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Product differentiation with consumer arbitrage

Iñaki Aguirre[a,*], María Paz Espinosa[b]

[a] Departamento de Fundamentos del Análisis Económico I, Universidad del País Vasco, Avenida Lehendakari Aguirre 83, 48015 Bilbao, Spain

[b] Departamento de Fundamentos del Análisis Económico II, Universidad del País Vasco, Bilbao, Spain

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Abstract

We determine the equilibrium to a game where two firms set delivered price schedules and consumers simultaneously declare where they want to take delivery of the product. Equilibrium pricing policies provide incentives for consumers not to demand their preferred varieties but rather to purchase more standard varieties. This behavior may decrease market diversity: the demanded varieties tend to agglomerate around the market center. We also show that it is efficient for the cost of adapting the product to consumers to be shared through arbitrage, but oligopoly gives rise to an inefficient level of personal arbitrage.

JEL classification: D82; L13; R32

Keywords: Spatial price discrimination; Product differentiation; Personal arbitrage

1. Introduction

In some markets, we observe that products are standardized: only one type of product is sold even though consumers' preferences may differ as to the best variety. In other markets, several varieties are produced but not as many as to satisfy the preferences of all consumers, so that some of them end up buying products that are close to, but not exactly, their preferred varieties. Examples include the acquisition of software: many of us do not buy an application that is the best suited to our needs, rather, we have to settle for something that is close enough; note that it would be possible for a consumer to order a

* Corresponding author. Tel.: +34-94-601-2000; fax: +34-94-601-3891.
E-mail address: jepagpei@bs.ehu.es (I. Aguirre).

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custom-made application, but in most cases this is not done because it would be too expensive. Another example is transport: we do not find a bus stop at every doorstep; people walk a few minutes to the nearest one.[1] We account for these phenomena on the basis of *personal arbitrage*.[2] We show that in equilibrium, consumers and firms share the costs of adapting the product to the consumers' needs, and the part of the cost paid by consumers may be in terms of utility loss. We also show that it is efficient for cost to be shared through arbitrage, but oligopoly gives rise to an inefficient level of personal arbitrage.

A common feature of most papers on spatial price discrimination is that consumers' locations are assumed to be known and firms take the product to those locations. However, some authors have addressed the problem faced by firms when demand is not known. Stole (1995), Hamilton and Thisse (1997) and Pires and Sarkar (2000) consider second degree price discrimination through the use of nonlinear pricing in an oligopoly setting where firms are spatially differentiated. In these papers the optimal pricing policies are obtained under the assumption that firms may commit themselves to a pricing strategy before consumers make their purchase decision, and this assumption may be appropriate in many circumstances (firms might commit themselves to prices through announcements in newspapers, catalogs, etc.). Nevertheless, changing pricing policies is not very costly and therefore commitment may be weak. This would be the case for some products with so many varieties that the price list is not publicly available. Rather, the consumer asks for a price quote specifying the required product characteristics (for example, a piece of furniture made to order). In this paper, we deal with the problem of firms' pricing under unknown locations when there is no commitment to pricing policies. In practice, this absence of commitment implies that firms' policies should be best responses to consumers' behavior and vice versa. In this context consumers are given an active role[3] and personal arbitrage appears under a new light, although some arbitrage would also be present with firms' commitment to the pricing schedules.

When transportation costs are quadratic (see, for example, D'Aspremont et al., 1979)[4] and firms set their basic varieties at both ends of the market, if consumers are not less efficient than firms in transportation we find an extreme personal arbitrage behavior: in a subgame perfect equilibrium, all consumers decide to demand the same variety at the center of the market, even though their preferences may differ. The demanded variety is such that

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[1] Rhee et al. (1992) study standardization in the presence of attributes that are unobservable by firms.

[2] This type of arbitrage is associated with the transferability of demand between different packages or bundles (for example, price-location packages) offered to consumers. See, for example, Tirole (1988).

[3] Other papers giving active roles to consumers are Fujita and Thisse (1986) and García et al. (1996). In these papers consumers decide their location in the market.

