\frac35(Ze)^2/R$

Término de asimetría: $B_4=-a_A\frac{(Z_A/2)^2}{A}$

Pairing term: $B_5=\begin{cases}\delta & \text{par par}\\ 0 & \text{impar}\\ -\delta & \text{impar impar}\end{cases}$ $\quad \delta\simeq A^{-1/2}a_p$

$B(Z,A)=a_vA-a_sA^{2/3}-a_c\frac{Z^2}{A^{1/3}}-a_A\frac{(Z-A/2)^2}{A}+$

$+\frac{(-1)^Z+(-1)^N}{2}a_pA^{-1/2}$

$m(Z,A)=Zm_p+(A-Z)m_n-a_vA+a_sA^{2/3}+a_c\frac{Z^2}{A^{1/3}}+a_A\frac{(Z-A/2)^2}{A}-\frac{(-1)^Z+(-1)^N}{2}a_pA^{-1/2}$

$a_V=15.85$ MeV$/c^2$, $\quad a_S=18.34$ MeV$/c^2$

$a_C=0.71$ MeV$/c^2$, $a_A=92.86$ MeV$/c^2$,

$a_P=11.46$ MeV$/c^2$

$\left.\frac{\partial m(Z,A)}{\partial Z}\right|_{A=\text{const}}=0\Rightarrow Z_0=\frac A2\left(\frac{m_n-m_p+a_A}{a_cA^{2/3}+a_A}\right)=\frac{A}{1.98+0.015A^{2/3}}$

$E_\alpha=[m(Z,A)-m(Z-2,A-4)-m_\alpha]c^2>0$ $A>150$

$E_f=[m(Z,A)-2m(Z/2,A/2)]c^2>0$ $A>90$

Modelo del gas de Fermi:

$dn(k)=\frac{a^3}{(2\pi)^3}4\pi k^2dk$ $\quad dn(E)=\frac{\sqrt2m^{3/2}a^3}{2\pi^2\hbar^3}E^{1/2}dE$

$2\int_0^{E_F^{(p)}}dn(E)=\frac{2\sqrt2m^{3/2}a^3}{3\pi^2\hbar^3}\left(E_F^{(p)}\right)^{3/2}=Z,$ $\quad \rho_p=Z/a^3$

$2\int_0^{E_F^{(n)}}dn(E)=\frac{2\sqrt2m^{3/2}a^3}{3\pi^2\hbar^3}\left(E_F^{(n)}\right)^{3/2}=N.$ $\quad \rho_n=N/a^3$

$E_F(p)=\frac{\hbar^2}{2m}\left(3\pi^2\rho_p\right)^{2/3}$, $\quad E_F(n)=\frac{\hbar^2}{2m}\left(3\pi^2\rho_n\right)^{2/3}$.

$k_F=\frac{\sqrt{2mE_F}}{\hbar}$ $\quad \langle E_k\rangle=\frac35E_F$ $\quad E_T=\langle E_k\rangle A$

Modelo de capas:

Orbital	Acumulado	Energía	Orbital	Acumulado	Energía
$1s_{1/2}$	2	0	$2p_{3/2}$	32	$3\hbar\omega$
$1p_{3/2}$	6	$\hbar\omega$	$1f_{5/2}$	38	$3\hbar\omega$
$1p_{1/2}$	8	$\hbar\omega$	$2p_{1/2}$	40	$3\hbar\omega$
$1d_{5/2}$	14	$2\hbar\omega$	$1g_{9/2}$	50	$3\hbar\omega$
$2s_{1/2}$	16	$2\hbar\omega$	$1g_{7/2}$	58	$4\hbar\omega$
$1d_{3/2}$	20	$2\hbar\omega$	$2d_{5/2}$	64	$4\hbar\omega$
$1f_{7/2}$	28	$3\hbar\omega$	$1h_{11/2}$	76	$4\hbar\omega$

Núcleos impar-impar → Número de Nordheim:

$\mathcal N=j_p-\ell_p+j_n-\ell_n$ Si $\mathcal N=0$ $j=\left|j_p-j_n\right|$, Si $\mathcal N=\pm1$

$j=j_p+j_n$ o $\left|j_p-j_n\right|$

TEMA 3.1: Radiación

Número de átomos: $N(t)=N_0e^{-\lambda t}\ dN(t)=-\lambda N dt$

Actividad: $A=-\frac{dN}{dt} \quad \lambda=\lambda_1+\lambda_2+\ldots$

Semivida: $N(t_{1/2})=N_0/2 \quad t_{1/2}=\frac{ln2}{\lambda}$

Mean lifetime: $\tau=\frac{1}{\lambda}$

Actividad modo k: $A_k=-\left(\frac{dN_k}{dt}\right)_k=\lambda_k N=\lambda_k N_0 e^{-\lambda t}$

$N_k=\frac{\lambda_k}{\lambda}N_0(1-e^{-\lambda t}) \quad \lambda_k=\lambda\cdot P_k$

