Economic Modelling 62 (2017) 82–89

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/econmod

Modelling countervailing incentives in adverse selection models: A synthesis☆

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ARTICLE INFO

JEL:
D82
L50

keywords:
Principal-agent relationships
Adverse selection
Countervailing incentives

ABSTRACT

This paper is concerned with countervailing incentives in the adverse selection problems that typically arise in principal-agent relationships when the agent has private information. These incentives are present when the agent is tempted to either overstate or understate his private information depending upon the specific realization of his type. These problems were first analyzed by Lewis and Sappington (1989) and have been characterized and extended by Maggi and Rodríguez-Clare (1995a) and Jullien (2000). In this paper we propose a simple method of characterizing countervailing incentives in which the key element is the analysis of the properties of the full information problem. Our method for solving the principal problem, once identified the presence of countervailing incentives, follows closely the Baron's (1989) approach, which does not require using optimal control theory. The methodology we present can be easily applied to many different economic settings. For example, in health economics, an insurer (or a hospital manager) might act as a principal and a physician as an agent. In labor settings, an employer may play the role of principal and a worker may act as the agent. In regulated industries, the regulatory agency might act as a principal designing incentive schemes for firms (the agents). In environmental regulation or resource exploitation, the principal might be an international agency dealing with national governments or firms.

1. Introduction

This paper is concerned with countervailing incentives in the adverse selection problems that typically arise in principal-agent relationships when the agent has private information. We propose a simple method of characterizing countervailing incentives in adverse selection problems. The key element in our proposal is to analyze the properties of the full information problem. One relevant advantage of our methodology is that it allows the resolution of the principal problem without using optimal control theory. This paper may be seen as a step-by-step guide to apply adverse selection models, characterized by countervailing incentives, to models of health economics, monopoly regulation, environmental regulation and others.

Most of the existing principal-agent models under adverse selection concern settings where the agent (he) has a systematic incentive to always overstate or to always understate his private information. The results are well known in the literature: the principal (she) deviates from the full information contract (either below or above the full information levels) in order to reduce informational rents. This incentive to exaggerate private information may, in certain circumstances, be tempered by a countervailing incentive to understate private information. That is, the agent might be tempted either to overstate or to understate his private information depending upon the specific realization of his type. When countervailing incentives arise, performance is distorted both above and below the levels under full information, and the agent's informational rents typically increase with the realization of his private information over some ranges, and decrease over other ranges.

Much research has analyzed the way countervailing incentives affect some specific agency problems, including Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995a, 1995b) and Jullien (2000). However, to the best of our knowledge, there are no general results in the literature characterizing the presence of countervailing incentives in a general framework. This is the contribution of this paper. We characterize the existence of countervailing incentives under adverse selection through the analysis of necessary and sufficient conditions. This new modelling of the principal-agent problem with countervailing incentives allows us to solve this problem without the

☆ Financial support from the Ministerio de Economía y Competitividad (ECO2012-31626, ECO2015-66027-P), and from the Departamento de Educación, Política Lingüística y Cultura del Gobierno Vasco (IT869-13) is gratefully acknowledged. We would like to thank the Editor, two anonymous referees and Ignacio Palacios-Huerta for helpful comments.

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http://dx.doi.org/10.1016/j.econmod.2017.01.007
Received 18 March 2016; Received in revised form 16 December 2016; Accepted 12 January 2017
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need to use optimal control, as well as to identify ex ante whether the contract is pooling for some types. As a result, the usual method followed in adverse selection models can be used to analyze a number of related issues that have attracted considerable attention in recent years such as partially altruistic agents in health economics, labor contracts, limited liability, environmental regulation and others.

The paper is organized as follows. Section II presents the general model. In Section III, we characterize the full information case. In Section IV we analyze the general contract under private information and state the main result of the paper. In Theorem 1 we identify the exact conditions under which general incentive problems are characterized by the existence of countervailing incentives. We also state a general and very simple method to obtain the optimal contract under private information. Then we illustrate how different economic problems analyzed in literature may be seen as particular cases of our general benchmark. Finally, Section V presents some concluding remarks.

2. The model

We consider that the relationship between the principal and the agent involves an action variable, denoted as l, which is observable to both, and a monetary transfer, denoted as t, from the principal to the agent. Moreover, there is a one-dimensional parameter, denoted as θ, which is known to the agent but unobservable to the principal. The principal's uncertainty about the parameter θ is represented by a probability distribution $F(\theta)$ with associated density function $f(\theta)$ strictly positive on the support $[\underline{\theta}, \overline{\theta}]$. This function is assumed to be common knowledge.

The agent's welfare is represented by a utility function $U(l, t, \theta)$ which depends upon the action variable l, the transfer t, and the unknown parameter θ. In particular, we assume that the agent's utility depends linearly on transfers:

$$U(l, t, \theta) = u(l, \theta)+t. \tag{1}$$

We consider a principal's welfare function that incorporates a linear cost of transfers:

$$W(l, t, \theta) = w(l, \theta)-\mu t, \tag{2}$$

where μ is a parameter that may incorporate both the shadow cost of public funds and distributive considerations. For example, if the principal is a regulatory agency which takes into account distributive concerns (through a coefficient $\alpha \in [0,1]$) [1] and public funds are costly ($\lambda > 0$), [2] then the principal's function can be represented as:

$$W(l, t, \theta) = CS(l)+\alpha U(l, t, \theta)-(1+\lambda)t = CS(l)+\alpha u(l, \theta)+\alpha t-(1+\lambda)t,$$

where $CS(.)$ denotes the consumer surplus. So in that case $\mu=1+\lambda-\alpha$ (Laffont and Tirole, 1990a, 1990b, consider $\alpha=1$ and $\lambda>0$, and Baron and Myerson, 1982, $0<\alpha<1$ and $\lambda=0$). If the principal does not take into account the agent's utility and public funds are not costly then $\mu=1$.

Finally, we assume that the principal is endowed with the power to set both l and t.

3. The full information case: a benchmark

Consider the benchmark case in which the regulator knows the parameter θ. The problem of the principal under full information is then given by:

$$\max_{l,t} W(l, t, \theta)$$

subject to $U(l, t, \theta)\geq 0$.

Solving (1) for t and substituting t in (2), the problem is equivalent to:

$$\max_{l,U} W(l, U, \theta)$$

subject to $U\geq 0$.

