Universidad del País Vasco — Euskal Herriko Unibertsitatea

COURSE GUIDE 2025/26

Faculty 345 - Faculty of Engineering - Bilbao **Cycle** .

Degree GITECI30 - Bachelor`s Degree in Industrial Technology Engineering **Year** First year

COURSE

27303 - Algebra **Credits, ECTS:** 9

COURSE DESCRIPTION

The aim of this course, which is part of the Basic Training Module, is to provide students a basic training in Mathematics that will enable them to access to the study of other disciplines of their curriculum.

Firstly, we begin with a review of matrices, determinants and systems of linear equations, to continue with the study of vector spaces and normed spaces. We work in depth the concepts of linear independence and basis. Inner products and induced norms are presented, emphasizing the concept of orthogonality and studying Gram-Smidth's method to get orthogonal bases. Then, linear transformations are discussed, defining the concepts of eigenvalue and eigenvector; diagonalization and triangularization by similarity are presented.

The last part of this course is focused on solving, from a numerical point of view, some problems studied during the course. Thus, Numerical Methods for solving systems of linear equations, and to get eigenvalues and eigenvectors are presented. Finally, and as a direct application of the theory studied in Chapter 3, we study the Least-Squares approximation method.

This subject provides students with the knowledge and skills needed to properly follow other disciplines of the curriculum both regarding Mathematics (Differential Equations and Numerical Methods) and other disciplines (Chemistry, Physics, Mechanics, Electrical Engineering, Thermodynamics, Structures, Fluid Mechanics, etc.) where a lot of models have a linear nature and where the matrix language is widely used.

COMPETENCIES/LEARNING RESULTS FOR THE SUBJECT

·Ability to solve mathematical problems that may arise in the field of engineering. ·Aptitude to apply their knowledge of linear algebra, geometry, differential geometry, differential and integral calculus, differential and partial differential equations, numerical methods, numerical algorithms, statistics and optimization (M01CM01 Competence).

Getting these competences will enable students:
-to present, in a clear way, the fundamental concepts of linear algebra.
-to solve mathematical problems applying their knowledge of linear algebra and numerical methods by using a mathematical software.

Theoretical and Practical Contents

I.LINEAR ALGEBRA

Chapter 1: MATRIX ALGEBRA AND SYSTEMS OF LINEAR EQUATIONS.
Definitions.- Matrix Operations.- Special Matrices.- Determinants and their properties.- The Inverse of a Matrix..- Partitioned Matrices.- Powers of a matrix.- Elementary Row Operations.- Systems of linear equations: Gauss Elimination.

Chapter 2: VECTOR SPACES.
Definition of vector space and some properties.- Vector Subspaces.- Linear dependence and Linear independence.- Basis, dimension and coordinates.- Change of Basis: The change-of-coordinates Matrix.- Dimension and Equations of a subspace.- Operations with vector subspaces.- Definition of Vector Norm.- Norms on Rn.

Chapter 3: INNER PRODUCT SPACES.
Definition and Properties of inner product..- Schwarz inequality.- Norm induced by the inner product.- Matrix Representation of Inner Product.- Change of Basis.- Orthogonal and Orthonormal Sets.- The Gram-Schmidt Orthogonalization Process.- Orthogonal Subspaces.

Chapter 4: LINEAR TRANSFORMATIONS.
Definition and Properties of Linear Transformations.- Image Space and Null Space of a Linear Transformation.- Classification of Linear Transformations.- Matrix Representation of Linear Transformations. The Matrix of a Linear Transformation.- Rank of a Linear Transformation.- Change of Basis.

Chapter 5: EIGENVALUES AND EIGENVECTORS: DIAGONALIZATION.
Introduction.- Eigenvalues and Eigenvectors: Definitions and basic properties.- Diagonalizing a Square Matrix.- Diagonalization of Symmetric Matrices.

Chapter 6: JORDAN FORM BY USING SIMILARITY TRANSFORMATIONS.
Introduction.- Jordan Form of 2x2 and 3x3 matrices.- Jordan´s classification Theorem.- Algorithm to obtain the Jordan Form of a Matrix.

II.LINEAR ALGEBRA (NUMERICAL METHODS)

Chapter 1: INTRODUCTION TO NUMERICAL ANALYSIS.
Introduction.- Types of Errors.- Computer arithmetic.- Aspects to analyze when choosing an algorithm.- Types of numerical methods.