Des. en cadena: $\frac{dN_i}{dt}=\lambda_{i-1}N_{i-1}-\lambda_i N_i$

Si $N_2(0)=0, N_3(0)=0... \rightarrow N_i(t)=N_1(0)(h_1e^{-\lambda_1 t}+h_2e^{-\lambda_2 t}+...+h_ie^{\lambda_i t})$

Equilibrio: $\frac{dN_1}{dt}=\frac{dN_2}{dt}=...\simeq 0\rightarrow \lambda_1 N_1=\lambda_2 N_2=...$

Datación geológica:

Número de isótopos padre e hija: $P+D=P_0+D_0$

$P=P_0e^{-\lambda(t-t_0)} \quad e^{-\lambda(t-t_0)}=\frac{P}{P_0}=\frac{1+D_0/P_0}{1+D/P}$

Isótopo descendiente D' $\quad \Delta t=t-t_0$

$\frac{dD'}{dt}=\lambda P-\lambda_{D'}D'$ si $\lambda\rightarrow 0 \quad \lambda_{D'}=0\ \frac{dD'}{dt}=0$

$\frac{P+D}{D'}=\frac{P_0+D_0}{D'_0}$ ya que $D'=D'_0\Rightarrow\frac{D}{D'}=\frac{P}{D'}\left[e^{\lambda(t-t_0)}-1\right]+\frac{D_0}{D'_0}$

Probabilidad de transición:

Energía: $W=E_0-\frac{\hbar\lambda}{2}$

TEMA 3.2: Regla de oro de Fermi

Probabilidad de transición:

Energía: $W=E_0-\frac{\hbar\lambda}{2}$

$\psi(\mathbf{r},t)=\psi(\mathbf{r})e^{-iWt/\hbar} \quad |\psi(\mathbf{r},t)|^2=|\psi(\mathbf{r},0)|^2e^{-\lambda t}$

$e^{-iWt/\hbar}=e^{-(iE_0/\hbar+\lambda/2)t}=\int_{-\infty}^{\infty}A(E)e^{-iEt/\hbar}dE, \quad t\geq 0$

$\psi(t)=\theta(t)e^{-(iE_0/\hbar+\lambda/2)t}$

$A(E)=\frac{1}{2\pi\hbar}\int_0^{\infty}e^{[i(E-E_0)/\hbar-\lambda/2]t}dt \quad \Rightarrow$

$\Rightarrow A(E)=\frac{1}{\hbar\lambda/2-2\pi i(E-E_0)} \quad A^*(E)A(E)=\frac{1}{\hbar^2\lambda^2/4+4\pi^2(E-E_0)^2}$

$\Gamma=\hbar\lambda=\hbar/\tau$

Hamiltoniano con perturbación: $H=H_0+V$

$\Psi=\sum_n a_n(t)\psi_n e^{-iE_nt/\hbar}$ usando $i\hbar\frac{d\Psi}{dt}=H\Psi$

$\dot{a}_k=-\frac{i}{\hbar}\sum_n a_nV_{kn}e^{i\frac{E_k-E_n}{\hbar}t}, \quad V_{kn}=\int\psi_k^*V\psi_n d\tau$

$a_k=\frac{V_{km}\left(1-e^{i\frac{E_k-E_m}{\hbar}T}\right)}{E_k-E_m} \quad a_k^*a_k=4|V_{km}|^2\frac{\sin^2\left(\frac{(E_k-E_m)T}{2\hbar}\right)}{(E_k-E_m)^2}$

Regla de oro:

$\lambda(m\rightarrow k)=\frac{|a_k(T)|^2}{T}=\frac{2\pi}{\hbar}|V_{km}|^2\delta(E_k-E_m)$

$\lambda=\sum_{k\neq m}\lambda(m\rightarrow k)$

TEMA 3.3: Desintegración alfa y fisión

α decay:

Probabilidad de decay: $P_\alpha=\lambda\simeq\frac{v}{R}T \quad \tau_0\sim\frac{v}{R}$

$t_{nuc}=\frac{2R}{v_F}\simeq 2.6\cdot 10^{-23}\cdot A^{1/3}$

Coef. transmisión: $T_a\propto e^{-2G}$

Factor Gamow: $G=\frac{1}{\hbar}\sqrt{\frac{2m}{E}}Z_1Z_2e^2\gamma\left(\frac{R}{R'}\right)$

$\tau=\tau_0\cdot e^{2G} \quad \lambda=\tau_0^{-1}\cdot e^{-2G}$

$T_\alpha\simeq\frac{p^2c^2}{2m_\alpha c^2} \quad \tau_0=\frac{R}{c}\sqrt{\frac{mc^2}{2(V_0+E)}} \quad R=r_0\left(A_{F_1}^{1/3}+A_{F_2}^{1/3}\right)$

$\gamma(x)=arccos\sqrt{x}-\sqrt{x(1-x)} \quad \frac{Z_1Z_2e^2}{R'}=E=\frac{1}{2}mv^2$