That is,

$$\max_{l,U} w(l, \theta)+\mu u(l, \theta)-\mu U \text{ subject to } U\geq 0. \tag{3}$$

The first order conditions (where subscripts denote partial derivatives) are given by:

$$W_l(l^*, U^*, \theta) = w_l(l^*, \theta)+\mu u_l(l^*, \theta) = 0, \tag{4}$$

$$U^* = 0. \tag{5}$$

Given that transfers are costly to the principal, the full information policy consists of $l^*(\theta)$ determined by (4) and payment transfers such that the agent obtains no utility, $t^*(\theta)=-u(l^*(\theta),\theta)$. Note that,

$$\frac{dl^*(\theta)}{d\theta} = -\frac{W_{l\theta}}{W_{ll}},$$

where $W_{l\theta}(l^*, U^*, \theta)=w_{l\theta}(l^*, \theta)+\mu u_{l\theta}(l^*, \theta)$. As a consequence, the sign of $\frac{dl^*(\theta)}{d\theta}$ is the same as the sign of $W_{l\theta}$.

4. Characterization of optimal contracts under private information

We now analyze the optimal policy when the agent has private information concerning the parameter θ. The parameter θ is continuously distributed on the support $\Theta=[\underline{\theta}, \overline{\theta}]$ according to the cumulative distribution function $F(\theta)$ and the strictly positive density $f(\theta)$. We assume that $F(\theta)$ satisfies the monotone hazard rate condition; that is, the ratios $\frac{f(\theta)}{1-F(\theta)}$ and $\frac{F(\theta)}{f(\theta)}$ are non-decreasing functions of θ.[3]

The single-crossing property, which states that the greater the parameter θ, the more systematically willing an agent is to forego transfer payments to obtain a higher value for l, holds if the firm's marginal rate of substitution (MRS) of the action variable for transfer payment grows with θ.[4] Given the agent's utility defined by (1), the marginal rate of substitution is $MRS_{lt}=-\frac{U_l}{U_t}=-u_l$. Without loss of generality we assume $\frac{\partial |MRS_{lt}|}{\partial\theta}=u_{\theta l}>0$.

To characterize the optimal regulatory policy under private information we first determine the class of feasible policies and then select the optimal policy from that class.[5] At the first stage, we restrict the analysis to direct revelation mechanisms by the revelation principle.[6] A direct revelation mechanism is composed of transfer functions and associated action variable levels given by $\{l(\theta),t(\theta)\}_{\theta\in\Theta}$. Therefore, we may be restricted to regulatory policies which require the agent to report his private information parameter truthfully, that is, incentive compatible policies, to determine the class of feasible policies. The principal maximizes the expected social welfare subject to the following incentive compatibility and individual rationality constraints:

Incentive compatibility constraints (*IC*): the agent reports θ

[1] For example, if the agent is a monopoly the parameter α may be such that it weighs more consumer surplus than firm profits.

[2] Raising and transferring \$1 through public channels costs society \$(1+λ). Transfers between a firm and either consumers or the state may involve administrative costs, tax distortions or inefficiencies that can be taken into account in the design of the regulatory mechanism. See, for example, Laffont and Tirole (1986, 1993) and Caillaud et al. (1988).

[3] These properties require the density function not to increase too rapidly. They are satisfied by distribution functions frequently used in the literature (for example, Uniform, Normal and Exponential).

[4] Araujo and Moreira (2010) study a class of adverse selection problems where the agent's utility function does not satisfy the Spence-Mirrlees Condition or, also named, the single-crossing property.

[5] We adopt the approach of Baron and Myerson (1982) and Guesnerie and Laffont (1984). In this paper, we follow closely the approach by Baron (1989) that is very intuitive from an economic viewpoint.

[6] The revelation principle was established by Myerson (1979) and Dasgupta, Hammond and Maskin (1979).

truthfully if the utility it expects to obtain by announcing his type is at least as great as the expected utility from any other report. That is,

(IC) $U(\theta) \geq U(\hat{\theta}, \theta) \quad \forall (\hat{\theta}, \theta) \in \Theta^2$,

where $U(\hat{\theta}, \theta) = u(l(\hat{\theta}), \theta) + t(\hat{\theta})$ and $U(\theta) = U(\theta, \theta)$.[7]

Individual rationality constraints (*IR*): the principal cannot force the agent to participate if it expects negative profits. That is,

(IR) $U(\theta) \geq 0 \quad \forall \theta \in \Theta$.

The problem of the principal (for instance, a regulator) can be written as:

$$\max_{l(\theta), t(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} W(l(\theta), t(\theta), \theta) d\theta \tag{6}$$

subject to IR and IC.

The following lemma characterizes the class of policies that satisfy *IC*.

Lemma 1. The necessary and sufficient conditions for *IC* are:

(i) $\frac{dU(\theta)}{d\theta} = u_\theta(l(\theta), \theta)$.

(ii) $u_{\theta l}(l(\theta), \theta)\frac{dl(\theta)}{d\theta} \geq 0$.[8].

Therefore, we can replace *IC* in problem (6) by conditions (i) and (ii) of Lemma 1. The resolution of this problem requires, in general, using optimal control theory. As Baron (1989) shows, in the basic adverse selection model where u_θ is either positive or negative for any θ, [9] optimal control theory can be avoided following a simple shortcut: Condition (i) can be written as a simple formulation of the agent's utility that incorporates the required informational rents to satisfy *IC* and that allows knowing where *IR* is binding.[10]

Lewis and Sappington (1989) reconsider the basic model and introduce the term "countervailing incentives" to refer to a model where the incentive of the agent to understate or overstate his type could be different for different types (that is, the sign of u_θ could be different for different types). The presence of countervailing incentives has some technical implications: if the agent incentives (the sign of u_θ) are not identified *a priori*, then we would not be able to rely on the classical Baron's approach and it would be necessary the use of optimal control theory to solve the problem. The resolution of the problem would determine the direction of informational rents (ex post).

Nevertheless a total characterization of countervailing incentives will allow ex ante the identification of the agent's incentives for any *IC* regulatory policy and, therefore, the optimal direction of the informational rents. As a consequence, it will be possible to solve the principal problem without using optimal control theory following closely Baron's (1989) approach.

Assume that the agent is responsive (see Cailleau et al., 1988) which implies that $l^*(\theta)$ is a non-decreasing function of θ and therefore can be implemented under private information through a transfer $t(l^*(\theta))$ such that $\frac{dU(\theta)}{d\theta} = u_\theta(l^*(\theta), \theta)$.

When $u_\theta(l^*(\theta), \theta) = 0 \ \forall \theta \in [\underline{\theta}, \bar{\theta}]$, the optimal allocation under complete information is implementable through the transfer $t(l^*(\theta)) = t^*(\theta)$, and therefore it would be optimal under private information (see Lewis and Sappington, 1988a). When $u_\theta(l^*(\theta), \theta) \neq 0$ for some $\theta \in [\underline{\theta}, \bar{\theta}]$ and transfers are costly, the optimal allocation under private information is, however, different from $l^*(\theta)$ because the expected value of the associated transfer $t(l^*(\theta))$ is too high. The principal faces a trade off between the cost of informational rents and the welfare loss generated by the departure from the complete information allocation. In order to solve this trade off, the principal would distort $l(\theta)$. The sign of this distortion is related to the sign of $u_\theta(l^*(\theta), \theta)$. Let $l^{PI}(\theta)$ be the optimal allocation under private information and let $\tilde{l}(\theta)$ the allocation such that $u_\theta(\tilde{l}(\theta), \theta) = 0 \ \forall \theta \in [\underline{\theta}, \bar{\theta}]$, that is the allocation that minimizes informational rents.