Chapter 2: NUMERICAL METHODS TO SOLVE SYSTEMS OF LIN. EQUATIONS.
Introduction.- Gaussian Elimination: LU Factorization.- Compact Elimination Methods: the Doolittle, the Crout and the Cholesky Methods.- Calculating the inverse of a matrix.- Gaussian Elimination with scaled partial pivoting.- Iterative Methods for solving linear systems: the Jacobi and the Gauss-Seidel Methods. Convergence of the Jacobi and the Gauss-Seidel Methods.- Iterative Methods for Eigenvalues: The Power Method.

Chapter 3: LEAST SQUARE APPROXIMATION.
Introduction to the problem. Characterization of the best approximating element.- Continuous Least Squares Approximation: Orthogonal Bases. Orthogonal Polynomials.- Discrete Least Squares Approximation.

TEACHING METHODS

* Lectures and exercises solving classes, in which the different topics covered by the subject are presented and dealt with. Some exercises will be given to the students to be solved as personal work.
* Seminars in which additional tasks will be carried out (tests, oral presentations of previously prepared exercises, etc.)
* Computer laboratories, in which students will use the Mathematica package to solve problems of linear algebra and will be introduced to Numerical Methods programming.
* eGela virtual platform, through which students have access to different resources related to the subject.
* Office hours, for students to solve questions with their teachers.

TYPES OF TEACHING

Types of teaching	M	S	GA	GL	GO	GCL	TA	TI	GCA
Hours of face-to-face teaching	60	6	15		9				
Horas de Actividad No Presencial del Alumno/a	90	9	22,5		13,5				

Legend:
M: Lecture-based
S: Seminar
GA: Applied classroom-based groups
GL: Applied laboratory-based groups
GO: Applied computer-based groups
GCL: Applied clinical-based groups
TA: Workshop
TI: Industrial workshop
GCA: Applied fieldwork groups

Evaluation methods

- Continuous evaluation
- End-of-course evaluation

Evaluation tools and percentages of final mark

- Written test, open questions 80%
- Exercises, cases or problem sets 10%
- Multiple choice tests, team-work, oral presentations,... 10%

ORDINARY EXAMINATION PERIOD: GUIDELINES AND OPTING OUT

1st Call: Regular Exam

The assessment of the learning results will be of a mixed type and will consist of:
1. Continuous assessment (20% of the final grade):
  - 10% corresponding to the Seminar marks (Mark Ns)
  - 10% corresponding to the "Mathematica" Labs (Mark Ni)
2. Final exam (80% of the final grade):
  - 40% corresponding to a written exam about the 1st Semester contents. (Mark Ne1).
    Exceptionally, if the circumstances require it, a part of the subject may be evaluated by performing different tasks
  - 40% corresponding to a written exam about the 2nd Semester contents.(Mark Ne2).
    Exceptionally, if the circumstances require it, a part of the subject may be evaluated by performing different tasks

The student's final grade (Mark Nf) will be obtained by applying the weighted average of the grades corresponding to all

the aforementioned exams and activities.

Remarks:

1)In general, during the development of assessment tests that will be carried out throughout the course, the use of books or notes, as well as telephone, electronic and computing devices will be prohibited, except:
-A calculator that is not graphic or programmable, during the written exam on the 2nd Semester contents.
-The corresponding computer assigned to the student by the teacher in the "Mathematica" quizzes.
Any modification of the material allowed in the different quizzes of the subject will be notified by the teacher well in advance of carrying out such quizzes.

2)A mid-term written exam will be held at the end of the first Semester, about this Semester contents. Those students who pass this exam may not sit for these contents again in the final exam.

3)The final grade corresponding to the written exams, Ne, will be the average grade of Ne1 and Ne2. To pass this subject it is compulsory to obtain a minimum grade of 4.5 out of 10 in each of the two written exams. Otherwise, the students will fail with a maximum grade of 4.5.

4)In the same way, to pass this subject it is compulsory to obtain a minimum grade of 5 out of 10 in the final grade, Nf. If not, the student will fail.

5)For those students that wish to decline to sit for this 1st Call, it is enough not taking this final exam.

In any case, students will have the right to be evaluated by means of the final assessment system, regardless of whether or not they have participated in the continuous assessment system. In order to do this, students must follow the instructions of Article 8 (Chapter 2) related to the Assessment Systems corresponding to the Rule of the UPV/EHU about the evaluation of students in official Degrees. In these cases, the following percentages will be applied:
-80% corresponding to a written exam about the contents of the Linear Algebra course. This exam will consist of two parts corresponding to the 1st and 2nd semester contents (Marks Ne1 and Ne2 respectively).
-20% corresponding to a quiz carried out to assess the learning results acquired through the Seminar contents and "Mathematica" labs.(Mark Ns/i).

