Regulating a Multiproduct Monopolist with Unknown Demand: Cross-Subsidization and Countervailing Incentives

by

IÑAKI AGUIRRE AND ARANTZA BEITIA*

This paper studies the regulation of a multiproduct monopolist that has private information about demand conditions. In particular, we consider the regulation of a two-product monopolist with interdependent demands when it has better information concerning the demand of one product than the regulator and public funds are costly. We show that the optimal regulation policy in this case crucially depends on whether goods are substitutes or complements. Cross-subsidization arises with demand complementarities, making it likely that countervailing incentives characterize the optimal contract. (JEL: D 82, L 50)

1 Introduction

It seems natural that regulated firms have better information about their operating environment than regulators do: "Because of its superior resources, its ongoing management of production, and its frequent direct contact with customers, a regulated firm will often be better informed that the regulator about both its operating technology and consumer demand" (ARMSTRONG AND SAPPINGTON [2007, p. 1564]). Most regulated firms supply a range of products. For instance, railroads offer freight and passenger services, electric utilities generate power at different times of the day or seasons of the year for both residential and industrial consumers, telephone companies serve residential consumers, small and big businesses, and so on (see, for example, CREW AND KLEINDORFER [1986] and LAFFONT AND TIROLE [1993]). Hence, the multiproduct framework seems the most appropriate scenario in which to analyze regulatory policies.[1]

* We are grateful to three anonymous referees, Ignacio Palacios-Huerta, and participants in the XX Jornadas de Economía Industrial (Granada, 2004) and the XXIX Simposio de Análisis Económico (Pamplona, 2004). Financial support from the Ministerio de Ciencia y Tecnología and FEDER (SEJ2006-05596), and from the Departamento de Educación, Universidades e Investigación del Gobierno Vasco (IT-223-07) is gratefully acknowledged.

1 While we consider a multiproduct monopolist, we do not mean that the firm necessarily sells different products. Our model also fits, for instance, contexts where a single-product firm sells the same good in different markets but demands are interdependent. In this context, it is important to understand whether the demands in the two markets are complements or substitutes (see LAYSON [1998] and ADACHI [2005] for interesting examples, many of them characterized by consumption externalities).

In particular, in this paper we are interested in studying the regulation of a multiproduct monopolist that has private information about demand conditions and whose regulator is allowed to make monetary transfers to the regulated firm. More precisely, we consider the regulation of a two-product monopolist with interdependent demands that has better information concerning the demand for one product than the regulator. The assumption of a single source of asymmetric information (for example, an idiosyncratic shock) affecting solely a market seems appropriate for industries, such as telecommunications, where either there is a new market or a new product, so that the regulator cannot predict its demand from past history, or where, because of technological progress or new information that affects consumers' valuation of the good, demand is stochastic and the evaluation of demand requires costly information that is cheaper to obtain for the regulator (see, for example, IOSSA [1999]). In these and similar cases, it is likely the case that the regulated firm will be better informed than the regulator about consumer demand.

We shall show that the optimal regulation policy depends crucially on whether goods are substitutes or complements. Although examples with demand substitutes arise naturally, demand complementarities appear in arguably rather common circumstances. For instance: (i) network externalities and switching cost: FARRELL AND KLEMPERER [2007]; (ii) bandwagon effects: ROHLFS [2007] considers two types: *(a)* network externalities, whereby existing subscribers benefit from being able to communicate with a larger user set, and *(b)* complementary bandwagon effects, whereby purchasers of the base product (e.g., hardware) benefit from the greater availability of competitively supplied complementary products (e.g., software) as the user set expands (iii) direct demand complementary; (iv) a single product sold in different markets (see ADACHI [2005]).

With demand substitutes, the firm's incentive is always to understate the market 1 demand (which is the one unknown to the regulator); and, in order to reduce informational rents, the regulator sets prices for both products below those that would take place in the full-information case. With demand complements, however, the characterization of the optimal regulatory policy is more complex and depends on whether product 1 is subsidized (sold below marginal cost) under full information. In this case, the firm's incentive is always to overstate the market 1 demand, and, in order to reduce informational rents, the regulator increases the price of product 1 and reduces the price of product 2 relative to the full-information case. When product 1 is not subsidized under full information, however, the firm's incentive is always to understate the market 1 demand, and to reduce informational rents the regulator reduces the price of product 1 and increases the price of product 2. Finally, we analyze the possibility that countervailing incentives, in the sense of LEWIS AND SAPPINGTON [1989], may arise with demand complements; in those cases the firm may be tempted either to overstate or to understate its private information, depending upon the realization of the demand parameter.

This paper is organized as follows. Section 2 briefly reviews the literature on the regulation of monopoly under asymmetric information. Section 3 develops the basic model. Section 4 presents the full-information case as a benchmark. In section 5 we analyze the design of the optimal regulatory policy under private information on demand conditions. Finally, section 6 offers concluding remarks.

2 Related Literature

For the case of a single product monopoly, there is an extensive literature focusing on the design of optimal regulatory mechanisms under asymmetric information about: (i) certain cost parameters (see, for example, BARON AND MYERSON [1982], LAFFONT AND TIROLE [1986], and BARON [1989]); (ii) market demand (RIORDAN [1984], LEWIS AND SAPPINGTON [1988a], and AGUIRRE AND BEITIA [2004]);[2] and (iii) both costs and demand conditions (LEWIS AND SAPPINGTON [1988b] and ARMSTRONG [1999]). The price of the product is established by the regulator, who is allowed to make monetary transfers to the regulated firm. The application of the principal–agent method leads to well-known results: asymmetry of information causes in general a loss of efficiency due to the necessity of limiting monopoly informational rents.

The literature on the regulation of a multiproduct monopoly has mainly focused on the case of asymmetric information concerning costs (SAPPINGTON [1983], LAFFONT AND TIROLE [1990a], [1990b], DANA [1993], and ARMSTRONG AND ROCHET [1999]). Despite its importance, however, the analysis of the design of regulatory policies when the regulated multiproduct firm is better informed about demand than the regulator has been mostly neglected in the literature. Notable exceptions are IOSSA [1999] and ARMSTRONG AND VICKERS [2000].

IOSSA [1999] addresses the issue of how to organize (with a multiproduct monopoly or with a differentiated duopoly) a two-product industry with interdependent demands when the regulator has worse information than the firm. As in her setting, we assume that there exists a single source of asymmetric information: the multiproduct firm observes privately the realization of an idiosyncratic shock affecting the demand of only one of its products. However, she assumes linear demands, whereas we allow for a more general demand configuration; and she analyzes the effects of asymmetric information on the optimal industry structure, whereas we focus on the optimal regulation of the multiproduct firm. Further, one more important difference rests in the treatment of complementary goods, in that she does not allow pricing one product below marginal cost. This pricing policy (cross-subsidization), however, is relevant theoretically and empirically to multiproduct firms, whether they are regulated or not.

---

[2] In AGUIRRE AND BEITIA [2004], we analyze the optimal regulation policy when a single-product firm has better information concerning the market demand than the regulator. This paper represents a generalization that fits better the multiproduct nature of regulated firms in the real world, where the interdependence between the demands of different products plays an important role.

The possibility of a multiproduct monopolist deciding to price one good below marginal cost when products are demand complements has been analyzed, for example, by TIROLE [1988] and more recently by DAVIS AND MURPHY [2000]. These authors show not only that a multiproduct firm may be interested in pricing below marginal cost, but also that the profit-maximizing outcome can involve a zero or negative price for one good.[3] On the other hand, CREW AND KLEINDORFER [1986] analyze the problem of multiproduct Ramsey pricing with interdependent demands and discuss in depth the case in which one product is subsidized (sold below marginal cost) because of the beneficial effects that it may have on the sales of the other product. They also include very interesting applications to the telecommunications industry. In fact, some authors, like SRINAGESH [1984], consider that cross-subsidization is so common in telecommunications that it might be labeled a stylized fact.[4] Further, much literature has documented the presence of cross-subsidization in the water, gas, and electricity utility industries (see, for example, SAWKINS AND REID [2007] for the water industry), in transportation, and in hospitals (HARRIS [1979]).

ARMSTRONG AND VICKERS [2000] discuss the regulation of a multiproduct monopolist when the firm has private information about cost or demand conditions. Contrary to the standard literature on the optimal regulation under asymmetric information, they do not permit the use of lump-sum transfer. They focus on the question of how much pricing discretion should be granted to a regulated multiproduct firm. Although we consider a standard model of regulation under adverse selection that allows transfers, we adopt the framework they proposed (in their Example 4) by considering that the source of asymmetric information is an additive shock privately observed by the firm.

3 The Model

We consider the following regulatory environment. The market demands for the products of the firm are given by $D^1(p_1, p_2, \theta)$ and $D^2(p_1, p_2)$ where p_i is the unit price of product i, $i = 1, 2$, and the parameter θ captures the firm's private information concerning the market demand of product 1. We assume that the greater the price of the good, the lower the demand ($\partial D^i / \partial p_i < 0$), that the own effect is greater than the cross effect,

$$\left|\partial D^i / \partial p_i\right| > \left|\partial D^i / \partial p_j\right|, \quad i, j = 1, 2, \; j \neq i,$$

[3] Their analysis shows that although several factors play a role in the design, pricing, and distribution of Microsoft's Internet Explorer, a key factor for this technology being included in Windows at no separate charge is the complementary nature of the demand for Windows and the demand for Web use.