We provide some examples for $\tilde{l}(\theta)$:

- In Baron and Myerson (1982), $u_\theta(p(\theta), \theta) = -q(p(\theta))$, then the pricing policy that cancels the informational rents is such that the monopoly does not produce.[11] Therefore, information rents always appear under private information, $p^{PI}(\theta) \neq \tilde{p}(\theta)$.
- In Lewis and Sappington (1988a), $u_\theta(p(\theta), \theta) = q_\theta(p(\theta))[p(\theta) - c]$, and given that there is no cost of public funds, $p^{PI}(\theta) = \tilde{p}(\theta) = p^*(\theta) = c \ \forall \theta \in [\underline{\theta}, \bar{\theta}]$. As a consequence, informational rents never exist. However, when the cost of public funds is considered (Aguirre and Beitia, 2004, 2008), $p^*(\theta) > p^{PI}(\theta) \geq \tilde{p}(\theta) = c \forall \theta \in [\underline{\theta}, \bar{\theta}]$.
- In Maggi and Rodríguez-Clare (1995a), $u_\theta(q(\theta), \theta) = -\bar{v}'(\theta) - q(\theta)$ (where $\bar{v}((\theta)$ is the agent reservation utility level and $q(\theta)$ the output produced by type θ), then $\tilde{q}(\theta) = -\bar{v}'(\theta)$ and the relationship between $q^{PI}(\theta)$, $\tilde{q}(\theta)$ and $q^*(\theta)$ depends on $\bar{v}''(\theta)$.

4.1. Necessary and sufficient conditions for countervailing incentives

We consider the following definition of countervailing incentives.

Definition 1. *Countervailing incentives*. There are countervailing incentives when the incentive of the agent to understate or overstate his type depends upon his realization. [12]

For any *IC* allocation, the incentive of the agent to understate or overstate his type is directly related to the direction of his informational rents. The next lemma states the direction of the informational rents.

Lemma 2. The direction of the informational rents.

(i) For any type $\theta \in [\underline{\theta}, \bar{\theta}]$ such that $u_\theta(l^*(\theta), \theta) \geq 0$, the optimal allocation under private information must be such that $u_\theta(l^*(\theta), \theta) \geq u_\theta(l^{PI}(\theta), \theta) \geq 0$. Consequently, such a type of agent would have an incentive to understate the true value of the private information parameter, θ. Further, we have that $\tilde{l}(\theta) \leq l^{PI}(\theta) \leq l^*(\theta)$.

(ii) For any type $\theta \in [\underline{\theta}, \bar{\theta}]$ such that $u_\theta(l^*(\theta), \theta) \leq 0$, the optimal allocation under private information must be such that $u_\theta(l^*(\theta), \theta) \leq u_\theta(l^{PI}(\theta), \theta) \leq 0$. Consequently, such a type of agent would have an incentive to overstate the true value of the private information parameter, θ. Further, $l^*(\theta) \leq l^{PI}(\theta) \leq \tilde{l}(\theta)$.

Lemma 2 states that the direction of the informational rents when the principal implements any *IC* allocation is necessarily the same as the direction of these rents when we implement $l^*(\theta)$ under private information. This means that the incentive of the agent to understate or overstate his type under private information is the same as his incentive when $l^*(\theta)$ is implemented through the necessary transfer to guarantee *IC*.

[7] For simplicity we omit some arguments of the agent utility function.

[8] The proof of Lemma 1 is standard and therefore omitted. See, for example, Baron and Myerson (1982) and Guesnerie and Laffont (1984). Note that part (ii) implies that a local optimum is always global.

[9] The sign of u_θ for any incentive compatible $l(\theta)$ is crucial to know the direction of informational rents. A positive (negative) sign implies that higher (smaller) types have to be compensated more because they have more incentives to announce smaller (higher) types.

[10] For example, in Baron and Myerson (1982) $u_\theta(p(\theta), \theta) = -q(p(\theta)) \leq 0 \quad \forall \theta \in [\underline{\theta}, \bar{\theta}]$. This condition might be replaced by the following formulation of the agent utility $U(\theta) = U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} q(p(\mu))d\mu$, where the second term represents the informational rents. The result of this problem is well known: given that transfers are costly, there are no informational rents at the top (IR is binding at $\bar{\theta}$, $U(\bar{\theta}) = 0$) and there is no distortion at the bottom.

[11] Under linear demand $q(p) = a - bp$, $\tilde{p}(\theta) = a/b$.

[12] See, for instance, Lewis and Sappington (1989) and Maggi and Rodríguez-Clare (1995a, 1995b).

Moreover, the single-crossing property, $u_{\theta l}>0$, implies that the optimal allocation under private information will always be between the two reference values, $l^{*}(\theta)$ and $\tilde{l}(\theta)$. The principal will move from $l^{*}(\theta)$ and $\tilde{l}(\theta)$ in order to solve the trade-off between maximizing welfare and minimizing informational rents.

The next theorem states almost tautologically the conditions under which the optimal policy under private information exhibits countervailing incentives. We only require the function u_{θ} to be a monotonic function of θ (that is, fully non-increasing or fully non-decreasing).[13]

Theorem 1. The principal's optimal policy under private information presents countervailing incentives if and only if under full information there exists a type $\tilde{\theta}\in(\underline{\theta},\overline{\theta})$ such that $u_{\theta}(l^{*}(\tilde{\theta}),\tilde{\theta})=0$, that is $\tilde{l}(\tilde{\theta})=l^{*}(\tilde{\theta})$.

Proof. Assume that u_{θ} is a non-decreasing monotone function of θ. (The proof when a non-increasing function is similar).

(i) Necessity is almost trivial. Assume that there does not exist a type $\tilde{\theta}\in(\underline{\theta},\overline{\theta})$ such that $u_{\theta}(l^{*}(\tilde{\theta}),\tilde{\theta})=0$. Then from monotony or $u_{\theta}(l^{*}(\theta),\theta)<0\ \forall\theta\in(\underline{\theta},\overline{\theta})$, and therefore from Lemma 2, any type would have an incentive to overstate his realization or $u_{\theta}(l^{*}(\theta),\theta)>0\ \forall\theta\in(\underline{\theta},\overline{\theta})$ and, consequently, from Lemma 2, any type would have an incentive to understate his realization. As a consequence, there are not countervailing incentives.