$x=R/R'=E/V_c \quad V_c=\frac{Z_1Z_2e^2}{R} \quad \gamma\approx\pi/2-2\sqrt{x}$

Fisión:

$\Delta B=2B(A/2,Z/2)-B(A,Z)=$

$=a_s\left(1-2^{1/3}\right)A^{2/3}-a_c\left(2^{-2/3}-1\right)\frac{Z^2}{A^{1/3}}$

$\Delta E=a_sA^{2/3}\left(\frac{2}{5}\epsilon^2+\ldots\right)+a_cZ^2A^{-1/3}\left(-\frac{1}{5}\epsilon^2\ldots\right)$

$\Delta E<0$ si $\quad\frac{Z^2}{A}>\frac{2a_s}{a_c}\simeq 51$

TEMA 3.4: Desintegración beta

$Q_{\beta^-}=\left(M_{Z,A}-M_{Z+1,A}\right)c^2$

$Q_{\beta^+}=\left(M_{Z,A}-M_{Z-1,A}-2m_e\right)c^2$

$\lambda=\frac{2\pi}{\hbar}|\mathcal{M}_{if}|^2\frac{dN}{dE_T}, \quad$ con $\mathcal{M}_{if}=\int\psi_f^*\mathcal{V}\psi_i d^3r$

$|M_{if}^F|^2=\sum_{m_f}\left|\int\psi_f^*\left(\sum_k t_\pm^k\right)\psi_i d^3r\right|^2$

$dN_e=\frac{p_e^2dp_e}{2\pi^2\hbar^3}V=\frac{E_e(E_e^2-m_e^2c^4)^{1/2}dE_e}{2\pi^2\hbar^3c^3}V$

$\frac{d\lambda}{dE_e}=\frac{F(Z,E_e)|\mathcal{M}_{if}|^2}{2\pi^3\hbar^7c^6}E_e(E_e^2-m_e^2c^4)^{1/2}(E_T-E_e)^2$

Plot Kurie: $\left[\frac{\lambda'(E)}{F(Z,E)E\left(E^2-m_e^2c^4\right)^{1/2}}\right]^{1/2}$

$\lambda=\frac{m^5g^2c^4|\mathcal{M}_{if}|^2}{2\pi^3\hbar^7}f(Z,E_T) \quad ft_{1/2}=\frac{(2\ln 2)\pi^3\hbar^7}{g^2m^5c^4}\cdot\frac{1}{|\mathcal{M}_{if}|^2}$

$f=\frac{1}{m^5c^{10}}\int_{mc^2}^{E_T}F(Z,E)E\left(E^2-m^2c^4\right)^{1/2}(E_T-E)^2dE$

Transiciones: $\vec{l}_i=\vec{l}_f+\vec{\ell}+\vec{S}$

$l_i=l_f+\ell$ (Fermi) $\quad l_i=l_f+\ell+1$ (Gamow–Teller)

Transición	ℓ	$\Delta\pi$	Δl Fermi	Δl Gam-Tell
super-permitida	0	+	0	0
permitida	0	+	0	0, 1
primera prohibida	1	−	0, 1	0, 1, 2
segunda prohibida	2	+	1, 2	1, 2, 3
tercera prohibida	3	−	2, 3	2, 3, 4

TEMA 3.4: Desintegración gamma

$|l_i-l_f|\leq l\leq l_i+l_f$

$\Delta\pi$=1→ E par y M impar

$\Delta\pi$=-1→ E impar y M par

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1 fermi= 1fm $=10^{-15}$ m$=10^{-13}$ cm

1 barn $\rightarrow 10^{-24}cm^2=10^2\ fm^2$

1 u= (1/12) m $({}^{12}_{6}C)$=1.660538921 10¯27 kg= 931.494061 MeV/c^2

Integrales armónicos esféricos:

$\int|Y_\ell^\ell(\theta,\phi)|^2(3\cos^2\theta-1)\sin\theta\, d\theta\, d\phi=-\frac{2\ell}{2\ell+3}$

$Y_\ell^{\ell-1}(\theta,\phi)=\sqrt{2\ell+1}\cos\theta\, Y_{\ell-1}^{\ell-1}(\theta,\phi)$

$\int|Y_\ell^{\ell-1}(\theta,\phi)|^2(3\cos^2\theta-1)\sin\theta\, d\theta\, d\phi=-1+\frac{9}{2\ell+3}$

Para $j=l-1/2$ $\left(-1+\frac{9}{2\ell+3}\right)\frac{1}{2\ell+1}+\left(-\frac{2\ell}{2\ell+3}\right)\frac{2\ell}{2\ell+1}=\frac{2-2\ell}{1+2\ell}=-\frac{2j-1}{2j+2}$

Unidades radiación:

→ Actividad:

- Curie (Ci): 1Ci= $3.7\cdot 10^{10}$ desintegraciones/s
- Becquerel (Bq) : 1Bq= 1 des/s= $0.27\cdot 10^{-10}Ci$
- Rutherford (Rd): 1Rd=10^6 desintegraciones/s