[4] ROHLFS [1979] shows that substantial cross-subsidization occurs between local service, which is priced approximately 50% below marginal cost, and long distance, which is priced at two or three times marginal cost.

and that the higher the realization of θ, the greater the quantity of output product 1 consumers demand at any nonnegative price ($\partial D^1 / \partial\theta > 0$). Moreover, we assume that

$$(\partial^2 D^1 / \partial p_1 \partial\theta) = (\partial^2 D^1 / \partial p_2 \partial\theta) = 0.$$

Note that any setting in which higher realizations of θ correspond to parallel outward shifts in demand constitutes a setting where this property is satisfied: that is, we might consider the demand of product 1 as

$$D^1(p_1, p_2, \theta) = d^1(p_1, p_2) + g(\theta) \qquad \text{with } g'(\theta) > 0.$$

We may see then the consumer surplus as

$$v(p_1, p_2) = \tilde{v}(p_1, p_2) + (\bar{p}_1 - p_1) g(\theta),$$

where $\tilde{v}$ is a known consumer surplus function and $\bar{p}_1$ is some (high) reference price, and

$$D^1 = -(\partial v / \partial p_1) = -(\partial \tilde{v} / \partial p_1) + g(\theta),$$

where $d^1 = -\partial \tilde{v} / \partial p_1$. That is, we consider an additive shock affecting market 1 similar to that in Example 4 in ARMSTRONG AND VICKERS [2000].[5]

The regulator's uncertainty about the parameter θ is represented by a probability distribution $F(\theta)$ with associated density function $f(\theta)$ strictly positive on the support $[\underline{\theta}, \bar{\theta}]$. The function is common knowledge, but the realization of θ is observed only by the firm. The cost (known) of producing q_1 and q_2 is given by $C(q_1, q_2) = c_1 q_1 + c_2 q_2 + F$, where c_1 and c_2 are the constant marginal costs and F is the fixed cost.

The regulator is endowed with the power to set unit prices p_1 and p_2 for the firm's outputs and to specify a transfer t from consumers to the firm. To simplify the analysis, we assume that the consumer demand depends only on prices p_1 and p_2.[6] The regulator can observe, and therefore enforce, the regulated prices. The quantities sold at the regulated prices are assumed to be too costly for the regulator to monitor directly (see, for example, LEWIS AND SAPPINGTON [1988a] for a discussion of this assumption). However, the regulator can still be sure that the firm fulfills its mandate to serve all demand at the regulated prices. The regulator need only invite consumers to report any incident in which they were either refused service at the established prices or charged higher prices, and penalize the firm for such rationing.

We consider a generalized social welfare function that incorporates both distributional considerations through a coefficient, $\alpha \in [0, 1]$, affecting firm's

---

[5] LEWIS AND SAPPINGTON [1988a], [1988b], ARMSTRONG [1999], IOSSA [1999], and AGUIRRE AND BEITIA [2004] also consider additive shocks.

[6] The total fixed charge may be thought of as apportioned among consumers in such a manner that no consumer is excluded from purchasing the good. The analysis can thus be conducted in terms of the aggregate fixed charges t paid to the firms by consumers.

profits, and costly public funds: raising and transferring \$1 through public channels costs society \$$(1 + \lambda)$:

(1) $$W(p_1, p_2, t) = v(p_1, p_2) + \alpha\Pi(p_1, p_2, t) - (1 + \lambda)t,$$

where $v(p_1, p_2)$ is the consumer surplus, and the profit of the firm is given by

(2) $$\Pi(p_1, p_2, t) = (p_1 - c_1)D^1(p_1, p_2, \theta) + (p_2 - c_2)D^2(p_1, p_2) - F + t.$$

It is standard in the literature to introduce distributional considerations through a coefficient $\alpha \in [0, 1]$. ARMSTRONG AND SAPPINGTON [2007, pp. 1562f.] stated: "The regulator's preference for consumer surplus over rent (indicated by $\alpha < 1$) reflects a greater concern with the welfare of consumers than the welfare of shareholders. This might be due to differences in their average income, or because the regulator cares about the welfare of local constituents and many shareholders reside in another jurisdictions."[7] The parameter λ is usually called the shadow cost of public funds. Transfers between a firm and either consumers or the state may involve administrative costs, tax distortions, or inefficiencies that must be taken into account in the design of the regulatory mechanism. See, for example, LAFFONT AND TIROLE [1993].

4 The Full-Information Case: A Benchmark

Consider the benchmark case in which the regulator knows all components of demand functions. The problem of the regulator under full information is given by

$$\max_{p_1, p_2, t} W(p_1, p_2, t)$$

subject to $\Pi(p_1, p_2, t) \geq 0$.

Solving (2) for t and substituting t in (1), the problem is seen to be equivalent to

$$\max_{p_1, p_2, \Pi} W(p_1, p_2, \Pi)$$

subject to $\Pi \geq 0$.

That is,

(3) $$\max_{p_1, p_2, \Pi} v(p_1, p_2) + (1+\lambda)\left[(p_1 - c_1)D^1(p_1, p_2, \theta) + (p_2 - c_2)D^2(p_1, p_2) - F\right] - (1 + \lambda - \alpha)\Pi$$

subject to $\Pi \geq 0$.

The first-order conditions are

(4) $$W_{p_1}(p_1^*, p_2^*, \Pi^*) = \lambda D^1(p_1^*, p_2^*, \theta)(1+\lambda)\sum_{i=1}^{2}(p_i^* - c_i)\frac{\partial D^i(\cdot)}{\partial p_1} = 0,$$

[7] CAILLAUD et al. [1988] present a complete discussion on welfare functions in the regulated context.

(5) $$W_{p_2}(p_1^*, p_2^*, \Pi^*) = \lambda D^2(p_1^*, p_2^*) + (1+\lambda)\sum_{i=1}^{2}(p_i^* - c_i)\frac{\partial D^i(\cdot)}{\partial p_2} = 0,$$

(6) $$W_\Pi(p_1^*, p_2^*, \Pi^*) = -(1 + \lambda - \alpha).$$

We can interpret the conditions (4) and (5) as follows: A marginal increase in p_i reduces consumer surplus in D^i, while the profit of the firm increases in

$$D^i + (p_i - c_i)(\partial D^i / \partial p_i) + (p_j - c_j)(\partial D^j / \partial p_i).$$

Given that the firm's profit is zero at the optimum, the regulator can reduce the transfer by the equivalent of that amount, which implies a saving in social cost of

$$\lambda[D^i + (p_i - c_i)(\partial D^i / \partial p_i) + (p_j - c_j)(\partial D^j / \partial p_i)].$$

At the optimal price, the marginal social cost of a change in price equals the marginal social benefit.

We can rewrite these conditions as

(7) $$\Pi^* = 0,$$

(8) $$\frac{p_i^* - c_i}{p_i^*} = \frac{\lambda}{(1+\lambda)}\left[\frac{1}{\varepsilon_{ii}} + \frac{(1+\lambda)}{\lambda}\frac{(p_i^* - c_i)D^j\varepsilon_{ji}}{R_i\varepsilon_{ii}}\right],$$

where ε_{ii} is the price elasticity of market i's demand,

$$\varepsilon_{ii} = -(\partial D^i / \partial p_i)(p_i / D^i),$$

ε_{ji} is the cross elasticity,

$$\varepsilon_{ji} = (\partial D^j / \partial p_i)(p_i / D^j),$$

and $R_i = p_i D^i$, $i, j = 1, 2, j \neq i$. The condition (8) can be expressed as

(9) $$\frac{p_i^* - c_i}{p_i^*} = \frac{\lambda}{(1+\lambda)}\frac{1}{\hat{\varepsilon}_{ii}},$$

where

(10) $$\hat{\varepsilon}_{ii} = \varepsilon_{ii}\frac{1 - \dfrac{\varepsilon_{ij}\varepsilon_{ji}}{\varepsilon_{ii}\varepsilon_{jj}}}{1 + \dfrac{R_j\varepsilon_{ji}}{R_i\varepsilon_{jj}}}, \quad i, j = 1, 2,\ j \neq i.$$

The *superelasticities* $\hat{\varepsilon}_{ii}$ were first derived by BOITEUX [1956]. We obtain that the optimal prices are given by the Ramsey formula: the Lerner index (or price–marginal-cost ratio) of each good is inversely proportional to its superelasticity of demand. In particular, the Lerner index is $\lambda/(1 + \lambda)$ times the reciprocal of the superelasticity of demand, where $\lambda/(1 + \lambda)$ is known as the Ramsey number. When λ is zero, taxation is not distortive and the optimal price of each good is therefore equal to its marginal cost. When the shadow cost of public funds becomes very high, prices tend to the monopoly prices.

When products 1 and 2 are demand substitutes ($\varepsilon_{ji} > 0$, $i, j = 1, 2$, $j \neq i$), the superelasticities are always positive ($\hat{\varepsilon}_{ii} > 0$, $i = 1, 2$), given that the own effects are greater than the cross effects ($\varepsilon_{ii}\varepsilon_{jj} > \varepsilon_{ij}\varepsilon_{ji}$). The optimal prices under full information are therefore higher than the marginal costs: $p_i^* - c_i > 0$, $i = 1, 2$.