[4] Models of product differentiation involving a quadratic utility loss function include Novshek and Sonnenschein (1979), Eaton and Wooders (1985), Neven (1985), Economides (1986) and Friedman and Thisse (1993). See Anderson et al. (1992, Chapter 4), for a more general discussion of this assumption. On the other hand, many ideal-point models used in the literature of marketing assume that preferences are negatively related to the squared (weighted) distance between location and the individual's ideal point (see, for example, Green and Srinivasan, 1978).

competition between firms is intense and the equilibrium delivered price for that variety is the lowest.[5] To obtain this result we do not need any kind of explicit or implicit coordination on the part of consumers; each consumer finds it optimal to demand the same variety, possibly different from his/her most preferred variety of the product.[6] The reason is just that his/her favorite variety is more expensive, since competition on that particular variety is weaker than on the standard variety; and the price difference is higher than the incurred utility loss.

More generally, when the locations of the firms are not fixed (in terms of product differentiation, they choose where to set the basic variety), we show that consumers tend to agglomerate around the center of the market and, therefore, the demand distribution is more concentrated than the distribution of preferences.[7] The equilibrium firms locations are inefficient because they do not minimize the social transportation cost. From a social point of view, it would be optimal for consumers and firms to share transportation costs; in other words for consumers to practise *efficient arbitrage*, but arbitrage behavior depends on firms' pricing policies and in equilibrium its level will not be optimal. Furthermore, even if locations were fixed at the efficient levels, in equilibrium there would be a welfare loss due to inefficient arbitrage associated with equilibrium pricing policies.

In this paper the shape of transportation costs plays an important role. In a product differentiation framework, the concavity or convexity of transport costs can be related to the shape of the customizing-cost curve.[8] In this case transportation cost is usually viewed as a convex function of distance (Thisse and Vives, 1988). In the geographical context, it is usually assumed that, due to economies of scale in transportation, transportation costs are concave (see, for example, Hoover, 1937, and Thisse and Vives, 1988), but for the sake of computational simplicity the standard assumption in the literature of spatial competition is that transportation costs are linear. However, some factors may penalize long-distance freight; for instance, there may be congestion along the transport route (for example, transportation downtown).[9] Taking into account these features, many works in the

[5] Recent work has shown that the key for the principle of Minimum Differentiation to hold is the moderation of price competition. Competition may be relaxed by introducing product differentiation in some other dimension (see, for example, De Palma et al., 1985), by considering unobservable attributes in consumer brand choice (Rhee et al., 1992), by fixing market price exogenously, by assuming competition in quantities rather than prices (Anderson and Neven, 1991), by allowing price matching policies (Zhang, 1995) or by considering collusion on price (Friedman and Thisse, 1993). Our result would provide an explanation of the principle of Minimum Differentiation based on consumer behavior.

[6] The possibility of arbitrage is also present in Eaton and Schmitt (1994). In that paper consumers and firms have different transportation costs and consumers are free to buy the product at any point in the space; consumers do not buy their ideal product if the price reduction on a different variety is more than the utility loss from not getting the preferred product.

[7] We obtain similar results under linear-quadratic transport cost (see Gabszewicz and Thisse, 1986) or under a more general family of convex transportation costs given by $t(d)=td^{\alpha}$, where d denotes distance and $1 < \alpha \le 2$ (see Economides, 1986 and Anderson et al., 1992, Chapter 6). In Appendix A, we generalize our results by allowing any convex transportation cost function.

[8] See the interpretation of spatial price discrimination in terms of product differentiation in Greenhut et al. (1987), MacLeod et al. (1988), Thisse and Vives (1988) or Aguirre et al. (1998).

[9] See Greenhut et al. (1987, p. 276), for other justifications of convex transportation costs. On the other hand, in some models of product differentiation freight costs are paid in terms of the good as 'iceberg transport costs' (see, Samuelson, 1954, and Martinez-Giralt and Usategui, 1997): the inverse of the rate at which the good loses its value when distance increases is an index for the transport cost and this index is considered a convex function of distance.

literature of spatial price competition assume convex (in particular, quadratic) transportation costs.

Under asymmetric transportation costs for consumers and firms, personal arbitrage is an equilibrium phenomenon whenever consumers are more efficient than firms in short-distance transportation. Nevertheless, it is under convex transportation technologies that the possibility of arbitrage becomes most relevant, not only because equilibrium pricing policies involve arbitrage by consumers, but also because it is optimal, even from a social welfare point of view, for there to be arbitrage.