→ Cantidad radiación:

- 1Roentgen= 1R= 1$esu/1cm^3$ de aire= $=2.58\cdot 10^{-4}$ C/kg de aire

→Dosis absorbida:

- 1rad=10^{-2} J/kg=100 ergs/g de material
- 1Gy=1 J/kg de material =100 rad

→ Dosis equivalente:

- 1rem=$w_R\times$ (dose in rads)
- Sievert (Sv): 1Sv=$w_R\times$ (dose in Gy)=100rem

Propiedades de las deltas:

$\delta(x)=\lim_{\lambda\rightarrow\infty}\sqrt{\frac{\lambda}{\pi}}e^{-\lambda x^2}=\lim_{\lambda\rightarrow\infty}\frac{\sin\lambda x}{\pi x}$

$=\lim_{\lambda\rightarrow\infty}\frac{1}{\lambda\pi}\frac{\sin^2\lambda x}{x^2}=\lim_{\varepsilon\rightarrow 0^+}\frac{\varepsilon}{\pi(x^2+\varepsilon^2)}$

$\delta(ax)=\frac{1}{|a|}\delta(x)$ $\quad$ a es una constante real cualquiera

$\delta(F(x))=\sum_j\frac{1}{|F'(x_j)|}\delta(x-x_j)$ $\quad x_j$0s simples de F(x)

$\lim_{\varepsilon\rightarrow 0^+}\frac{1}{x\pm i\varepsilon}=\mathrm{VP}\frac{1}{x}\mp i\pi\delta(x)$,

donde $\left\langle\mathrm{VP}\frac{1}{x},\phi\right\rangle=\lim_{\varepsilon\rightarrow 0^+}\int_{|x|\geq\varepsilon}dx\,\frac{\phi(x)}{x}$

TEMA 4.1: Relatividad especial

$\vec{a} \to M\vec{a}, \quad M = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$

$\vec{a}^{\mathrm{T}} \to (M\vec{a})^{\mathrm{T}} = \vec{a}^{\mathrm{T}} M^{\mathrm{T}}, \quad M^{\mathrm{T}} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$

$\vec{b}^{\mathrm{T}}\vec{a} \to (M\vec{b})^{\mathrm{T}} M\vec{a} = \vec{b}^{\mathrm{T}} M^{\mathrm{T}} M \vec{a} = \vec{b}^{\mathrm{T}} \mathbb{I} \vec{a} = \vec{b}^{\mathrm{T}} \vec{a}.$

Invarianza relativista: $E^2 = m^2 + |\vec{p}|^2$

$p = (E, \vec{p}) = (E, p_x, p_y, p_z) \quad p \cdot p = p^2 = E^2 - |\vec{p}|^2$

$p \cdot p \equiv p_\mu \eta^{\mu\nu} p_\nu \equiv p_\mu p^\mu = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} p_\mu \eta^{\mu\nu} p_\nu =$
$= p^\mu \eta_{\mu\nu} p^\nu$

$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}^{\mu\nu} \qquad p_\mu = \eta_{\mu\nu} p^\nu$

Transformadas de Lorentz: $p_\mu \to \Lambda_\mu{}^\nu p_\nu$

$\Lambda_\mu{}^\nu = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}$

$p \cdot q = p_\mu \eta^{\mu\nu} q_\nu \to p_\rho \Lambda^\rho{}_\mu \eta^{\mu\nu} \Lambda^\sigma{}_\nu q_\sigma = p_\mu \eta^{\mu\nu} q_\nu$

$p_\mu \Lambda^\mu{}_\rho \eta^{\rho\sigma} \Lambda^\nu{}_\sigma q_\nu = p_\mu \eta^{\mu\nu} q_\nu \quad \Lambda^\mu{}_\rho \eta^{\rho\sigma} \Lambda^\nu{}_\sigma = \eta^{\mu\nu}$

Ec. Klein-Gordon: $\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\phi(\vec{x}, t) = 0$

$\phi(\vec{x}, t) = e^{-ip\cdot x} \quad p \cdot x = Et - \vec{p} \cdot \vec{x}$

$(E^2 - |\vec{p}|^2 - m^2)\phi(\vec{x}, t) = 0$ sols: $E = \pm\sqrt{m^2 + |\vec{p}|^2}$

Lagrangiano KG: $\frac{\partial^2}{\partial t^2}\phi = \nabla^2\phi - m^2\phi$

E. cinética: $k = \frac{1}{2}\left(\frac{\partial}{\partial t}\phi\right)^2$ Hamil: H$= \int d\vec{x}(k+u)$

E. total: $k + u = \frac{1}{2}\left(\frac{\partial}{\partial t}\phi\right)^2 + \frac{1}{2}\vec{\nabla}\phi \cdot \vec{\nabla}\phi + \frac{m^2}{2}\phi^2$

Ec. Dirac: $(i\gamma_\mu \partial^\mu - m)\psi = 0$

$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}$ Clifford algebra: $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$