On the other hand, when products are demand complements ($\varepsilon_{ij} < 0$, $i, j = 1, 2$, $j \neq i$), the superelasticity of one (and only one) product may be negative, and therefore its price might be lower than its marginal cost.[8] In that case, one product would be subsidized (sold below its marginal cost) because of the positive effects it has on sales of the other product. However, both superelasticities cannot be simultaneously negative.[9]

The full-information policy consists of prices p_i^*, $i = 1, 2$, given by (11) and transfer payments t^* such that firms obtain zero profits. It is easy to check that the optimal price of product 1 is an increasing function of the consumer demand:

$$\frac{dp_1^*}{d\theta} = \frac{-W_{p_1\theta}W_{p_2p_2}}{W_{p_1p_1}W_{p_2p_2} - W_{p_2p_1}W_{p_1p_2}} > 0, \tag{11}$$

given that $W_{p_1\theta} = \lambda(\partial D^1 / \partial\theta) > 0$. On the other hand, the effect of θ on the optimal price of product 2 is given by

$$\frac{dp_2^*}{d\theta} = \frac{dp_2^*}{dp_1^*}\frac{dp_1^*}{d\theta} = -\frac{W_{p_2p_1}}{W_{p_2p_2}}\frac{dp_1^*}{d\theta}. \tag{12}$$

Therefore, the sign of $dp_2^* / d\theta$ depends crucially on the sign of $W_{p_2p_1}$ Note that when $W_{p_2p_1} > 0$ then $dp_2^* / dp_1^* > 0$. In consequence, in response to a change in θ it would be optimal to move both prices in the same direction: $dp_1^* / d\theta > 0$ and $dp_2^* / d\theta > 0$. In that case we say that prices are *social complements*. On the other hand, when $W_{p_2p_1} < 0$ then $dp_2^* / dp_1^* < 0$; hence in response to a change in θ it would be optimal to move the two prices in opposite directions: $dp_1^* / d\theta > 0$ and $dp_2^* / d\theta < 0$. In that case we say that prices are *social substitutes*. Note that if $(\partial^2 D^i / \partial p_i\, \partial p_j) = 0$, then prices are social complements (substitutes) when products are demand substitutes (complements).[10]

---

[8] Note that the superelasticity of product i is negative when $R_i\varepsilon_{jj} + R_j\varepsilon_{ji} < 0$, that is, when $D^j\,|\partial D^j / \partial p_i| > D^i\,|\partial D^j / \partial p_j|$.

[9] Suppose that the superelasticity of product j were also negative. Then, we would have that $D^i\,|\partial D^i / \partial p_j| > D^j\,|\partial D^i / \partial p_i|$. But these two conditions would then imply that $|\partial D^i / \partial p_i||\partial D^j / \partial p_j| > |\partial D^i / \partial p_j||\partial D^j / \partial p_i|$, which contradicts the fact that own effects are greater than cross effects.

[10] Of course, when products are demand-independent (or the regulated monopolist sells the same good in two perfectly separated markets), $W_{p_2p_1} = 0$ and therefore

5 Regulation under Private Information on Demand

We now study the regulation model when the firm has private information about the demand of product 1. The parameter θ is continuously distributed on the support $\Theta = [\underline{\theta}, \bar{\theta}]$ according to the cumulative distribution function $F(\theta)$ and strictly positive density $f(\theta)$. We assume that $F(\theta)$ satisfies the monotone-hazard-rate condition; that is, the ratios $f(\theta) / 1 - f(\theta)$ and $F(\theta) / f(\theta)$ are nondecreasing functions of θ.[11]

The single-crossing property, which states that the greater the demand, the more systematically willing a firm is to forgo transfer payments to obtain a higher unit price for product 1, holds if the firm's marginal rate of substitution (MRS) of product 1 price for transfer payment grows with θ. Given the firm's profit, defined by (2), we have

$$\mathrm{MRS}_{p_1 t} = -(\Pi_{p_1} / \Pi_t) = -\Pi_{p_1}.$$

Since we have assumed that $(\partial^2 D^1 / \partial p_1 \, \partial\theta) = 0$, then

$$\frac{\partial \left| \mathrm{MRS}_{p_1 t} \right|}{\partial \theta} = \Pi_{p_1 \theta} > 0.$$

On the other hand, the firm's MRS of product 2 price for transfer payment does not change with θ.

To characterize the optimal regulatory policy under private information we first determine the class of feasible policies and then select the optimal policy from within that class (that is, we adopt the approach of BARON AND MYERSON [1982] and GUESNERIE AND LAFFONT [1984]). At the first stage, we restrict the analysis to direct revelation mechanisms by the revelation principle.[12] A direct revelation mechanism is composed of transfer functions and associated price levels given by

$$\{p_1(\theta), p_2(\theta), t(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}.$$

Therefore, we may be restricted to regulatory policies that require the firm to report its private information parameter truthfully, that is, incentive-compatible regulatory policies, to determine the class of feasible policies. The regulator

---

$(dp_2^* / d\theta) = 0.$ Then the analysis would be similar to the regulation of a single monopoly under unknown demand (see, for example, AGUIRRE AND BEITIA [2004]).

[11] These properties require the density function not to increase too rapidly and are satisfied by frequently used distribution functions (for example, the uniform, normal, and exponential functions).

[12] The revelation principle was established by MYERSON [1979] and DASGUPTA, HAMMOND, AND MASKIN [1979].

maximizes the expected social welfare subject to the following incentive compatibility and individual rationality constraints:

Incentive compatibility constraints (IC): the firm reports θ truthfully if the profit it expects to obtain by announcing its type is at least as great as the expected profit from any other report. That is,

$$\text{(IC)} \qquad \Pi(\theta) \geq \Pi(\hat{\theta}, \theta) \quad \forall (\hat{\theta}, \theta) \in \Theta^2,$$

where

$$\Pi(\hat{\theta}, \theta) = \left[p_1(\hat{\theta}) - c_1\right] D^1\left(p_1(\hat{\theta}), p_2(\hat{\theta}), \theta\right) + \left[p_2(\hat{\theta}) - c_2\right] D^2\left(p_1(\hat{\theta}), p_2(\hat{\theta})\right) - F + t(\hat{\theta})$$

and $\Pi(\theta) = \Pi(\theta, \theta)$.

Individual rationality constraints (IR): the regulator cannot force the firm to participate if it expects negative profits. That is,

$$\text{(IR)} \qquad \Pi(\theta) \geq 0 \quad \forall \theta \in \Theta.$$

The regulator's problem can be written as

$$\text{(13)} \qquad \max_{p_1(\theta), p_2(\theta), t(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} W\left(p_1(\theta), p_2(\theta), t(\theta)\right) f(\theta) d\theta$$

subject to (IR) and (IC).

The following lemma characterizes the class of policies that satisfies (IC).

LEMMA 1 *Necessary and sufficient conditions for* (IC) *are:*

(i)

$$\frac{d\Pi(\theta)}{d\theta} = \Pi_\theta = \left[p_1(\theta) - c_1\right] \frac{\partial D^1}{\partial \theta}.$$

(ii) The regulated price $p_1(\theta)$ is a nondecreasing function of θ.

The optimal regulatory policy depends on whether the prices of the two products are social complements or social substitutes, which are concepts in general that are inversely related to the notions of demand substitutes and demand complements, respectively.

5.1 Social Complements

When prices are complements from a social welfare point of view ($SW_{p_i p_j} > 0$),[13] optimal prices under complete information are always higher than marginal costs. The next lemma restricts the optimal regulatory policy.

LEMMA 2 *Given that* $p_1^*(\theta) > c_1, \forall \theta \in [\underline{\theta}, \overline{\theta}]$, *the optimal regulatory price of product 1 under private information must cover marginal cost. That is,* $p_1(\theta) \geq c_1, \forall \theta \in [\underline{\theta}, \overline{\theta}]$.

Therefore, Lemma 2 implies that for any incentive-compatible policy, informational rents must increase with the demand parameter, that is,

$$d\Pi(\theta)/d\theta = \Pi_\theta > 0, \forall \theta \in [\underline{\theta}, \overline{\theta}].$$

As a consequence, the individual rationality constraint needs to be satisfied only at $\theta = \underline{\theta}$, and we can rewrite the firm's profits of type θ as

$$\Pi(\theta) = \Pi(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \Pi_\theta(p_1(\mu), p_2(\mu), \mu) d\mu. \tag{14}$$

The regulation problem can thus be rewritten as

$$\max_{p_1(\theta), p_2(\theta), \Pi(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} W\big(p_1(\theta), p_2(\theta), \Pi(\theta)\big) f(\theta) d\theta \tag{15}$$

subject to (a) $\Pi(\theta) = \Pi(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \Pi_\theta(p_1(\mu), p_2(\mu), \mu) d\mu,$

(b) $\Pi(\underline{\theta}) \geq 0,$

(c) $\dfrac{dp_1(\theta)}{d\theta} \geq 0,$

(d) $p_1(\theta) - c_1 \geq 0.$

By introducing condition (a) into the objective function and by taking into account that condition (b) is binding at the optimum, we can rewrite the social welfare in state θ as

[13] Recall that if $(\partial^2 D^i / \partial p_i \partial p_j) = 0$, prices are social complements when products are demand substitutes.

$$\begin{aligned}(16)\quad &\widetilde{W}\big(p_1(\theta),p_2(\theta)\big)\\ &= v\big(p_1(\theta),p_2(\theta)\big)+(1+\lambda)\Big[\big(p_1(\theta)-c_1\big)D^1\big(p_1(\theta),p_2(\theta),\theta\big)\\ &\qquad +\big(p_2(\theta)-c_2\big)D^2\big(p_1(\theta),p_2(\theta),\theta\big)-F\Big]\\ &\qquad -(1+\lambda-\alpha)\frac{1-F(\theta)}{f(\theta)}\Pi_\theta\big(p_1(\theta),p_2(\theta),\theta\big).\end{aligned}$$

The regulator's maximization problem under incomplete information is given by

$$(17)\qquad \max_{p_1(\theta),p_2(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \widetilde{W}\big(p_1(\theta),p_2(\theta)\big)f(\theta)d\theta$$

subject to (c) and (d).