(ii) It is also straightforward to show sufficiency. Assume that there exists a type $\tilde{\theta}\in(\underline{\theta},\overline{\theta})$ such that $u_{\theta}(l^{*}(\tilde{\theta}),\tilde{\theta})=0$. If u_{θ} is a strictly monotone increasing function, then $u_{\theta}(l^{*}(\theta),\theta)<0\ \forall\theta\in(\underline{\theta},\tilde{\theta})$ (and, from Lemma 2, these types would have incentives to overstate) and $u_{\theta}(l^{*}(\theta),\theta)>0\ \forall\theta\in(\tilde{\theta},\overline{\theta})$ (and the incentive would be, therefore, to understate). As a consequence, the incentive of the agent to understate or overstate his type depends on his realization. So there are countervailing incentives.∎

Theorem 1 provides a complete characterization of the existence of countervailing incentives that will allow solving the principal problem under private information in a simple way without using optimal control.

4.2. Characterization of the optimal policy under private information

Once identified the presence of countervailing incentives (that is, there exists a type $\tilde{\theta}\in(\underline{\theta},\overline{\theta})$ such that $u_{\theta}(l^{*}(\tilde{\theta}),\tilde{\theta})=0$)) we proceed to solve the principal problem following closely the approach by Baron (1989). The optimal policy under private information depends crucially on the curvature of U for any implementable policy. This curvature is given by:

$$\frac{d^{2}U(\theta)}{d\theta^{2}}=\frac{du_{\theta}}{d\theta}=u_{\theta l}\frac{dl}{d\theta}+u_{\theta\theta}.$$

Given that $u_{\theta l}\frac{dl}{d\theta}\geq0$ for any implementable policy, the sign of $\frac{d^{2}U(\theta)}{d\theta^{2}}$ depends on both the sign and the relative magnitude of $u_{\theta\theta}$.[14] In order to solve the problem under countervailing incentives, we distinguish three cases:[15]

a) $u_{\theta\theta}\leq0$ (small enough in absolute value)

In this case, U is convex for any implementable allocation and U reaches its minimum at $\tilde{\theta}$ when there exist countervailing incentives. Further, $\tilde{l}(\theta)$ is a non-decreasing function of θ and, therefore, is implementable under private information. Note that in this case we have that under private information $u_{\theta}(l(\tilde{\theta}),\tilde{\theta})=0$, $u_{\theta}(l(\theta),\theta)\leq0$ $\forall\theta\in[\underline{\theta},\tilde{\theta})$ and $u_{\theta}(l(\theta),\theta)\geq0$ $\forall\theta\in(\tilde{\theta},\overline{\theta}]$, given that $u_{\theta}(l^{*}(\tilde{\theta}),\tilde{\theta})=0$ and $\frac{d^{2}U(\theta)}{d\theta^{2}}=\frac{du_{\theta}}{d\theta}\geq0$. Integrating both sides of $\frac{dU(\theta)}{d\theta}=u_{\theta}(l(\theta),\theta)$ over $[\theta,\tilde{\theta})$ and $(\tilde{\theta},\theta]$ yields, respectively:

$$U(\theta)=U(\tilde{\theta})-\int_{\theta}^{\tilde{\theta}}u_{\theta}(l(v),v)dv\quad\forall\theta\in(\underline{\theta},\tilde{\theta}),$$

and

$$U(\theta)=U(\tilde{\theta})+\int_{\tilde{\theta}}^{\theta}u_{\theta}(l(v),v)dv\quad\forall\theta\in(\tilde{\theta},\overline{\theta}),$$

$$U(\tilde{\theta})\geq0.$$

The principal's problem can be written as:

$$\max_{l(\theta),U(\theta)}\int_{\underline{\theta}}^{\overline{\theta}}W(l(\theta),t(\theta),\theta)f(\theta)d\theta\tag{7}$$

$$\textit{subject to }(a)\ U(\theta)=U(\tilde{\theta})-\int_{\theta}^{\tilde{\theta}}u_{\theta}(l(v),v)dv\quad\forall\theta\in(\underline{\theta},\tilde{\theta}),$$

$$(b)\ U(\theta)=U(\tilde{\theta})+\int_{\tilde{\theta}}^{\theta}u_{\theta}(l(v),v)dv\quad\forall\theta\in(\tilde{\theta},\overline{\theta}),$$

$$(c)\ U(\tilde{\theta})\geq0,$$

$$(d)\ \frac{dl}{d\theta}\geq0,$$

$$(e)\ u_{\theta}(l(\tilde{\theta}),\tilde{\theta})=0,$$

$$(f)\ u_{\theta}(l(\theta),\theta)\leq0\quad\forall\theta\in[\underline{\theta},\tilde{\theta}),$$

$$(g)\ u_{\theta}(l(\theta),\theta)\geq0\quad\forall\theta\in(\tilde{\theta},\overline{\theta}].$$

Plugging (a) and (b) into the objective function and taking into account that (c) is binding at the optimum we can rewrite the social welfare in state θ as:

$$\widetilde{W}(l(\theta),\theta)=\begin{cases}w(l(\theta),\theta)+\mu u(l(\theta),\theta)+\mu\frac{F(\theta)}{f(\theta)}u_{\theta}(l(\theta),\theta)&\forall\ \theta\in(\underline{\theta},\tilde{\theta})\\ w(l(\theta),\theta)+\mu u(l(\theta),\theta)-\mu\frac{1-F(\theta)}{f(\theta)}u_{\theta}(l(\theta),\theta)&\forall\ \theta\in(\tilde{\theta},\overline{\theta}).\end{cases}$$

The principal's problem becomes:

$$\max_{l(\theta)}\int_{\underline{\theta}}^{\overline{\theta}}\widetilde{W}(l(\theta),\theta)f(\theta)d\theta\tag{8}$$

subject to (d), (e), (f) and (g).

The next proposition characterizes the principal's optimal policy under private information.