$\sigma_\mu = (\mathbb{I}, \sigma_1, \sigma_2, \sigma_3), \quad \bar{\sigma}_\mu = (\mathbb{I}, -\sigma_1, -\sigma_2, -\sigma_3).$

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\sigma_i \sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k \quad det(p \cdot \sigma) = p^2$

$\vec{\nabla} \cdot \vec{E} = \rho, \quad \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J}, \quad \vec{\nabla} \cdot \vec{B} = 0, \quad \frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} = 0$

$J^\mu = (\rho, \vec{J})^\mu \quad F_{\mu\nu} = -F_{\nu\mu}$

$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}^{\mu\nu} \qquad F_{\mu\nu} \to \Lambda^\rho_\mu \Lambda^\sigma_\nu F_{\rho\sigma}$

$\partial_\mu F^{\mu\nu} = J^\nu$

$\partial_\mu F^{\mu 0} = \partial_0 F_{00} - \partial_1 F_{10} - \partial_2 F_{20} - \partial_3 F_{30} = \vec{\nabla} \cdot \vec{E} = \rho$

$\partial_\mu F^{\mu\nu} - J^\nu = 0 \quad \Lambda^\sigma_\nu (\partial_\mu F^{\mu\nu} - J^\nu) = 0$

Identidad Bianchi: $\partial_\mu F_{\nu\rho} + \partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho_\mu} = 0$

Potencial : $A_\mu = (V, \vec{A})_\mu \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$x'^\mu = \Lambda^\mu_\nu x^\nu \quad J = det\frac{\partial x'^\mu}{\partial x^\nu}$

$E = \sqrt{p^2 + m^2} \simeq m + \frac{p^2}{2m}$

Partícula sin masa: $|\vec{p}| = E \quad \vec{p}_1 \cdot \vec{p}_2 = |\vec{p}_1||\vec{p}_2|cos\theta$

TEMA 4.2: Teoría de grupos

1. Elemento identidad: $a \cdot 1 = 1 \cdot a = a$
2. Inversa: $a \cdot a^{-1} = a^{-1} \cdot a = 1$
3. Grupo cerrado: $a \cdot b = c$ en el grupo
4. Operaciones asoaciativas: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Algebra de Lie: $\mathbb{M} = e^{i\alpha^a T^a}$

Conmutador: $[T^a, T^b] = if^{abc}T^c$

$[\hat{L}_i, \hat{L}_j] = i\epsilon_{ijk}\hat{L}_k \quad [\hat{L}_x, \hat{L}_y] = i\hat{L}_z \quad [\frac{\sigma_i}{2}, \frac{\sigma_j}{2}] = i\epsilon_{ijk}\frac{\sigma_k}{2}$

Isoespín: $\langle p|p\rangle = \langle n|n\rangle = 1 \quad \langle p|n\rangle = \langle n|p\rangle = 0$

Singlete: $\frac{1}{\sqrt{2}}(|pn\rangle - |np\rangle)$

Triplete: $|pp\rangle \quad \frac{1}{\sqrt{2}}(|pn\rangle + |np\rangle) \quad |nn\rangle$

TEMA 4.3: Regla oro Fermi y diagramas Feynmann

Prob scattering of $p_1, p_2 ... p_n = |_{out}\langle p_1 p_2 ... p_n | p_A p_B\rangle_i\, n|^2$

De Broglie: $\lambda_{dB} = \frac{\hbar}{|\vec{p}|}$ Compton: $\lambda_C = \frac{\hbar}{mc} \quad v = \frac{p^2}{E}$

pp scattering cross section:

$\sigma = \frac{1}{2E_A}\frac{1}{2E_B}\frac{1}{|v_A - v_B|}|_{out}\langle f|p_A p_B\rangle_i\, n|^2$

Regla de oro de Fermi:

$\sigma = \frac{1}{2E_A}\frac{1}{2E_B}\frac{1}{|\vec{v}_A - \vec{v}_B|}\int \prod_{i=1}^{n}\left[\frac{d^4 p_i}{(2\pi)^4} 2\pi\delta(p_i^2 - m_i^2)\right] |\mathcal{M}(A + B \to 1 + 2 + \cdots + n)|^2 (2\pi)^4 \delta^{(4)}\left(p_A + p_B - \sum_{i=1}^{n} p_i\right)$

$\mathcal{M}(A + B \to 1 + 2 + ... + n) =_{out} \langle p_1 p_2 ... p_n | p_A p_B\rangle_i\, n|^2$

TEMA 5.1: Quantum electrodinamics

Massless Dirac Ec. : $i\gamma \cdot \partial\psi = 0$

$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ Weyl eqs: $i\bar{\sigma} \cdot \partial\psi_L = 0 \quad i\sigma \cdot \partial\psi_R = 0$

$\psi_R = u_R e^{-ip\cdot x} \quad p = E(1, sin\theta cos\phi, \sin\theta sun\phi, cos\theta)$