First, we solve the problem without taking into account constraints (c) and (d). Then, we analyze the implications of these restrictions on the optimal regulatory policy. The first-order conditions are

$$(18)\quad \widetilde{W}_{p_1}(\hat{p}_1(\theta),\hat{p}_2(\theta)) = f(\theta)W_{p_1}(\hat{p}_1(\theta),\hat{p}_2(\theta)) - (1+\lambda-\alpha)\big[1-F(\theta)\big]\Pi_{\theta p_1} = 0,$$

$$(19)\qquad \widetilde{W}_{p_2}(\hat{p}_1(\theta),\hat{p}_2(\theta)) = W_{p_2}(\hat{p}_1(\theta),\hat{p}_2(\theta)) = 0.$$

The following proposition states the main result of this subsection.

PROPOSITION 1 *The optimal regulatory price policy under private information, $p_1^{PI}(\theta)$ and $p_2^{PI}(\theta)$, is given by*

$$p_1^{PI}(\theta)=\begin{cases}\hat{p}_1(\theta) & \overline{\theta}\ge\theta\ge\theta_0,\\ c_1 & \theta_0\ge\theta,\end{cases}\qquad p_2^{PI}(\theta)=\begin{cases}\hat{p}_2(\theta) & \overline{\theta}\ge\theta\ge\theta_0,\\ \hat{p}_2(\theta_0) & \theta_0\ge\theta,\end{cases}$$

where $\hat{p}_1(\theta)$ and $\hat{p}_2(\theta)$ solve (18) *and* (19)*, that is,*

$$\begin{aligned}(20)\quad \frac{\hat{p}_1(\theta)-c_1}{\hat{p}_1(\theta)} &= \frac{\lambda}{(1+\lambda)}\left[\frac{1}{\varepsilon_{11}}+\frac{(1+\lambda)}{\lambda}\frac{(\hat{p}_2(\theta)-c_2)D^2\varepsilon_{21}}{R_1\varepsilon_{11}}\right]\\ &\quad -\frac{1+\lambda-\alpha}{(1+\lambda)}\left[\frac{1-F(\theta)}{f(\theta)}\right]\frac{\Pi_{\theta p_1}}{\varepsilon_{11}D^1},\end{aligned}$$

$$(21)\qquad \frac{\hat{p}_2(\theta)-c_2}{\hat{p}_2(\theta)} = \frac{\lambda}{(1+\lambda)}\left[\frac{1}{\varepsilon_{22}}+\frac{(1+\lambda)}{\lambda}\frac{(\hat{p}_1(\theta)-c_1)D^1\varepsilon_{12}}{R_2\varepsilon_{22}}\right],$$

and θ_0 is such that $\hat{p}_1(\theta_0)=c_1$.

By solving (20) and (21) jointly we get

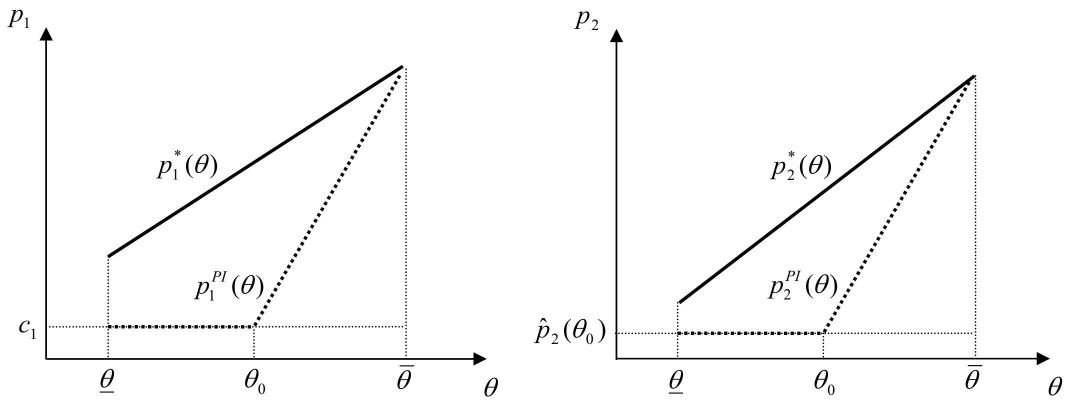
$$(22)\quad \frac{\hat{p}_1(\theta)-c_1}{\hat{p}_1(\theta)} = \frac{\lambda}{(1+\lambda)}\frac{1}{\hat{\varepsilon}_{11}} - \frac{(1+\lambda-\alpha)}{(1+\lambda)}\frac{\varepsilon_{22}}{\varepsilon_{11}\varepsilon_{22}-\varepsilon_{12}\varepsilon_{21}}\left[\frac{1-F(\theta)}{f(\theta)}\right]\frac{\Pi_{\theta p_1}}{D^1},$$

$$(23)\quad \frac{\hat{p}_2(\theta)-c_2}{\hat{p}_2(\theta)}=\frac{\lambda}{(1+\lambda)}\frac{1}{\hat{\varepsilon}_{22}}-\frac{(1+\lambda-\alpha)}{(1+\lambda)}\frac{R_1\varepsilon_{22}}{R_2(\varepsilon_{11}\varepsilon_{22}-\varepsilon_{12}\varepsilon_{21})}\left[\frac{1-F(\theta)}{f(\theta)}\right]\frac{\Pi_{\theta p_1}}{D^1}.$$

By comparing the complete-information solution $(p_1^*(\theta), p_2^*(\theta))$ and the asymmetric information pricing policy $(p_1^{PI}(\theta), p_2^{PI}(\theta))$ in Figure 1, we see that for all $\theta \in [\underline{\theta}, \overline{\theta})$, we have $p_1^*(\theta) > p_1^{PI}(\theta)$ and $p_2^*(\theta) > p_2^{PI}(\theta)$.[14]

Figure 1

Regulatory Policy with Social Complements under Full and under Asymmetric Information



When the regulator chooses the optimal price levels under private information, she maximizes the expected social welfare subject to the incentive compatibility constraint. In order to reduce the informational rents of the monopoly, she will make this constraint as small as possible, while still being compatible with the maximization of the social surplus. To reduce this rent the regulator must reduce Π_θ for all $\theta \in [\underline{\theta}, \overline{\theta})$. Given that $\Pi_{\theta p_1} > 0$ and $\Pi_{\theta p_2} = 0$, she achieves this objective by decreasing p_1. Moreover, given that prices are complements from a social welfare point of view, it is optimal to move these variables in the same direction, and as a consequence p_2 is reduced. Figure 1 also shows how the two product prices change with θ under both full information and asymmetric information.

We next discuss the effect of distributional considerations, measured through α, and the cost of public funds, λ, on the optimal regulatory policy under private information. It must be stressed that if the cost of public funds is zero, then marginal-cost pricing would be optimal under both complete and incomplete information, independently of distributional considerations. Therefore, the results of LEWIS AND SAPPINGTON [1988a] may be generalized to a multiproduct firm

---

[14] For the sake of simplicity we represent in the figures the case of linear functions. This would be the case, for example, when the distribution function is uniform and $(\partial^2 D^1 / \partial \theta^2) = g''(\theta) = 0$.

setting.[15] Under costly public funds, Ramsey pricing characterizes the optimal policy under full information, which implies that informational rents necessarily appear in order to guarantee incentive compatibility.

5.2 Social Substitutes

When prices are substitutes from a social welfare point of view, the optimal prices under complete information may be smaller than marginal costs, depending on the value of θ. We shall distinguish three cases:

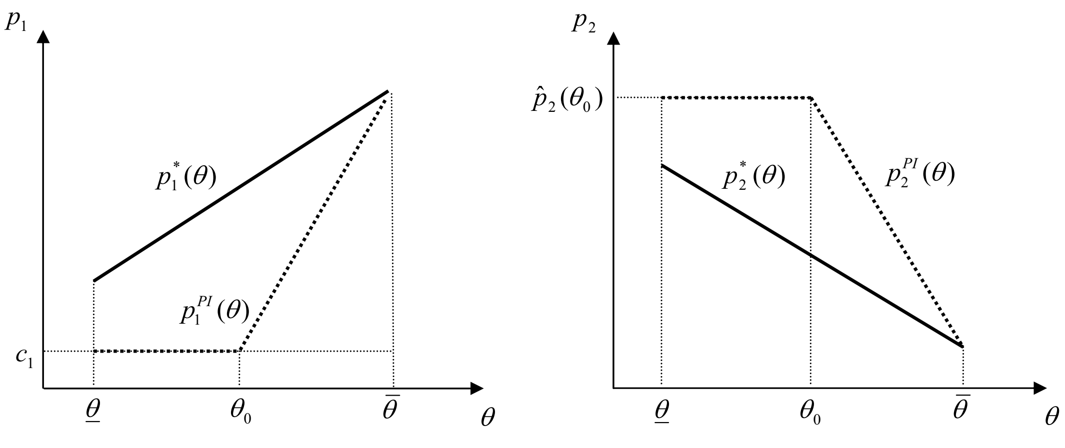
5.2.1 Demand Complements when Product 1 is Not Cross-Subsidized

The first case occurs when product 1 is not subsidized by product 2, in the sense that under full information the optimal price of product 1 is above marginal cost for any realization of θ; that is, $p_1^*(\theta) > c_1, \forall\, \theta \in [\underline{\theta}, \overline{\theta}]$. Given that Lemma 2 applies, the characterization of the optimal regulatory prices under private information is similar to the case of social complements.