Even in a geographical interpretation, transport cost may be different for firms and consumers. Consumers may transport goods themselves as a non-market activity so that the cost (in terms of time, wear and tear, gasoline, etc.) coincides neither with the equilibrium price in the market for transport services nor with the transport cost for the firm (for instance, a worker's time cost includes taxes while consumers' time spent in non-market activities is not taxed; the transport of products by the firm may also require insurance against freight damage). With respect to this point, Lewis (1945) states:

> We must also note precisely what transport cost means in this context. MTC (marginal transport cost) is a measure of all the inconvenience associated with buying at a distant shop. It is not the same as what it would cost the shop in money to deliver one's purchases. A customer who shops in the centre of the town may be quite capable of carrying home a pair of shoes without much extra trouble or inconvenience, and if asked to pay an extra sum equal to what delivery would cost the shop, might prefer to carry for himself (pp. 208–209).

The consequences of arbitrage have been largely ignored in the literature on spatial price discrimination. Usually, optimal pricing policies satisfy a restriction to prevent arbitrage by consumers or arbitrage is simply assumed away.[10] However, as Machlup (1949), Scherer (1970, pp. 270–271) and Phlips (1983, p. 28) illustrate *personal arbitrage* may be a *real economic phenomenon*, for example, under the basing-point system. That arbitrage was a concern in the US steel industry until 1924 can be confirmed, for instance, in Machlup (1949) (The Basing-point System, pp. 139–142) which dedicates a few pages to the problem of 'diversion of shipments' in this industry and also in the cement industry. Here are some excerpts:[11]

> Even if sellers under the basing-point system refuse to quote prices f.o.b. place of shipment and insist on quoting delivered prices, buyers may get wise to the fact that

[10] An exception is a recent paper dealing with arbitrage and convex transportation costs by Barros and Martinez-Giralt (1996), who study the implications for the location of firms of convex transportation costs and arbitrage under FOB pricing; in their paper arbitrage is allowed only for firms and to a limited extent.

[11] In a pure location context, convex transportation costs may not be convincing in many circumstances. Under linear or concave transport cost, our model would account for these arbitrage phenomena as long as consumers were more efficient than firms on short distance transportation.

> producers are willing to absorb large amounts of freight on shipments to distant destinations. A smart buyer might try to buy in the guise of a distant customer and divert the shipment to a destination much closer to the producing mill.The buyer can divert shipments from one destination to another most easily if he picks up the products at the mill in his own truck. Let us assume that a steel consumer has two establishments, one close to and the other distant from a steel mill. Either the latter consuming point or both are governed by a basing-point away from that mill. Knowing that the mill will absorb all or a part of the freight to the distant consuming point, he might order the steel needed for both his establishments as if he wanted it all in the distant place.

This paper is organized as follows. Section 2 sets up the model. In Section 3, we solve the game to obtain the subgame perfect equilibria and present the main results. The equilibrium price schedule is such that the demanded varieties tend to agglomerate around the center of the market. We show that personal arbitrage is an equilibrium phenomenon whenever consumers have convex transportation costs or are more efficient than firms in short-distance transportation. Section 4 briefly analyzes welfare and policy implications. Section 5 offers concluding remarks.

2. The model

We consider a model with a continuum of consumers and two firms denoted by A and B. The two firms produce a homogeneous product but may have different locations on [0,1]. Buyers are uniformly distributed with a unit density on the interval [0, 1]. The location of a consumer is denoted by x and defined as the distance to the left point of the market. We will refer to x as the *type* of consumer located at x. Firms know the distribution of consumer types, but are unable to identify the type of an individual buyer: x is private information of each buyer. Consumer x may order delivery of the product to a location $\hat{x}$, possibly different from x. The reservation value for the good, R, is the same for all consumers and each one purchases precisely one unit of the product from the firm providing the lowest final (delivered) price including the transport cost incurred by the consumer. When the two firms have the same delivered price at a given location, the consumer chooses the supplier with the lower transportation cost.[12]

The location of firm A is denoted by a, the distance from the firm to the left point of the market, and the location of firm B is b, the distance from the firm to the right endpoint of the market. With no loss of generality we assume $a \leq 1 - b$. Marginal costs are constant and identical for both firms; for the sake of notational simplicity prices are expressed net of marginal cost. We assume that firms sell at a constant unit price at a given location, although they may spatially price discriminate: firms use delivered pricing policies.