$u_R(p) = v_L(p) = \sqrt{2E}\begin{pmatrix} e^{-i\phi/2}\cos\frac{\theta}{2} \\ e^{i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}$

$u_L(p) = v_R(p) = \sqrt{2E}\begin{pmatrix} e^{-i\phi/2}\sin\frac{\theta}{2} \\ -e^{i\phi/2}\cos\frac{\theta}{2} \end{pmatrix}$

$u_R^\dagger u_R = u_L^\dagger u_L = v_R^\dagger v_R = v_L^\dagger v_L = 2E \quad u_R^\dagger u_L = v_L^\dagger v_R = 0$

$e_L^+ e_R^- \to \mu_L^+ \mu_R^-, \quad e_L^+ e_R^- \to \mu_R^+ \mu_L^-,$

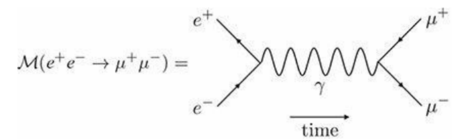
$e_R^+ e_L^- \to \mu_L^+ \mu_R^-, \quad e_R^+ e_L^- \to \mu_R^+ \mu_L^-$

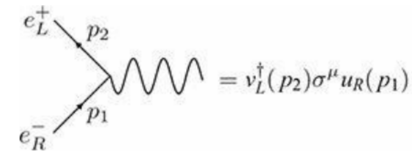
$\sigma = \frac{\pi}{2E_{\mathrm{cm}}^2}\left(\frac{e^2}{4\pi}\right)^2 \int d\cos\theta\,(1 + \cos^2\theta)$

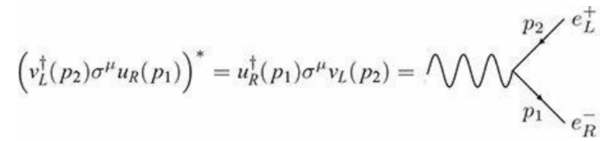
Cte de estructura fina: $\alpha = \frac{e^2}{4\pi}$

Differential cross section:

$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\cos\theta} = \frac{\pi\alpha^2}{2E_{\mathrm{cm}}^2}(1 + \cos^2\theta)$







$\mathcal{M}(e_L^+ e_R^- \to \mu_L^+ \mu_R^-)^* = \mathcal{M}(\mu_R^+ \mu_L^- \to e_R^+ e_L^-) = \mathcal{M}(e_R^+ e_L^- \to \mu_R^+ \mu_L^-).$

$|\mathcal{M}(e_R^+ e_L^- \to \mu_R^+ \mu_L^-)|^2 = |\mathcal{M}(e_L^+ e_R^- \to \mu_L^+ \mu_R^-)|^2 = e^4(1 + \cos\theta)^2,$

$|\mathcal{M}(e_R^+ e_L^- \to \mu_L^+ \mu_R^-)|^2 = |\mathcal{M}(e_L^+ e_R^- \to \mu_R^+ \mu_L^-)|^2 = e^4(1 - \cos\theta)^2.$

TEMA 5.2: Quarks y gluones

$p_1 + p_2 = p_3 + p_4$

Variables de Mandelstam:

$s \equiv (p_1 + p_2)^2 = (p_3 + p_4)^2,$

$t \equiv (p_1 - p_4)^2 = (p_2 - p_3)^2,$

$u \equiv (p_1 - p_3)^2 = (p_2 - p_4)^2.$

$s + t + u = 0$

$\frac{d\sigma(e^+e^- \to q\bar{q})}{d\cos\theta} = \frac{3\pi\alpha^2}{2E_{\mathrm{cm}}^2} Q_q^2 (1 + \cos^2\theta).$

Drell-Yan process: pp$\to e^+e^- + X \quad q\bar{q} = \to ee^-$

$\hat{s} = (p + p')^2 = (k + k')^2, \quad \hat{t} = (p - k)^2 = (p' - k')^2,$

$\hat{u} = (p - k')^2 = (p' - k)^2. \quad \hat{s} = E_{cm}^2$

$\sigma(e^+e^- \to q\bar{q}) = 4\frac{\pi\alpha^2}{\hat{s}}\sum_{q \text{ quarks}} Q_q^2.$

$\sigma(q\bar{q} \to e^+e^-) = \frac{4}{9}\frac{\pi\alpha^2}{\hat{s}}\sum_{q \text{ quarks}} Q_q^2$