Proposition 1. The optimal policy under private information $\{l^{PI}(\theta),t^{PI}(\theta)\}_{\theta\in[\underline{\theta},\overline{\theta}]}$ is given by:

$$l^{PI}(\theta)=\begin{cases}\hat{l}_{1}(\theta)\ \underline{\theta}\leq\theta\leq\theta_{1}\\ \tilde{l}(\theta)\ \theta_{1}\leq\theta\leq\theta_{2},\\ \hat{l}_{2}(\theta)\ \theta_{2}\leq\theta\leq\overline{\theta}\end{cases}$$

$$t(l^{PI}(\theta))=\begin{cases}-\int_{\theta}^{\tilde{\theta}}u_{\theta}(l^{PI}(v),v)dv-u(l^{PI}(\theta),\theta)\ \underline{\theta}\leq\theta\leq\tilde{\theta}\\ \int_{\tilde{\theta}}^{\theta}u_{\theta}(l^{PI}(v),v)dv-u(l^{PI}(\theta),\theta)\ \tilde{\theta}\leq\theta\leq\overline{\theta}\end{cases},$$

where $\hat{l}_{1}(\theta)$ solves $w_{l}(\hat{l}_{1}(\theta),\theta)+\mu u_{l}(\hat{l}_{1}(\theta),\theta)+\mu\frac{F(\theta)}{f(\theta)}u_{\theta l}(\hat{l}_{1}(\theta),\theta)=0$, and $\hat{l}_{2}(\theta)$ solves $w_{l}(\hat{l}_{2}(\theta),\theta)+\mu u_{l}(\hat{l}_{2}(\theta),\theta)-\mu\frac{1-F(\theta)}{f(\theta)}u_{\theta l}(\hat{l}_{2}(\theta),\theta)=0$. The types

[13] That is, $u_{\theta\theta}$ does not switch signs across the interval of types $[\underline{\theta},\overline{\theta}]$.

[14] Note that the curvature of $U(\theta)$ determines who obtains the highest informational rent in equilibrium.

[15] Note that our classification of cases is similar of that used by Maggi and Rodríguez-Clare (1995a) even though our approach considers a general utility function for the agent.

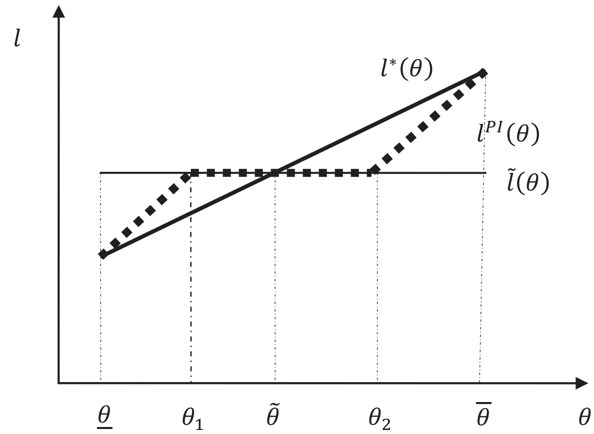

Fig. 1. Optimal Policies under full information and under private information when $u_{\theta\theta}$=0. Pooling equilibrium appears under private information.

θ_1 and θ_2 are such that constraints (f) and (g) are binding, respectively, and therefore $\hat{l}_1(\theta_1)=\tilde{l}(\theta_1)$ and $\hat{l}_2(\theta_2)=\tilde{l}(\theta_2)$.

Note that the constraint (*d*), $\frac{dl}{d\theta}\geq0$, and the single-crossing property, $u_{\theta l}>0$, imply that when the constraint (*f*) is binding for a type $\theta_1\in(\underline{\theta},\tilde{\theta})$, then it will be binding for any type in $(\theta_1,\tilde{\theta})$. Alternatively, if the constraint (*g*) is binding for a type $\theta_2\in(\tilde{\theta},\overline{\theta}]$, then it will be binding for any type in $(\tilde{\theta},\theta_2)$. As a consequence, $l^{PI}(\theta)=\tilde{l}(\theta)\forall\ \theta\in[\theta_1,\theta_2]$ and there are no informational rents for types in this interval.

Note that when $u_{\theta\theta}$=0 then $\tilde{l}(\theta)=l^*(\tilde{\theta})\forall\ \theta\in[\underline{\theta},\overline{\theta}]$. As a consequence, in this case there would be a pooling equilibrium in the interval $[\theta_1,\theta_2]$, as illustrated in Fig. 1.

Nevertheless, when $u_{\theta\theta}<0$, $\tilde{l}(\theta)$ is a strictly increasing function and therefore the optimal solution under private information depends on θ even though some types do not receive informational rents, as illustrated in Figs. 2 and 3.

Fig. 3 represents the agent's utility under private information when the optimal level of the action variable under full information, $l^*(\theta)$, is implemented through the transfer needed to guarantee incentive compatibility, $t(l^*(\theta))$. Given that the implementation $l^*(\theta)$ under private information generates high informational rents, the principal deviates from this level and in order to reduce these rents, they will be zero in the interval $[\theta_1,\theta_2]$.

This kind of optimal contract has characterized many incentive problems in the literature. In a labor economics context, Kübler (2002) analyzes the optimal contract between an employer and a worker when the productivity of the worker is not observable by the principal (the employer). In a context in which the more productive a worker is, the higher is his reservation utility, she shows that optimal contracts may be characterized by countervailing incentives (compare for example her Fig. 1 with our Fig. 1). Aguirre and Beitia (2008) consider the regulation of a multiproduct monopolist with unknown demand and show that when the firm sells demand complements then countervailing incentives characterize the optimal contract in contexts where the firm would want to practice cross-subsidization under full information. Finally, in a context of health economics, Choné and Ma (2011) discuss how countervailing incentives of this type may characterize the optimal contract.

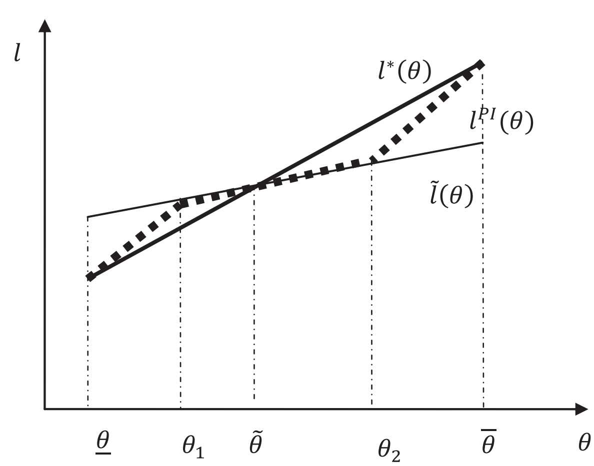

Fig. 2. Optimal Policies under full information and under private information when *u* is not too concave.

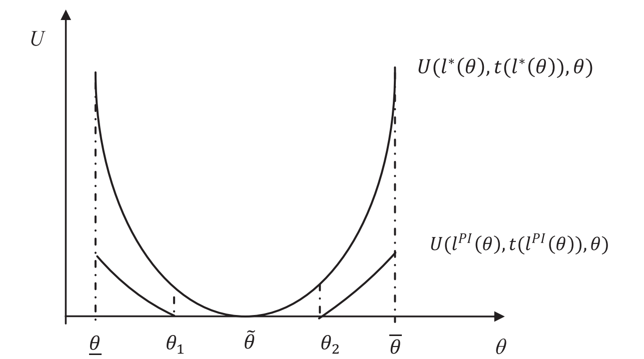

Fig. 3. Agent's utility when $l^*(\theta)$ and $l^{PI}(\theta)$ are implemented under private information and $u_{\theta\theta}\leq0$.

b) $u_{\theta\theta}>0$.