By comparing the complete-information solution $(p_1^*(\theta), p_2^*(\theta))$ and the asymmetric information pricing policy $(p_1^{PI}(\theta), p_2^{PI}(\theta))$ in Figure 2, we see that for all $\theta \in [\underline{\theta}, \overline{\theta})$, we have $p_1^*(\theta) > p_1^{PI}(\theta)$ and $p_2^*(\theta) < p_2^{PI}(\theta)$.

Figure 2

Regulatory Policy with Social Substitutes under Full Information and Asymmetric Information



When the regulator chooses the optimal price levels under private information, she maximizes the expected social welfare taking into account the incentive compatibility constraint, and in order to reduce informational rents the regulator

[15] If there is no cost of public funds, then the full-information policy (which coincides with the first-best policy) is implementable, and private information is inconsequential for regulation.

must reduce Π_θ for all $\theta \in [\underline{\theta}, \overline{\theta})$. Given that $\Pi_{\theta p_1} > 0$ and $\Pi_{\theta p_2} = 0$, she again achieves this objective by decreasing p_1. Moreover, given that prices are substitutes from a social welfare point of view, it is optimal to move these variables in opposite directions, and as a consequence p_2 is increased.

5.2.2 Demand Complements When Product 1 Is Cross-Subsidized

The second case occurs when product 1 is subsidized by product 2 in the sense that under full information the optimal price for product 1 is below its marginal cost for any realization of θ. That is, $p_1^*(\theta) < c_1, \forall \theta \in [\underline{\theta}, \overline{\theta}]$. The next lemma restricts the optimal regulatory policy.

LEMMA 3 *Given that* $p_1^*(\theta) < c_1, \forall \theta \in [\underline{\theta}, \overline{\theta}]$, *the optimal regulatory price of product 1 under private information must be lower than or equal to marginal cost. That is,* $p_1(\theta) \leq c_1, \forall \theta \in [\underline{\theta}, \overline{\theta}]$.

Therefore, Lemma 3 implies that for any incentive-compatible policy informational rents must not increase with the demand parameter:

$$d\Pi(\theta)/d\theta = \Pi_\theta \leq 0, \forall \theta \in [\underline{\theta}, \overline{\theta}].$$

The individual rationality constraint needs to be satisfied only at $\theta = \overline{\theta}$, and we can rewrite the firm's profits of type θ as

$$\Pi(\theta) = \Pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} \Pi_\theta(p_1(\mu), p_2(\mu), \mu)d\mu. \tag{24}$$

The regulation problem may be written as

$$\max_{p_1(\theta), p_2(\theta), \Pi(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} W\big(p_1(\theta), p_2(\theta), \Pi(\theta)\big) f(\theta) d\theta \tag{25}$$

subject to

(a) $\Pi(\theta) = \Pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} \Pi_\theta(p_1(\mu), p_2(\mu), \mu)d\mu,$

(b) $\Pi(\overline{\theta}) \geq 0,$

(c) $\dfrac{dp_1(\theta)}{d\theta} \geq 0,$

(d) $p_1(\theta) - c_1 \leq 0.$

By introducing (a) into the objective function and by taking into account that (b) is binding at the optimum, we rewrite the social welfare in state θ as

(26) $$\widetilde{W}\big(p_1(\theta), p_2(\theta)\big) = v\big(p_1(\theta), p_2(\theta)\big) + (1+\lambda)\Big[\big(p_1(\theta)-c_1\big)D^1\big(p_1(\theta), p_2(\theta), \theta\big) + \big(p_2(\theta)-c_2\big)D^2\big(p_1(\theta), p_2(\theta)\big) - F\Big] + (1+\lambda-\alpha)\frac{F(\theta)}{f(\theta)}\Pi_\theta\big(p_1(\theta), p_2(\theta), \theta\big).$$

The regulator's problem under incomplete information is therefore given by

(27) $$\max_{p_1(\theta), p_2(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \widetilde{W}\big(p_1(\theta), p_2(\theta)\big) f(\theta) d\theta$$

subject to (c) and (d).

The first-order conditions [without taking into account constraints (c) and (d)] are then

(28) $$\widetilde{W}_{p_1}\big(\hat{p}_1(\theta), \hat{p}_2(\theta)\big) = f(\theta) W_{p_1}\big(\hat{p}_1(\theta), \hat{p}_2(\theta)\big) - (1+\lambda-\alpha)F(\theta)\Pi_{\theta p_1} = 0,$$

(29) $$\widetilde{W}_{p_2}\big(\hat{p}_1(\theta), \hat{p}_2(\theta)\big) = W_{p_2}\big(\hat{p}_1(\theta), \hat{p}_2(\theta)\big) = 0.$$

The next proposition states the optimal regulatory price policy.

PROPOSITION 2 *The regulatory policy under private information,* $p_1^{PI}(\theta)$ *and* $p_2^{PI}(\theta)$*, is*

$$p_1^{PI}(\theta) = \begin{cases} c_1 & \theta_1 \le \theta, \\ \hat{p}_1(\theta) & \underline{\theta} \le \theta \le \theta_1, \end{cases} \qquad p_2^{PI}(\theta) = \begin{cases} \hat{p}_2(\theta_1) & \theta_1 \le \theta, \\ \hat{p}_2(\theta) & \underline{\theta} \le \theta \le \theta_1, \end{cases}$$

where $\hat{p}_1(\theta)$ *and* $\hat{p}_2(\theta)$ *solve (28) and (29), that is,*

(30)
$$\frac{\hat{p}_1(\theta)-c_1}{\hat{p}_1(\theta)} = \frac{\lambda}{(1+\lambda)}\left[\frac{1}{\varepsilon_{11}} - \frac{(1+\lambda)}{\lambda}\frac{\big(\hat{p}_2(\theta)-c_2\big)D^2\varepsilon_{21}}{R_1\varepsilon_{11}}\right]' + \frac{1+\lambda-\alpha}{(1+\lambda)}\left[\frac{F(\theta)}{f(\theta)}\right]\frac{\Pi_{\theta p_1}}{\varepsilon_{11}D^1},$$

(31) $$\frac{\hat{p}_2(\theta)-c_2}{\hat{p}_2(\theta)} = \frac{\lambda}{(1+\lambda)}\left[\frac{1}{\varepsilon_{22}} - \frac{(1+\lambda)}{\lambda}\frac{\big(\hat{p}_1(\theta)-c_1\big)D^1\varepsilon_{12}}{R_2\varepsilon_{22}}\right],$$

and θ_1 *is such that* $\hat{p}_1(\theta_1) = c_1$.

By solving (30) and (31) jointly we get

(32) $$\frac{\hat{p}_1(\theta)-c_1}{\hat{p}_1(\theta)} = \frac{\lambda}{(1+\lambda)}\frac{1}{\hat{\varepsilon}_{11}} + \frac{(1+\lambda-\alpha)}{(1+\lambda)}\frac{\varepsilon_{22}}{\varepsilon_{11}\varepsilon_{22}-\varepsilon_{12}\varepsilon_{21}}\left[\frac{F(\theta)}{f(\theta)}\right]\frac{\Pi_{\theta p_1}}{D^1},$$

$$(33)\quad \frac{\hat{\hat{p}}_2(\theta)-c_2}{\hat{\hat{p}}_2(\theta)} = \frac{\lambda}{(1+\lambda)}\frac{1}{\hat{\varepsilon}_{22}} + \frac{(1+\lambda-\alpha)}{(1+\lambda)}\frac{R_1\varepsilon_{12}}{R_2(\varepsilon_{11}\varepsilon_{22}-\varepsilon_{12}\varepsilon_{21})} + \left[\frac{F(\theta)}{f(\theta)}\right]\frac{\Pi_{\theta p_1}}{D^1}\,.$$

Figure 3

Regulatory Policy with Social Substitutes and Cross-Subsidization

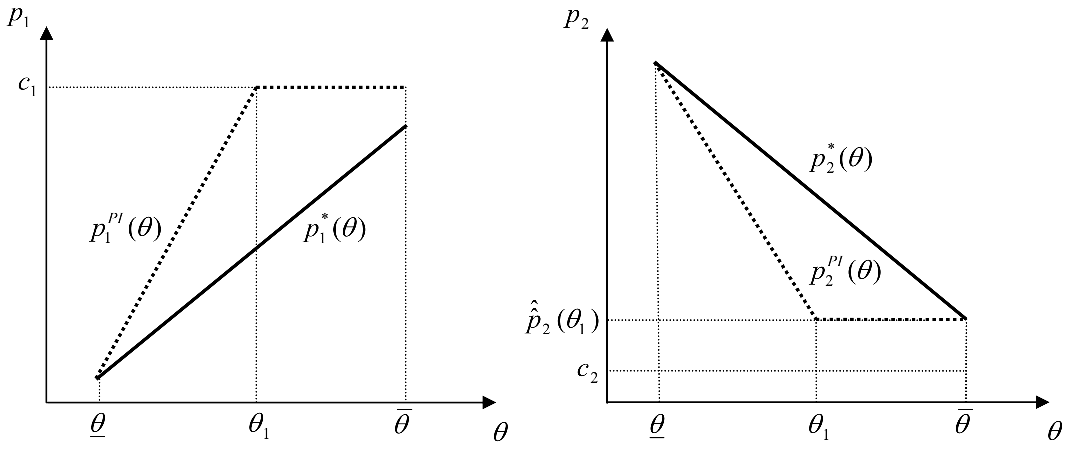


Figure 3 illustrates the comparison between regulatory policies under both full information and private information: for all $\theta \in (\underline{\theta}, \overline{\theta}]$, we have $p_1^*(\theta) < p_1^{PI}(\theta)$ and $p_2^*(\theta) > p_2^{PI}(\theta)$ In order to reduce informational rents the regulator reduces the difference $p_1 - c_1$ with respect to the complete-information case for all θ in $(\underline{\theta}, \overline{\theta}]$. Given that this difference is negative, the regulator increases p_1, and, since prices are social substitutes, this implies a decrease in p_2.