$s \equiv (P_1+P_2)^2 = 2P_1\cdot P_2 \quad \hat{s} = x_1 x_2 s = 2(x_1P_1)\cdot(x_2P_2)$
$\frac{d^2\sigma(q\bar{q}\to e^+e^-)}{dx_1\,dx_2} = \frac{4}{9}\frac{\pi\alpha^2}{\hat{s}}\sum_{\text{q quarks}} Q_q^2 f_q(x_1) f_{\bar{q}}(x_2),$
$\frac{d^2\sigma(pp\to e^+e^-)}{dx_1\,dx_2} = \frac{4}{9}\frac{\pi\alpha^2}{s}\sum_{\text{q quarks}} Q_q^2 \frac{f_q(x_1)}{x_1}\frac{f_{\bar{q}}(x_2)}{x_2} \quad Q^2 = \hat{s} = x_1x_2$
Rapidity: $y = \frac{1}{2}log\frac{E+p_z}{E-p_z} = \frac{1}{2}log\frac{x_1}{x_2}$
$E = x_1\frac{E_{\text{cm}}}{2} + x_2\frac{E_{\text{cm}}}{2} = \frac{x_1+x_2}{2}E_{\text{cm}}, \quad x_1 = \sqrt{\frac{Q^2}{s}}e^y$
$p_z = x_1\frac{E_{\text{cm}}}{2} - x_2\frac{E_{\text{cm}}}{2} = \frac{x_1-x_2}{2}E_{\text{cm}}. \quad x_2 = \sqrt{\frac{Q^2}{s}}e^{-y}$
$J = \begin{vmatrix} \frac{\partial x_1}{\partial Q^2} & \frac{\partial x_1}{\partial y} \\ \frac{\partial x_2}{\partial Q^2} & \frac{\partial x_2}{\partial y}\end{vmatrix} = \begin{vmatrix} \frac{e^y}{2\sqrt{s}Q^2} & \sqrt{\frac{Q^2}{s}}e^y \\ \frac{e^{-y}}{2\sqrt{s}Q^2} & -\sqrt{\frac{Q^2}{s}}e^{-y}\end{vmatrix} = \frac{1}{s}.$
$\frac{d^2\sigma(pp\to e^+e^-)}{dQ^2\,dy} = \frac{d^2\sigma(pp\to e^+e^-)}{dx_1\,dx_2}\cdot J$
$= \frac{4\pi\alpha^2}{9Q^2s}\sum_{\text{q quarks}} Q_q^2 f_q\left(\sqrt{\frac{Q^2}{s}}e^y\right) f_{\bar{q}}\left(\sqrt{\frac{Q^2}{s}}e^{-y}\right).$

TEMA 5.3: Cromodinámica cuántica

$\psi \to \mathbb{U}\psi \quad \mathbb{U} = e^{i\alpha^aT^a} \quad [T^a,T^b] = if^{abc}T^c \quad \psi = \mathbb{U}\psi'$
$\mathscr{L} = \bar{\psi}(i\gamma\cdot\partial + g\gamma\cdot A^aT^a)\psi \quad A_\mu^a \to A_\mu^a + \Delta A_\mu^a$
Covariant dervivative: $D_\mu = \partial_\mu - igA_\mu^a T_\mu^a$
$D_\mu \to \mathbb{U}D_\mu\mathbb{U}^\dagger \quad \mathscr{L} = \bar{\psi}i\gamma\cdot D\psi$
$D_\mu D_\nu - D_\nu D_\mu = [D_\mu, D_\nu]$
$= \partial_\mu\partial_\nu - ig\partial_\mu A_\nu^a T^a - igA_\mu^a T^a\partial_\nu - g^2A_\mu^aA_\nu^bT^aT^b - \partial_\nu\partial_\mu$
$\quad + ig\partial_\nu A_\mu^a T^a + igA_\nu^a T^a\partial_\mu + g^2A_\mu^aA_\nu^bT^bT^a$
$= -ig\left(\partial_\mu A_\nu^a T^a - \partial_\nu A_\mu^a T^a - igA_\mu^aA_\nu^b[T^a,T^b]\right)$
$\equiv -igF_{\mu\nu}^a T^a. \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^bA_\nu^c,$
$tr[T^aT^b] = \frac{1}{2}\delta^{ab}$
Killing form: $\mathscr{L}_{QCD} = -\frac{1}{4}F_{mu\nu}F_{\mu\nu}F^{\mu\nu a} + \bar{\psi}i\gamma\cdot D\psi$
Gluon: $F_{\mu\nu}^a|_{g=0} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$
$\beta(\alpha) = Q\frac{d\alpha}{dQ} \quad \beta_{\text{SM}}(\alpha) = \frac{2}{3}\frac{\alpha^2}{\pi}\sum_{\text{fermions } i} Q_i^2,$
$\alpha(Q) = \frac{\alpha(Q_0)}{1-\frac{2}{3\pi}\alpha(Q_0)\log\left(\frac{Q}{Q_0}\right)}.$
Strong coupling: $\alpha_s = \frac{g^2}{4\pi}$
$\alpha_s(Q) = \frac{\alpha_s(Q_0)}{1+\frac{\alpha_s(Q_0)}{2\pi}\left(11-\frac{2}{3}n_f\right)\log\left(\frac{Q}{Q_0}\right)},$
Bianchi: $(D_\mu F_{\nu\rho})^a + (D_\rho F_{\mu\nu})^a + (D_\nu F_{\rho\mu})^a = 0.$
$f^{abd}f^{dce} + f^{bcd}f^{dae} + f^{cad}f^{dbe} = 0.$
$[T^a,[T^b,T^c]] + [T^b,[T^c,T^a]] + [T^c,[T^a,T^b]] = 0.$