[12] The assumption that price ties are broken in the socially efficient way is fairly standard in the literature. See, for example, Lederer and Hurter (1986) for a justification.

We consider asymmetric transportation costs for consumers and firms: $t_f(d)$ is the transportation cost for firms and $t_c(d)$ the transportation cost for consumers, where d is the Euclidean distance between two locations in the market.[13] Transportation costs are strictly increasing functions of distance, $t_f'(d) > 0$ and $t_c'(d) > 0$. Some of the main results of the paper are obtained by assuming quadratic transportation costs: $t_f(d) = t_f d^2$ and $t_c(d) = t_c d^2$.[14] We assume consumers have a reservation value high enough for one firm to find it profitable to serve the whole market: $R > t_f(1)$.

The timing of the game is as follows. At stage 1, firms choose their locations in the market simultaneously and independently. At stage 2, the two firms decide on the price level for each possible variety and consumers decide which variety to demand, simultaneously and independently.[15] Denote by $\hat{x}$ the variety demanded by consumer located at x. Then, if consumer x finally buys the good, the supplier will deliver it at location $\hat{x}$. Finally, firms' price schedules and demanded varieties are observed and consumers buy the product from the supplier offering the lowest price at the requested location.

Since firms and consumers decide price schedule and product variety simultaneously, neither has any ability to precommit. This description fits well in markets where goods are produced to order and the consumer has to specify the design of the product. This is the case for instance of custom-made (clothing, etc.) or custom-built (houses, etc.) products; other examples include the acquisition of software, furniture and so on.[16] Our model predicts that in this context, if transportation costs are convex buyers will not demand their preferred product design, but will rather tend to ask for more standard varieties.

3. Equilibrium analysis

We now solve the sequential game by backward induction to obtain the subgame perfect equilibria.

3.1. Second stage: optimal pricing policies and consumers' behavior

3.1.1. Equilibrium pricing policies

We obtain the optimal price schedule (the best response) for any set of requested varieties or announced locations. Let $\hat{X} = \{\hat{x} \in [0,1]\}$ be a set of announced locations. Let $p_A(\hat{x})$ and $p_B(\hat{x})$ be the delivered prices at location $\hat{x} \in \hat{X}$. The delivered price at $\hat{x}$ must

[13] See Gronberg and Meyer (1981) for a model with different transportation cost for consumers and firms.

[14] In Appendix A we allow more general transportation technologies for firms and consumers and we show that our main results do not depend on this assumption.

[15] We could allow consumers to announce different types to the two firms, but consumers have nothing to gain by doing this, so we ignore the possibility.

[16] This description of market behavior fits also many industrial procurement procedures. In fact, the model can be interpreted as a model for a non-final (intermediate) product, so that consumers are firms and the product is used as input in the production of a final good.

cover the transportation cost.[17] Define x_{AB} as the location such that $t_f(|a-x_{AB}|)=t_f(|(1-b)-x_{AB}|)$, i.e., the midpoint location between a and b:

$$x_{AB} = \frac{1-b+a}{2} \tag{1}$$

At a given location $\hat{x}\in\hat{X}$, competition is à la Bertrand: with cost asymmetries if $\hat{x}\neq x_{AB}$ and with the same cost if $\hat{x}=x_{AB}$. When $\hat{x}<x_{AB}$, firm A's transportation cost is lower than firm B's. The opposite is true when $\hat{x}>x_{AB}$. This implies that in equilibrium the delivered price at $\hat{x}$ will equal the transportation cost of the firm located further from $\hat{x}$.