The principal faces the same maximization problem as in the previous case (*U* is strictly convex for any implementable allocation) but now $\tilde{l}$ is a decreasing function of θ and, as a consequence, it is not implementable under private information. The next proposition summarizes the optimal contract under private information.

Proposition 2. The optimal policy under private information $\{l^{PI}(\theta),t^{PI}(\theta)\}_{\theta\in[\underline{\theta},\overline{\theta}]}$ is:

$$l^{PI}(\theta)=\begin{cases}\hat{l}_1(\theta)\ \underline{\theta}\leq\theta\leq\theta_1\\ l^*(\tilde{\theta})\ \theta_1\leq\theta\leq\theta_2,\\ \hat{l}_2(\theta)\ \theta_2\leq\theta\leq\overline{\theta}\end{cases}$$

where $\hat{l}_1(\theta)$ solves $w_l(\hat{l}_1(\theta),\theta)+\mu u_l(\hat{l}_1(\theta),\theta)+\mu\frac{F(\theta)}{f(\theta)}u_{\theta l}(\hat{l}_1(\theta),\theta)=0$, and $\hat{l}_2(\theta)$ solves $w_l(\hat{l}_2(\theta),\theta)+\mu u_l(\hat{l}_2(\theta),\theta)-\mu\frac{1-F(\theta)}{f(\theta)}u_{\theta l}(\hat{l}_2(\theta),\theta)=0$. The types θ_1 and θ_2 are such that $\hat{l}_1(\theta_1)=\hat{l}_2(\theta_2)=l^*(\tilde{\theta})$.

By Lemma 2 we know that $l^{PI}(\tilde{\theta})=l^*(\tilde{\theta})=\tilde{l}(\tilde{\theta})$. Given that $\tilde{l}(\theta)$ is a decreasing function of θ and $l^{PI}(\theta)$ is necessarily non-decreasing (by constraint (d)), we know that the optimal policy under private information is such that $l^{PI}(\theta)\leq l^{PI}(\tilde{\theta})=l^*(\tilde{\theta})=\tilde{l}(\tilde{\theta})<\tilde{l}(\theta)\ \forall\ \theta\in(\underline{\theta},\tilde{\theta})$ and $\tilde{l}(\theta)<\tilde{l}(\tilde{\theta})=l^*(\tilde{\theta})=l^{PI}(\tilde{\theta})\leq l^{PI}(\theta)\ \forall\ \theta\in(\tilde{\theta},\overline{\theta}]$, which implies that constraints (e) and (f) are never binding. Pooling equilibrium appears at an interior interval of types $[\theta_1,\theta_2]$. At the interval $(\underline{\theta},\tilde{\theta})$, given that $u_\theta(l(\theta),\theta)\leq0$, the principal deviates from the optimal allocation under complete information by increasing l in order to reduce informational rents, but once he reaches $l^{PI}(\tilde{\theta})=l^*(\tilde{\theta})$ at θ_1, $l^{PI}(\theta)$ remains constant until reaching $\tilde{\theta}$. A similar argument follows in the interval $(\tilde{\theta},\overline{\theta})$.

Fig. 4 represents the optimal policy under full information and under private information and Fig. 5 illustrates the agent's utility when $l^*(\theta)$ and $l^{PI}(\theta)$ are implemented under private information. Proposition 2 states that as in the case where $u_{\theta\theta}$=0, there is pooling equilibrium in the interval $[\theta_1,\theta_2]$ (see Fig. 4) but now the only type that does not receive informational rents is $\tilde{\theta}$ (see Fig. 5).

This kind of contract is obtained in Lewis and Sappington (1989). They consider the regulation of a single product firm with unknown total cost. The firm has private information on constant marginal cost and on fixed cost, and they assume that both variables move in opposite direction: the higher the marginal cost of a firm is, the lower is its fixed

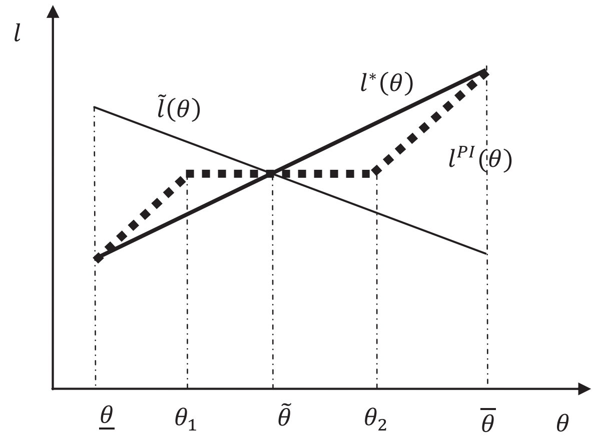

Fig. 4. Optimal policies when u is strictly convex.

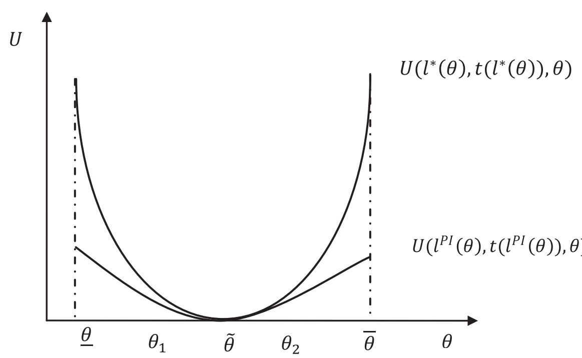

Fig. 5. Agent's utility when $l^{*}(\theta)$ and $l^{PI}(\theta)$ are implemented under private information when $u_{\theta\theta}>0$.

cost. The optimal regulatory policy exhibits countervailing incentives: for low realizations of θ the firm's incentive to overstate θ will dominate its incentive to understate θ, while for higher realizations the dominant incentive will be to understate θ. Under private information, the optimal contract is such that an interior type obtains zero profits and there is pooling equilibrium for an interior interval of types.

c) $u_{\theta\theta}<0$ (high enough in absolute value).