TEMA 6.1: Violación de la paridad

Paridad: $\mathbb{P}\vec{x} = -\vec{x} \quad \mathbb{P}\vec{v} = -\vec{v} \quad \mathbb{P}\vec{L} = \vec{L}$
Pseudoscalars: $\mathbb{P}\pi^0 = -\pi^0$
$\mathbb{P}\vec{E} = -\vec{E}, \quad \mathbb{P}\vec{\nabla} = -\vec{\nabla}, \quad \mathbb{P}\rho = \rho, \quad \mathbb{P}\vec{B} = \vec{B}$
$\mathbb{P}\vec{J} = -\vec{J}, \quad \mathbb{P}\frac{\partial}{\partial t} = \frac{\partial}{\partial t} \quad \mathbb{P}\theta = \pi - \theta, \quad \mathbb{P}\phi = \phi + \pi.$
$\mathbb{P}\begin{pmatrix} u_R(p) \\ u_L(p)\end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix} u_R(p) \\ u_L(p)\end{pmatrix} = \begin{pmatrix} u_L(p) \\ u_R(p)\end{pmatrix}.$
Time reversal: $\mathbb{T}t = -t \quad \mathbb{T}\vec{p} = -\vec{p}$
$\mathbb{T}\vec{E} = \vec{E}, \quad \mathbb{T}\vec{\nabla} = \vec{\nabla}, \quad \mathbb{T}\rho = \rho, \quad \mathbb{T}\vec{B} = -\vec{B},$
$\mathbb{T}\vec{J} = -\vec{J}, \quad \mathbb{T}\frac{\partial}{\partial t} = -\frac{\partial}{\partial t} \quad \mathbb{T}\hat{H} = \hat{H} \quad \mathbb{T}i = -i$
$\mathbb{T}u_R = \epsilon u_R = \begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix}u_R.$
Conjugador de carga: $\mathbb{C}q = -q$
$\mathbb{C}\vec{E} = -\vec{E}, \quad \mathbb{C}\vec{\nabla} = \vec{\nabla}, \quad \mathbb{C}\rho = -\rho, \quad \mathbb{C}\vec{B} = -\vec{B},$
$\mathbb{C}\vec{J} = -\vec{J}, \quad \mathbb{C}\frac{\partial}{\partial t} = \frac{\partial}{\partial t} \quad \mathbb{C}u = v \quad \mathbb{C}u_R = v : L$
$\mathbb{CPT}J = H^\dagger$

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$e^{iX} = \mathbb{I} + iX - \frac{X^2}{2} - i\frac{X^3}{6} + ...$
$\int dx\,\delta(y - f(x)) = \int dx\,\frac{1}{\left|\frac{df}{dx}\right|}\delta(x - f^{-1}(y))$
$f(x) = \tau \quad x = f^{-1}(\tau) \quad d\tau = \frac{df}{dx}dx \quad dx = \frac{1}{\frac{df}{dx}}d\tau$
$log_{10}x = \frac{lnx}{ln10}$

MASAS Y CARGAS

Quarks			
Símbolo	**Masa (MeV/c^2)**	**Carga**	**Espín**
u	≈ 2.2	$+\frac{2}{3}$	$\frac{1}{2}$
d	≈ 4.7	$-\frac{1}{3}$	$\frac{1}{2}$
c	≈ 1280	$+\frac{2}{3}$	$\frac{1}{2}$
s	≈ 96	$-\frac{1}{3}$	$\frac{1}{2}$
t	≈ 173100	$+\frac{2}{3}$	$\frac{1}{2}$
b	≈ 4180	$-\frac{1}{3}$	$\frac{1}{2}$

Leptones			
Símbolo	**Masa (MeV/c^2)**	**Carga**	**Espín**
e	0.511	-1	$\frac{1}{2}$
μ	105.66	-1	$\frac{1}{2}$
τ	1776.86	-1	$\frac{1}{2}$
ν_e	< 1	0	$\frac{1}{2}$
ν_μ	< 0.17	0	$\frac{1}{2}$
ν_τ	< 18.2	0	$\frac{1}{2}$

Bosones de Gauge			
Símbolo	**Masa (MeV/c^2)**	**Carga**	**Espín**
γ	0	0	1
g	0	0	1
Z^0	91190	0	1
$W^\pm$	80390	± 1	1

Bosón Escalar			
Símbolo	**Masa (MeV/c^2)**	**Carga**	**Espín**
H	124970	0	0

Hadrones Compuestos			
Símbolo	**Masa (MeV/c^2)**	**Carga**	**Espín**
p	938.272	$+1$	$\frac{1}{2}$
n	939.565	0	$\frac{1}{2}$
π^+	139.570	$+1$	0
π^0	134.977	0	0
π^-	139.570	-1	0