5.2.3 Countervailing Incentives

The third case occurs when product 1 subsidizes product 2 only for low realizations of θ; that is, $c_1 \in [p_1^*(\underline{\theta}), p_1^*(\overline{\theta})]$. As we shall show, this case is characterized by the existence of countervailing incentives. Let $\tilde{\theta}$ be the value of the demand parameter such that $p_1^*(\tilde{\theta}) = c_1$. Given that $dp_1^*(\theta)/d\theta > 0$, then $p_1^*(\theta) < c_1$ for all $\theta \in [\underline{\theta}, \tilde{\theta})$, and $p_1^*(\theta) > c_1$ for all $\theta \in (\tilde{\theta}, \overline{\theta}]$. The next lemma restricts the optimal regulatory policy.

LEMMA 4 *The optimal regulatory price of product 1 under private information may be above or below than marginal cost, depending on θ. In particular,* $p_1(\theta) \le c_1, \forall\, \theta \in [\underline{\theta}, \tilde{\theta}]$, *and* $p_1(\theta) \ge c_1, \forall\, \theta \in [\tilde{\theta}, \overline{\theta}]$.

Lemma 4 implies that $d\Pi(\theta)/d\theta = \Pi_\theta \le 0$ for all $\theta \in [\underline{\theta}, \tilde{\theta}]$ and $d\Pi(\theta)/d\theta = \Pi_\theta \ge 0$ for all $\theta \in [\tilde{\theta}, \overline{\theta}]$. Countervailing incentives are then unavoidable (see, for example, LEWIS AND SAPPINGTON [1989], MAGGI AND

RODRIGUEZ-CLARE [1995], JULLIEN [2000], or ARMSTRONG AND SAPPINGTON [2007]). The individual rationality constraint needs to be satisfied only at $\theta = \tilde{\theta}$, and we can rewrite the firm's profits of type θ as

(34) $$\Pi(\theta) = \Pi(\tilde{\theta}) - \int_{\theta}^{\tilde{\theta}} \Pi_\theta\big(p_1(\mu), p_2(\mu), \mu\big) d\mu \quad \forall\, \theta \in [\underline{\theta}, \tilde{\theta}],$$

(35) $$\Pi(\theta) = \Pi(\tilde{\theta}) + \int_{\tilde{\theta}}^{\theta} \Pi_\theta\big(p_1(\mu), p_2(\mu), \mu\big) d\mu \quad \forall\, \theta \in [\tilde{\theta}, \bar{\theta}].$$

The regulation problem can be written as

(36) $$\max_{p_1(\theta), p_2(\theta), \Pi(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} W\big(p_1(\theta), p_2(\theta), \Pi(\theta)\big) f(\theta) d\theta$$

subject to (a) $\Pi(\theta) = \Pi(\tilde{\theta}) - \int_{\theta}^{\tilde{\theta}} \Pi_\theta\big(p_1(\mu), p_2(\mu), \mu\big) d\mu \quad \forall\, \theta \in [\underline{\theta}, \tilde{\theta}]$,

(a1) $\Pi(\theta) = \Pi(\tilde{\theta}) + \int_{\tilde{\theta}}^{\theta} \Pi_\theta\big(p_1(\mu), p_2(\mu), \mu\big) d\mu \quad \forall\, \theta \in [\tilde{\theta}, \bar{\theta}]$,

(b) $\Pi(\tilde{\theta}) \geq 0$,

(c) $\dfrac{dp_1(\theta)}{d\theta} \geq 0$,

(d) $p_1(\theta) - c_1 \leq 0, \quad \forall\, \theta \in [\underline{\theta}, \tilde{\theta}]$,

(d1) $p_1(\theta) - c_1 \geq 0, \quad \forall\, \theta \in [\tilde{\theta}, \bar{\theta}]$.

By introducing (a) and (a1) into the objective function and by taking into account that (b) is binding at the optimum, we can rewrite the social welfare in state θ as

(37) $$\begin{aligned} &\tilde{W}\big(p_1(\theta), p_2(\theta)\big) \\ &= v\big(p_1(\theta), p_2(\theta)\big) + (1+\lambda)\Big[\big(p_1(\theta) - c_1\big) D^1\big(p_1(\theta), p_2(\theta), \theta\big) \\ &\qquad + \big(p_2(\theta) - c_2\big) D^2\big(p_1(\theta), p_2(\theta)\big) - F\Big] \\ &\quad + \begin{cases} (1+\lambda-\alpha)\dfrac{F(\theta)}{f(\theta)} \Pi_\theta\big(p_1(\theta), p_2(\theta), \theta\big) & \forall\, \theta \in [\underline{\theta}, \tilde{\theta}], \\ -(1+\lambda-\alpha)\dfrac{1-F(\theta)}{f(\theta)} \Pi_\theta\big(p_1(\theta), p_2(\theta), \theta\big) & \forall\, \theta \in [\underline{\theta}, \tilde{\theta}]. \end{cases} \end{aligned}$$

Thus the regulator's problem is

(38)
$$\max_{p_1(\theta),p_2(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \widetilde{W}\big(p_1(\theta), p_2(\theta)\big) f(\theta) d\theta$$

subject to (c), (d), and (d1).

The next proposition characterizes the optimal regulatory price policy.

PROPOSITION 3 *The optimal regulatory price policy under private information, $p_1^{PI}(\theta)$ and $p_2^{PI}(\theta)$, is given by*

$$p_1^{PI}(\theta) = \begin{cases} \hat{\hat{p}}_1(\theta) & \underline{\theta} \le \theta \le \theta_1, \\ c_1 & \theta_1 \le \theta \le \theta_0, \\ \hat{p}_1(\theta) & \theta_0 \le \theta \le \overline{\theta}, \end{cases} \qquad p_2^{PI}(\theta) = \begin{cases} \hat{\hat{p}}_2(\theta) & \underline{\theta} \le \theta \le \theta_1, \\ p_2^*(\widetilde{\theta}) & \theta_1 \le \theta \le \theta_0, \\ \hat{p}_2(\theta) & \theta_0 \le \theta \le \overline{\theta}, \end{cases}$$

where $\hat{p}_1(\theta)$, $\hat{p}_2(\theta)$, $\hat{\hat{p}}_1(\theta)$, $\hat{\hat{p}}_2(\theta)$, and $p_2^*(\widetilde{\theta})$ are defined by (20)*,* (21)*,* (30)*,* (31)*, and* (9)*, respectively; θ_0 is such that $\hat{p}_1(\theta_0) = c_1$, and θ_1 is such that $\hat{\hat{p}}_1(\theta_1) = c_1$.*

Figure 4
Countervailing Incentives

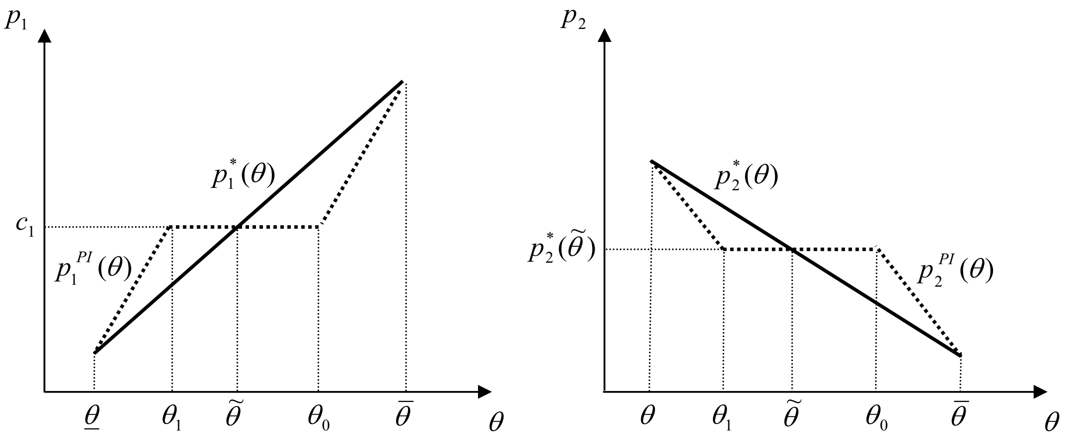


Figure 4 illustrates the relationship between optimal prices under the two informational regimes (full information and asymmetric information) as a function of the type θ. Note that there are countervailing incentives under the optimal regulatory policy: for low realizations of θ the firm's incentive to overstate θ will dominate its incentive to understate θ, while for higher realizations the dominant incentive will be to understate θ. The optimal regulatory policy will prescribe product 1's prices (product 2's prices) below (above) marginal cost for low realizations of θ, $\theta \le \theta_1$, and product 1's prices (product 2's prices) above (below) marginal cost for high realizations, $\theta \ge \theta_0$. For intermediate realizations, $\theta_1 \leq \theta \leq \theta_0$, the countervailing incentives compel the regulator to set a price for each product that does not vary with the realized demand of product 1. As a consequence, no pricing authority is delegated to the firm, and the regulator relies on her prior beliefs to establish a single regulated price for each product.