In this case U is concave for any implementable allocation and reaches its maximum at $\tilde{\theta}$ when there exist countervailing incentives. Moreover, $\tilde{l}$ is an increasing function of θ and, therefore, is implementable under private information. Given that $u_{\theta}(l^{*}(\tilde{\theta}), \tilde{\theta})=0$ and $\frac{d^2U(\theta)}{d\theta^2}=\frac{du_{\theta}}{d\theta}\leq 0$, then $u_{\theta}(l(\tilde{\theta}), \tilde{\theta})=0$, $u_{\theta}(l(\theta),\theta)\geq 0 \quad \forall\ \theta\in[\underline{\theta}, \tilde{\theta})$ and $u_{\theta}(l(\theta),\theta)\leq 0\ \forall\ \theta\in(\tilde{\theta}, \overline{\theta}]$. Integrating both sides of $\frac{dU(\theta)}{d\theta}=u_{\theta}(l(\theta),\theta)$ over $[\underline{\theta},\theta)$ and $(\theta,\overline{\theta}]$ yields, respectively:

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} u_{\theta}(l(v), v)dv \quad \forall\ \theta\in(\underline{\theta}, \tilde{\theta}),$$

and

$$U(\theta) = U(\overline{\theta}) - \int_{\theta}^{\overline{\theta}} u_{\theta}(l(v), v)dv \quad \forall\ \theta\in(\tilde{\theta}, \overline{\theta}),$$

where $U(\underline{\theta})\geq 0$ and $U(\overline{\theta})\geq 0$.

The principal's problem can be written as:

$$\max_{l(\theta),U(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} W(l(\theta), t(\theta), \theta)f(\theta)d\theta \tag{9}$$

subject to (a') $U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} u_{\theta}(l(v), v)dv \quad \forall\ \theta\in(\underline{\theta}, \tilde{\theta}),$

(b') $U(\theta) = U(\overline{\theta}) - \int_{\theta}^{\overline{\theta}} u_{\theta}(l(v), v)dv \quad \forall\ \theta\in(\tilde{\theta}, \overline{\theta}),$

(c′) $U(\underline{\theta})\geq 0$,

(d') $U(\overline{\theta})\geq 0$,

(e′) $\frac{dl}{d\theta}\geq 0$,

(f') $u_{\theta}(l(\tilde{\theta}), \tilde{\theta})=0$,

(g') $u_{\theta}(l(\theta), \theta)\geq 0 \quad \forall\theta\in(\underline{\theta}, \tilde{\theta})$,

(h') $u_{\theta}(l(\theta), \theta)\leq 0 \quad \forall\ \theta\in(\tilde{\theta}, \overline{\theta}]$.

Plugging (a') and (b') into the objective function and taking into account that (c') and (d') are binding at the optimum, we can rewrite the social welfare in state θ as:

$$\widetilde{W}(l(\theta), \theta)=w(l(\theta), \theta)+\mu u(l(\theta), \theta)-\mu\frac{F(\tilde{\theta})-F(\theta)}{f(\theta)}u_{\theta}(l(\theta), \theta) \quad \forall\ \theta\in[\underline{\theta}, \overline{\theta}].$$

The principal's problem becomes:

$$\max_{l(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \widetilde{W}(l(\theta), \theta)f(\theta)d\theta \tag{10}$$

subject to (e'), (f'), (g') and (h').

The next proposition characterizes the principal's optimal policy under private information.

Proposition 3. The optimal policy under private information is $\{l^{PI}(\theta),t^{PI}(\theta)\}_{\theta\in[\underline{\theta},\overline{\theta}]}$ where $l^{PI}(\theta)$ solves $w_l(l^{PI}(\theta),\theta)+\mu u_l(l^{PI}(\theta),\theta)-\mu\frac{F(\tilde{\theta})-F(\theta)}{f(\theta)}u_{\theta l}(l^{PI}(\theta),\theta)=0$.

Proposition 3 implies that for all types different from $\tilde{\theta}$ the principal deviates from the first best $l^{*}(\theta)$ in order to reduce informational rents. The direction and the amount of the deviation is given by $\mu\frac{F(\tilde{\theta})-F(\theta)}{f(\theta)}u_{\theta l}(l^{PI}(\theta),\theta)$, which is minimum at $\tilde{\theta}$, maximum at the end of the interval and, by constraints (g') and (h'), it has the limit at $\tilde{l}(\theta)$.

Fig. 6 illustrates the optimal policies both under full information and under private information, and Fig. 7 shows the agent's utility when l^{*}(θ) and $l^{PI}(\theta)$ are implemented under private information.

Maggi and Rodríguez-Clare (1995a) consider a situation where a principal contracts with an agent to produce a certain amount of output

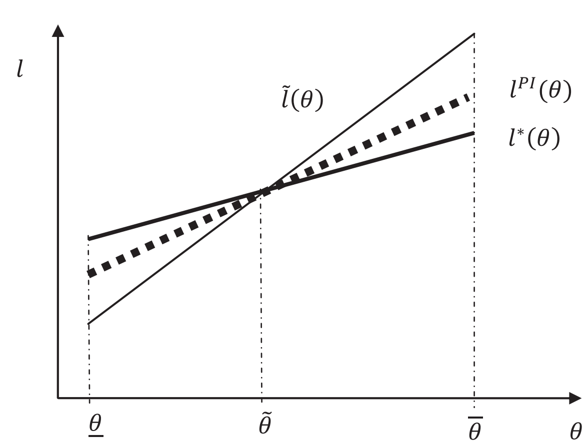

Fig. 6. Optimal policies when U is strictly concave.

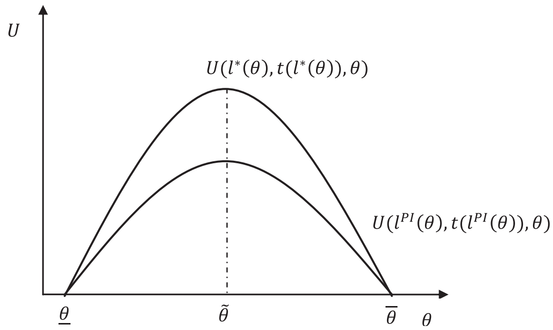

Fig. 7. Agent's utility when $l^*(\theta)$ and $l^{PI}(\theta)$ are implemented under private information when u is strictly concave.

and compensates him with a monetary transfer. Constant marginal cost (increasing with the type) is privately observed by the agent and it is assumed that the agent also has an outside opportunity that provides him with a reservation utility decreasing with marginal costs. When the reservation utility is highly convex then $u_{\theta\theta}<0$ (high enough in absolute value) and the analysis of Maggi and Rodríguez-Clare (1995a) is well described by Fig. 7 where informational rents are bell-shaped with both extreme types earning no rents. In an environmental economics context, Sheriff (2008) considers the socially optimal policy for reducing emissions in politically influential sectors. He shows that countervailing incentives can exist if high productivity is correlated with high foregone profits from abatement, and the incentive of over-state or under-state productivity depends crucially of the realization of private information.