The student's final grade (Mark Nf) will be obtained by applying the weighted average of the grades corresponding to the two aforementioned exams and activities, being in this case also applicable remarks 1), 3), 4) and 5) indicated above.

EXTRAORDINARY EXAMINATION PERIOD: GUIDELINES AND OPTING OUT

2nd Call: Resit Exam

In this case, the final assessment system will be based on the following percentages:
-90% corresponding to a written exam about the course contents in the classroom (Mark Ne)
-10% corresponding to a "Mathematica" exam in the computer lab (Mark Ni)

The student's final grade (Mark Nf) will be obtained by applying the weighted average of the grades corresponding to the two aforementioned exams, being in this case also applicable remarks 1), 4) and 5) indicated above.

Those students who, having attended the first call and failing it, got an average grade greater than or equal to 5 over 10 in the computer lab part, do not need to take this part again in the second call. That is, in that case they can maintain the lab grade for the extraordinary call, unless they want to sit the lab again, in which case the valid grade will be the last one, regardless of whether it is better or not.

MANDATORY MATERIALS

BIBLIOGRAPHY

Basic bibliography

In Spanish:
· Profesores de Algebra Lineal. Dpto. de Matemática Aplicada de la ETSI Bilbao, 2013: Algebra Lineal: Notas de Clase. Servicio de publicaciones de la Escuela Técnica Superior de Ingeniería de Bilbao.
· Profesores de Algebra Lineal. Dpto. de Matemática Aplicada de la ETSI Bilbao, 2013: Algebra Lineal (Métodos Numéricos): Notas de Clase. Servicio de publicaciones de la Escuela Técnica Superior de Ingeniería de Bilbao.
· Bravo, E. y otros, 2002: Exámenes resueltos de Algebra Lineal y Matemáticas I (1996-2000). Servicio Editorial de la Universidad del País Vasco.
· Bravo, E. y otros, 2006: Exámenes resueltos de Algebra Lineal y Matemáticas I (2001-2005). Servicio Editorial de la

Universidad del País Vasco.
· De Burgos, J, 2007: Álgebra Lineal. Definiciones, Teoremas y Resultados, García-Maroto Editores, Madrid.
· Hill, R., 1997: Álgebra Lineal Elemental con Aplicaciones, Prentice Hall Hispanoamericana, México.

In English:
· Linear Algebra: Course Notes (English Version of Algebra Lineal- Notas de Clase), 2017. eGela Platform.
· Linear Algebra (Numerical Methods):Course Notes(English Version of Algebra Lineal(Métodos Numéricos): Notas de Clase), 2017. eGela Platform.
·Exam Exercise Sets (2010-2017). eGela Platform.

Detailed bibliography

In Spanish:
· Burgos, J., 2006: Algebra Lineal y geometría cartesiana, 3ª Edición, Mc Graw-Hill, Madrid.
· Grossman, S.I., 2008: Álgebra lineal, 6ª Edición, Mc Graw Hill Interamericana Editores.
· Hernández, E., 1999: Algebra y geometría, Addison-Wesley.
· Rojo, J., Martín, I., 2005: Ejercicios y problemas de álgebra lineal, McGraw-Hill.
· Rojo, J., 2007: Algebra lineal, McGraw-Hill Interamericana.
· Burden, R.L., Faires,J.D., 2009: Análisis Numérico, Grupo Ed. Iberoamérica.
· Conte, S.D. y de Boor, C., 2005: Análisis Numérico, McGraw-Hill.
· Kincaid, D. y Cheney, W.,2007: Análisis Numérico. Las matemáticas del cálculo científico, Addison-Wesley Iberoamericana.

In English:
· Lay, D.C.,2006: Linear Algebra and its applications, 3rd Edition (Update),Pearson Education.
· Leon, S.J., 2010: Linear Algebra with applications.8th Edition.Pearson Education International.
· Noble, B., Daniel, J.W., 1988: Applied linear algebra. 3th Edition, Pretince-Hall, New Jersey
· Strang, G., 2009: Introduction to Linear Algebra, 4th Edition, Wellesley-Cambridge Press.
· Burden, R.L., Faires,J.D.: Numerical Analysis, 8th Edition. International Student Edition.

Journals

Web sites of interest

· http://www.ehu.eus
· http://egela.ehu.eus
· http://www.divulgamat.net/
· http://www.wolfram.com/products/mathematica
· http://ocw.mit.edu
· http://www.laylinalgebra.com

OBSERVATIONS