The type of countervailing incentives in our model is similar to that in LEWIS AND SAPPINGTON [1989]. Informational rents are decreasing for low types (low realizations of θ), equal to zero for an intermediate interval of types (intermediate realizations of θ), and increasing for high types (high realizations of θ).[16]

6 Concluding Remarks

In this paper we have studied the optimal regulation policy of a multiproduct monopolist when the regulated firm has better information about demand conditions than the regulator. We find that the optimal regulation policy depends crucially on whether goods are substitutes or complements. With demand substitutes, the firm's incentive is always to understate the market 1 demand; to reduce informational rents the regulator sets prices for both products below those under full information. With demand complements, the characterization of the optimal regulatory policy is more complex and depends on whether product 1 is subsidized under full information. In that case, the firm's incentive is always to overstate the market 1 demand, and, in order to reduce informational rents, the regulator increases the price of product 1 and reduces the price of product 2. When product 1 is not subsidized under full information, the firm's incentive is always to understate the market 1 demand; and to reduce informational rents, the regulator reduces the price of product 1 and increases the price of product 2. Finally, we analyze the possibility that countervailing incentives may arise with demand complements. In those cases the firm may be tempted either to overstate or to understate its private information, depending upon the realization of the parameter of demand.

Our results may be generalized in several directions. The assumption of constant marginal cost could be relaxed in order to allow increasing (and separable) marginal cost without altering our main results. The analysis might also be extended to consider the regulation of a monopolist selling *n* substitute products (the case with complements would be very complex) under private information concerning one market.[17] One further possible extension that we leave for further research is to consider the optimal regulation of a multiproduct

[16] MAGGI AND RODRIGUEZ-CLARE [1995] identify other type of optimal control where informational rents are bell-shaped, with both extreme types earning no rents.

[17] The case of multidimensional private information affecting the demand of different products would involve considerable technical difficulties. This situation can be analyzed only, to the best of our knowledge, in very simple settings where either there is perfect correlation between the private information parameters (in this setting our results could be generalized to the case of demand substitutes) or the type space is discrete rather than continuous.

firm with private information concerning the degree of substitutability between its products.

Appendix

A.1 Proof of Lemma 1

The proof of Lemma 1 is standard (see, for example, BARON AND MYERSON [1982] and GUESNERIE AND LAFFONT [1984]). The profit of the firm of type θ when it reports $\hat{\theta}$ is given by

$$\Pi(\hat{\theta},\theta) = \left[p_1(\hat{\theta}) - c_1\right] D^1\left(p_1(\hat{\theta}), p_2(\hat{\theta}), \theta\right) + \left[p_2(\hat{\theta}) - c_2\right] D^2\left(p_1(\hat{\theta}), p_2(\hat{\theta})\right) - F + t(\hat{\theta}).$$

If truth-telling is a local optimal response for the monopolist, then the following two conditions are satisfied: $\Pi_{\hat{\theta}}(\theta,\theta) = 0$ (the first-order condition) and $\Pi_{\hat{\theta}\hat{\theta}}(\theta,\theta) \leq 0.$ By differentiating the profit function with respect to θ we obtain that

$$\frac{d\Pi(\hat{\theta},\theta)}{d\theta} = \Pi_{\hat{\theta}}(\hat{\theta},\theta)\frac{d\hat{\theta}}{d\theta} + \Pi_{\theta}(\hat{\theta},\theta).$$

If we evaluate this expression at $\hat{\theta} = \theta,$ from the first-order condition we have

$$\frac{d\Pi(\theta)}{d\theta} = \Pi_{\theta}(\hat{\theta},\theta) = \left(p_1(\theta) - c_1\right)\frac{\partial D^1}{d\theta},$$

which corresponds to part (i) in Lemma 1.

By differentiating the first-order condition with respect to θ we have

$$\Pi_{\hat{\theta}\hat{\theta}}(\theta,\theta) + \Pi_{\hat{\theta}\theta}(\theta,\theta) = 0.$$

Note that if the second-order condition $\left(\Pi_{\hat{\theta}\hat{\theta}}(\theta,\theta) \leq 0\right)$ is satisfied, then

$$\Pi_{\hat{\theta}\theta}(\theta,\theta) = \Pi_{\hat{\theta}p_1}\left[dp_1(\theta)/d\theta\right] \geq 0.$$

Given that $\Pi_{\hat{\theta}p_1} \geq 0,$ then $dp_1(\theta)/d\theta \geq 0$ is equivalent to the sufficient local order condition.

It is easy to show that $dp_1(\theta)/d\theta \geq 0$ also implies global optimality. If truth-telling, $\hat{\theta} = \theta,$ is a global optimum for type θ, we have that

$$\Pi(\theta,\theta) \geq \Pi(\hat{\theta},\theta), \quad \forall(\hat{\theta},\theta) \in \Theta^2,$$

or, equivalently,

$$\int_{\hat{\theta}}^{\theta} \Pi_{\hat{\theta}}(x,\theta)dx \geq 0.$$

By using the first-order condition, we have

$$\int_{\hat{\theta}}^{\theta}\left[\Pi_{\hat{\theta}}(x,\theta)-\Pi(x,x)\right]dx \geq 0 \quad \text{or} \quad \int_{\hat{\theta}}^{\theta}\int_{x}^{\theta}\Pi_{\hat{\theta}\theta}(x,y)dxdy \geq 0.$$

Therefore, $p_1(\theta) / d\theta \geq 0$ implies

$$\Pi_{\hat{\theta}\theta}(\theta,\theta) = \Pi_{\theta p_1}[dp_1(\theta)/d\theta] \geq 0,$$

and given that $x \geq \theta$ when $\hat{\theta} > \theta$ and $x \leq \theta$ when $\hat{\theta} < \theta$, a local optimum is always global. *Q.E.D.*

A.2 Proof of Lemma 2

Assume that $\breve{p}_1(\theta)$ is a solution to the regulatory problem under private information such that $\breve{p}_1(\theta) < c_1$ for $\theta \in \left[\underline{\theta},\breve{\theta}\right]$. (Note that $\breve{p}_1(\breve{\theta}) < c_1$ implies that $\breve{p}_1(\theta) < c_1$ for $\theta \in \left[\underline{\theta},\breve{\theta}\right]$, given part (ii) of Lemma 1.) At $\theta \in \left[\underline{\theta},\breve{\theta}\right]$ the slope of $\Pi(\theta)$ required to induce truth-telling, given Lemma 1, part (i), is

$$\frac{d\Pi(\theta)}{d\theta} = \Pi_\theta = \left(p_1(\theta) - c_1\right)\frac{\partial D^1}{\partial \theta} < 0,$$

which specifies how informational rents must change with the product 1 demand parameter. Individual rationality (IR) and $d\Pi(\theta)/d\theta < 0$ imply that any type $\theta \in \left[\underline{\theta},\breve{\theta}\right]$ would obtain a strictly positive information rent. Consider a price policy $\breve{\breve{p}}_1(\theta)$ such that

$$\breve{\breve{p}}_1(\theta) = \begin{cases} c_1 & \text{for } \theta \in [\underline{\theta},\breve{\theta}], \\ \breve{p}_1(\theta) & \text{for } \theta \in (\breve{\theta},\overline{\theta}]. \end{cases}$$

Note that when moving from $\breve{p}_1(\theta)$ to $\breve{\breve{p}}_1(\theta)$ total surplus increases, since from a social welfare point of view the price of product 1 must be greater than its marginal cost because of the cost of public funds, and informational rents also decrease. This argument, therefore, means that $\breve{p}_1(\theta)$ is not a solution to the regulatory problem. *Q.E.D.*

A.3 Proof of Lemma 3

Assume that $\widehat{p}_1(\theta)$ is a solution to the regulatory problem under private information such that $\widehat{p}_1(\theta) > c_1$ for $\theta \in \left[\widehat{\theta},\overline{\theta}\right]$. (Note that $\widehat{p}_1(\widehat{\theta}) > c_1$ implies that $\widehat{p}_1(\theta) > c_1$ for $\theta \in \left[\widehat{\theta},\overline{\theta}\right]$, given part (ii) of Lemma 1.) At $\theta \in \left[\widehat{\theta},\overline{\theta}\right]$ the slope of $\Pi(\theta)$ required to induce truth-telling, given Lemma 1, part (i), is

$$\frac{d\Pi(\theta)}{d\theta} = \Pi_\theta = \left(p_1(\theta) - c_1\right)\frac{\partial D^1}{\partial \theta} > 0,$$

which specifies how informational rents must change with the product 1 demand parameter. Individual rationality (IR) and $d\Pi(\theta)/d\theta > 0$ imply that any type $\theta \in \left[\hat{\theta},\overline{\theta}\right]$ would obtain a strictly positive information rent. Consider a price policy $\hat{\hat{p}}_1(\theta)$ such that

$$\hat{\hat{p}}_1(\theta) = \begin{cases} \hat{p}_1(\theta) & \text{for } \theta \in [\underline{\theta},\hat{\theta}), \\ c_1 & \text{for } \theta \in [\hat{\theta},\overline{\theta}]. \end{cases}$$

Note that when moving from $\hat{p}_1(\theta)$ to $\hat{\hat{p}}_1(\theta)$ total surplus increases, since from a social welfare point of view the price of product 1 must be lower than its marginal cost because of the cost of public funds, and informational rents also decrease. This argument, therefore, contradicts $\hat{p}_1(\theta)$ as a solution to the regulatory problem. *Q.E.D.*

A.3 Proof of Lemma 4

The proof is straightforward, taking into account the proofs of Lemma 2 and Lemma 3. The type $\tilde{\theta}$ is the value of the demand parameter such that $p_1^*(\tilde{\theta}) = c_1$ and $p_1^*(\tilde{\theta}) < c_1$ for all $\theta \in \left[\underline{\theta},\tilde{\theta}\right)$, and $p_1^*(\theta) < c_1$ for all $\theta \in \left(\tilde{\theta},\overline{\theta}\right]$. Lemma 2 applies when $p_1^*(\theta) > c_1$, and Lemma 3 when $p_1^*(\theta) < c_1$. Therefore, from Lemma 3 it follows that $p_1(\theta) \le c_1, \forall\, \theta \in \left[\underline{\theta},\tilde{\theta}\right]$, and from Lemma 2 that $p_1(\theta) \ge c_1, \forall\, \theta \in \left[\tilde{\theta},\overline{\theta}\right]$. *Q.E.D.*