5. Concluding remarks

This paper develops a simple method of characterizing countervailing incentives in adverse selection problems. The key element in our characterization consists in the analysis of the properties of the full information action profile. This allows us to solve the principal problem without using optimal control theory. Our methodology can be applied to a variety of adverse selection problems in different areas such as health economics, monopoly regulation, labor contracts, limited liabilities and environmental regulation.

There is also a literature on multidimensional screening that considers incentive mechanisms when private information concerns more than one variable (see, for instance, Rochet and Stole, 2003) in which countervailing incentives have also been studied. Boone and Schottmüller (2013) analyze optimal procurement mechanisms when firms are specialized. They assume that the procurement agency has incomplete information concerning the firms' cost functions and values high quality as well as low price. They analyze a two-dimensional screening model with countervailing incentives. Szalay (2013) considers the regulation of a two-product monopolist when private information concerns two variables and characterizes the optimal policy that exhibits countervailing incentives. Future research may consider extending the analysis to such multidimensional problems. Finally, it would be also interesting to analyze in future research the application of our methodology to dynamic agency problems (see, for instance, Li et al., 2016).

Appendix A

A.1 Proof of Lemma 2

(i) Proof by contradiction.

Assume that $l^{\overline{PI}}(\theta)$ is the solution under private information such that for a type $\theta_0 \in [\underline{\theta}, \overline{\theta}]$ we have that $u_\theta(l^*(\theta_0), \theta_0) \geq 0 > u_\theta(l^{\overline{PI}}(\theta_0), \theta_0)$. Then the single-crossing property, $u_{\theta l}>0$, implies that $l^*(\theta_0) \geq \tilde{l}(\theta_0) > l^{\overline{PI}}(\theta_0)$.

At θ_0 the slope of $U(\theta)$ required to induce truth-telling, given Lemma 1, part (i), is:

$$\frac{dU(\theta_0)}{d\theta} = u_\theta(l^{\overline{PI}}(\theta_0), \theta_0) < 0$$

which specifies how informational rents must change with θ. Individual rationality (IR) and the monotonicity of $u_\theta$imply that any type $\theta \in (\underline{\theta}), \theta_0]$ would obtain a strictly positive information rent.

Consider an allocation $\check{\tilde{l}}(\theta)$ such that $\check{\tilde{l}}(\theta) = l^{\overline{PI}}(\theta) \forall\ \theta \neq \theta_0$ and $\check{\tilde{l}}(\theta_0) = \tilde{l}(\theta_0)$. A move from $l^{\overline{PI}}$ to $\check{\tilde{l}}(\theta)$ increases the principal's welfare since the distortion is reduced and informational rents also decrease. Thus we have reached a contradiction.

(ii) The proof is quite similar to part (i), and hence omitted.∎

References

Aguirre, Iñaki, Beitia, Arantza, 2004. Regulating a monopolist with unknown demand: costly public funds and the value of private information. J. Public Econ. Theory 6 (5), 691–704.
Aguirre, Iñaki, Beitia, Arantza, 2008. Regulating a multiproduct monopolist with unknown demand: cross-subsidization and countervailing incentives. J. Inst. Theor. Econ. 164, 652–675.
Araujo, Aloisio, Moreira, Humberto, 2010. Adverse selection problems without the Spence-Mirless condition. J. Econ. Theory 145 (3), 1113–1141.
Baron, David, 1989. Design of regulatory mechanisms and institutions. In: Schmalensee, Richard, Willig, Robert (Eds.), Handbook of Industrial Organization 2. North-Holland, Amsterdam.
Baron, David, Myerson, Roger, 1982. Regulating a monopolist with unknown costs. Econometrica 50 (4), 911–930.
Boone, Jan, Schottmüller, Christoph., 2013. Procurement with Specilized Firms. Unpublished manuscript.
Caillaud, Bernard, Guesnerie, Roger, Rey, Patrick, Tirole, Jean, 1988. Government intervention in production and incentives theory: a review of recent contributions. Rand J. Econ. 19 (1), 1–26.
Choné, Philippe, Ma, Ching-to Albert, 2011. Optimal health care contract under physician agency. Ann. Econ. Stat., 229–256.
Dasgupta, Partha, Hammond, Peter, Maskin, Eric, 1979. The implementation of social choices rules: some general results on incentive compatibility. Rev. Econ. Stud. 46 (2), 185–216.
Guesnerie, Roger, Laffont, Jean-Jacques, 1984. A complete solution to a class of principal-agent problems with an application to the control of a self-managed firm. J. Public Econ. 25, 329–369.
Jullien, Bruno, 2000. Participation constraints in adverse selection models. J. Econ. Theory 93 (1), 1–47.
Kübler, Dorothea, 2002. Who benefits from workers with general skill? Countervailing skills is labour contracts. Metroeconomica 53 (3), 290–314.
Laffont, Jean-Jacques, Tirole, Jean, 1986. Using cost observation to regulate firms. J. Polit. Econ. 94, 614–641.
Laffont, Jean-Jacques, Tirole, Jean, 1990a. The regulation of multi-product firms. Part I: theory. J. Public Econ. 43, 1–36.
Laffont, Jean-Jacques, Tirole, Jean, 1990b. The regulation of multi-product firms. Part II: applications to competitive environments and policy analysis. J. Public Econ. 43, 37–66.
Laffont, Jean-Jacques, Tirole, Jean, 1993. A Theory of Incentives in Procurement and Regulation. MIT Press, Cambridge, MA.

Lewis, Tracy, Sappington, David, 1988a. Regulating a monopolist with unknown demand. Am. Econ. Rev. 78 (5), 986–998.
Lewis, Tracy, Sappington, David, 1989. Countervailing incentives in agency problems. J. Econ. Theory 49 (2), 294–313.
Li, Hong, Mu, Congming, Yang, Jinqiang, 2016. Optimal contract theory with time-inconsistent preferences. Econ. Model. 52, 519–530.
Maggi, Giovanni, Rodríguez-Clare, Andrés, 1995a. On countervailing incentives. J. Econ. Theory 66, 238–263.
Maggi, Giovanni, Rodríguez-Clare, Andrés, 1995b. Costly distortion of information in agency problems. Rand J. Econ. 26 (4), 675–689.
Myerson, Roger, 1979. Incentive-compatibility and the bargaining problem. Econometrica 47 (1), 61–74.
Rochet, Jean-Charles, Stole, Lars, 2003. The Economics of Multidimensional Screening, Advances in Economics and Econometrics: Theory and Application, Eighth World Congress 1. Cambridge University Press, 150–197.
Sheriff, Glenn, 2008. Optimal environmental regulation of politically influential sectors with asymmetric information. J. Environ. Econ. Manag. 55, 72–89.
Szalay, Dezsö, 2013. Regulating a Multiproduct and Multitype Monopolist (GESY Discussion Paper No. 397). University of Bonn.