A.4 Proof of Proposition 1

Firstly, if we do not take constraint (d) into account, then it is straightforward to obtain the price policy $\left(\hat{p}_1(\theta),\hat{p}_2(\theta)\right)$ from the conditions (19) and (20). If we include constraint (d), then the optimal regulatory policy for product 1 may be written as $p_1^{PI}(\theta) = \max\,\{\hat{p}_1(\theta),c_1\}$. Denote by θ_0 the type such that the conditions (19) and (20) are satisfied but the price of product 1 is equal to marginal cost, that is, $\hat{p}_1(\theta_0) = c_1$. By evaluating (19) and (20) at $\left(c_1,\hat{p}_2(\theta_0)\right)$ we obtain

$$\tilde{W}_{p_1}\left(c_1,\hat{p}_2(\theta_0)\right) < 0 \quad \forall\, \theta \in \left[\underline{\theta},\theta_0\right),$$

$$\tilde{W}_{p_1}\left(c_1,\hat{p}_2(\theta_0)\right) > 0 \quad \forall\, \theta \in \left(\theta_0,\overline{\theta}\right],$$

$$\tilde{W}_{p_2}\left(c_1,\hat{p}_2(\theta_0)\right) = 0 \quad \forall\, \theta \in \left[\underline{\theta},\overline{\theta}\right],$$

because $\tilde{W}_{p_1\theta} > 0$ and $\tilde{W}_{p_2\theta} = 0$. Given that prices are social complements, ($\tilde{W}_{p_1p_2} > 0$) and that second-order conditions must be satisfied, the above inequalities imply that for any $\theta \in \left(\theta_0,\overline{\theta}\right]$ there is a social benefit (a saving of public funds) from increasing the price of product 1 beyond its marginal cost and from increasing the price of product 2 over $\hat{p}_2(\theta_0)$, which implies $p_1^{PI}(\theta_0) = \hat{p}_1(\theta)$ and $p_2^{PI}(\theta) = \hat{p}_2(\theta)$ for all $\theta \in (\theta_0, \overline{\theta}]$. However, for types in the range $[\underline{\theta}, \theta_0)$, an increase in p_1 beyond its marginal cost, along with the corresponding increase in p_2, generates a loss of social welfare. Given Lemma 2, p_1 must be equal to c_1, and consequently p_2 must be at the optimal level when the price of product 1 equals its marginal cost, that is, $\hat{p}_2(\theta_0)$. Therefore, when $\theta < \theta_0$, the optimal price policy is $p_1^{PI}(\theta_0) = c_1$ and $p_2^{PI}(\theta) = \hat{p}_2(\theta_0)$. *Q.E.D.*

A.5 Proof of Proposition 2 and Proof of Proposition 3

The proofs are almost identical to the proof of Proposition 1, and hence omitted.

References

ADACHI, T. [2005], "Third-Degree Price Discrimination, Consumption Externalities and Social Welfare," *Economica*, 72, 171–178.

AGUIRRE, I., AND A. BEITIA [2004], "Regulating a Monopolist with Unknown Demand: Costly Public Funds and the Value of Private Information," *Journal of Public Economic Theory*, 6, 693–706.

ARMSTRONG, M. [1999], "Optimal Regulation with Unknown Demand and Cost Functions," *Journal of Economic Theory*, 84, 196–215.

–– AND R. PORTER (eds.) [2007], *Handbook of Industrial Organization,* Vol. 3, North-Holland: Amsterdam.

–– AND J.-C. ROCHET [1999], "Multi-Dimensional Screening: A User's Guide," *European Economic Review*, 43, 959–979.

–– AND D. E. M. SAPPINGTON [2007], "Recent Developments in the Theory of Regulation," pp. 1557–1700 in: ARMSTRONG AND PORTER (eds.) [2007].

–– AND J. VICKERS [2000], "Multi-Product Price Regulation under Asymmetric Information," *Journal of Industrial Economics*, XLVIII, 137–160.

BARON, D. [1989], "Design of Regulatory Mechanisms and Institutions," pp. 1347–1447 in: R. Schmalensee and R. Willig (eds.), *Handbook of Industrial Organization,* Vol. 2, North-Holland: Amsterdam.

–– AND R. MYERSON [1982], "Regulating a Monopolist with Unknown Costs," *Econometrica*, 50, 911–930.

BOITEUX, M. [1956], "Sur la gestion des monopoles publics astreints à l'equilibre budgétaire," *Econometrica*, 24, 22–40.

CAILLAUD, B., R. GUESNERIE, P. REY, AND J. TIROLE [1988], "Government Intervention in Production and Incentives Theory: A Review of Recent Contributions," *RAND Journal of Economics*, 19, 1–26.

CREW, M. A., AND P. R. KLEINDORFER [1986], *The Economics of Public Utility Regulation*, Macmillan: London.

DANA, J. D. [1993], "The Organization and Scope of Agents: Regulating Multi-Product Industries," *Journal of Economic Theory*, 59, 288–310.

DASGUPTA, P., P. HAMMOND, AND E. MASKIN [1979], "The Implementation of Social Choices Rules: Some General Results on Incentive Compatibility," *Review of Economic Studies*, 46, 185–216.

DAVIS, S. J., AND K. M. MURPHY [2000], "A Competitive Perspective on Internet Explorer," *American Economic Review: Papers and Proceedings*, 90, 184–187.[18]

FARRELL, J., AND P. KLEMPERER [2007], "Coordination and Lock-In: Competition with Switching Costs and Networks Effects," pp. 1967–2072 in: ARMSTRONG AND PORTER (eds.) [2007].

GUESNERIE, P., AND J.-J. LAFFONT [1984], "A Complete Solution to a Class of Principal–Agent Problems with an Application to the Control of a Self-Managed Firm," *Journal of Public Economics*, 25, 329–369.

HARRIS, J. E. [1979], "Pricing Rules for Hospitals," *Bell Journal of Economics*, 10, 224–243.

IOSSA, E. [1999], "Informative Externalities and Pricing in Regulated Multi-Product Industries," *Journal of Industrial Economics*, XLVII, 195–219.

JULLIEN, B. [2000], "Participation Constraints in Adverse Selection Models," *Journal of Economic Theory*, 93, 1–47.

LAFFONT, J.-J., AND J. TIROLE [1986], "Using Cost Observation to Regulate Firms," *Journal of Political Economy*, 94, 614–641.

– – AND – – [1990a], "The Regulation of Multi-Product Firms, Part I: Theory," *Journal of Public Economics*, 43, 1–36.

– – AND – – [1990b], "The Regulation of Multi-Product Firms, Part II: Applications to Competitive Environments and Policy Analysis," *Journal of Public Economics*, 43, 37–66.

– – AND – – [1993], *A Theory of Incentives in Procurement and Regulation*, MIT Press: Cambridge, MA.

LAYSON, S. K. [1998], "Third-Degree Price Discrimination with Interdependent Demands," *Journal of Industrial Economics*, 46, 511–524.

LEWIS, T. R., AND D. E. M. SAPPINGTON [1988a], "Regulating a Monopolist with Unknown Demand," *American Economic Review*, 78, 986–998.

– – AND – – [1988b], "Regulating a Monopolist with Unknown Demand and Cost Functions," *RAND Journal of Economics*, 19, 438–457.

– – AND – – [1989], "Countervailing Incentives in Agency Problems," *Journal of Economic Theory*, 49, 294–313.

MAGGI, G., AND A. RODRIGUEZ-CLARE [1995], "On Countervailing Incentives," *Journal of Economic Theory*, 66, 238–263.

MYERSON, R. [1979], "Incentive-Compatibility and the Bargaining Problem," *Econometrica*, 47, 61–74.

RIORDAN, M. [1984], "On Delegating Price Authority to a Regulated Firm," *RAND Journal of Economics*, 15, 108–115.

ROHLFS, J. [1979], "Economically Efficient Bell-System Pricing," Bell Laboratory Discussion Paper No. 138.

– – [2007], "Bandwagon Effects in Telecommunications," pp. 80–117 in: S. K. Majumdar, I. Vogelsang, and M. E. Cave (eds.), *Handbook of Telecommunications Economics,* Vol. 2, North-Holland: Amsterdam.

SAPPINGTON, D. E. M. [1983], "Optimal Regulation of a Multiproduct Monopoly with Unknown Technological Capabilities," *The Bell Journal of Economics*, 14, 453–463.

SAWKINS, J. W., AND S. REID [2007], "The Measurement and Regulation of Cross Subsidy: The Case of Scottish Water Industry," *Utilities Policy*, 15, 36–48.

[18] A mathematical supplement is available at http://gsbwww.uchicago.edu/fac/steven.davis/research.

SRINAGESH, P. [1984], "Mixed Linear–Nonlinear Pricing with Bundling," *Journal of Regulatory Economics*, 3, 251–263.
TIROLE, J. [1988], *The Theory of Industrial Organization*, MIT Press: Cambridge, MA.

Iñaki Aguirre
Arantza Beitia
Departamento de Fundamentos del Análisis Económico I
University of the Basque Country
Avda. Lehendakari Aguirre 83
48015 Bilbao
Spain
E-mail:
inaki.aguirre@ehu.es
arantza.beitia@ehu.es