Given the previous argument, when firms A and B are located at a and b, respectively, the equilibrium pricing policies are given by:[18]

$$p_A(\hat{x}) = p_B(\hat{x}) = \max\{t_f(|a-\hat{x}|), t_f(|(1-b)-\hat{x}|)\} \quad \text{for all} \;\; \hat{x}\in\hat{X} \tag{2}$$

When $\hat{X}$ does not coincide with [0,1], there are varieties which are not demanded by any consumer. At those locations any price quote is a best response since no sales will be made at that price. However, we impose the restriction that firms choose for each location a price that would be optimal if a consumer decided to demand that variety. This restriction selects a unique equilibrium for the game. Note that if firms could perfectly discriminate among consumers' locations, (2) would be also the equilibrium price schedule with $\hat{x}=x$. Denote by $\hat{X}_f$ ($f=A$, B) the set of announced locations for which either firm f ($f=A$, B) quotes the lowest delivered price or, if both firms quote the same price, firm f is not the high transportation cost firm. Note that $\hat{X}_f$ is a subset of $\hat{X}$. Denote by $\hat{F}$ the cumulative distribution function of announced locations. Profits for the two firms are:

$$\int_{\hat{x}\in\hat{X}_A} \{t_f(|(1-b)-\hat{x}|) - t_f(|a-\hat{x}|)\}\mathrm{d}\hat{F}(\hat{x}) \tag{3}$$

$$\int_{\hat{x}\in\hat{X}_B} \{t_f(|a-\hat{x}|) - t_f(|(1-b)-\hat{x}|)\}\mathrm{d}\hat{F}(\hat{x}) \tag{4}$$

3.1.2. Consumers' optimal behavior

The surplus of consumer x when he/she announces $\hat{x}$ and buys from firm f ($f=A$, B) is given by:

$$S_x(\hat{x}) = R - p_f(\hat{x}) - t_c(|\hat{x}-x|) \tag{5}$$

[17] We assume that firms do not price below cost. If a firm were to price below transport cost at a given location, it could do at least as well by pricing at transport cost for any given price of the other firm. This is a usual assumption in the literature of spatial price discrimination. See, for example, Lederer and Hurter (1986), Thisse and Vives (1988), De Fraja and Norman (1993) and Aguirre and Martín (2001).

[18] See Lederer and Hurter (1986) for a formal proof.

Given firms' locations and pricing policies, consumer x chooses $\hat{x}$ so as to maximize his/her surplus. To maximize surplus, the requested variety $\hat{x}$ (by consumer x) minimizes the sum of delivered price at the announced location plus the transportation cost to the true location. Consumer x requests his/her preferred variety, $\hat{x}=x$, when:

$$p_f(x)\leq p_f(\hat{x})+t_c(|\hat{x}-x|),\quad \forall\hat{x}\neq x \tag{6}$$

i.e., given firms' pricing policies in equilibrium, (2), when:

$$\max\{t_f(|a-x|),t_f(|(1-b)-x|)\}-\max\{t_f(|a-\hat{x}|),t_f(|(1-b)-\hat{x}|)\}\leq t_c(|\hat{x}-x|) \tag{7}$$

Suppose that consumer $x\in[0,\ x_{AB}]$, is considering whether to be served at location x or $\hat{x}$; consumer x does not have any incentive to demand a less preferred variety:

$$t_f(|(1-b)-x|)-t_f(|(1-b)-\hat{x}|)\leq t_c(|\hat{x}-x|)\quad\text{for any }\hat{x}\in\hat{X}_A \tag{8}$$

and

$$t_f(|(1-b)-x|)-t_f(|a-\hat{x}|)\leq t_c(|\hat{x}-x|)\quad\text{for any }\hat{x}\in\hat{X}_B \tag{9}$$

When transportation costs are convex, consumer x is better off buying the good at some location other than his/her own. Fig. 1 shows the incentives to arbitrage under convex transportation costs. In this case, there are locations at which the difference in expected delivered prices makes up for the utility loss from not consuming the most preferred variety of the product. Under convex transportation costs, given firms' locations and anticipated pricing policies, consumer x demands the product at a location $\hat{x}^*$ such that his surplus, (5), is maximized:

$$\hat{x}^*=\operatorname{argmin}_{\hat{x}}[\max\{t_f(|a-\hat{x}|),t_f(|(1-b)-\hat{x}|)\}+t_c(|\hat{x}-x|)] \tag{10}$$

where x is the true location. Notice that if $x\in[0,\ x_{AB}]$ then $\hat{x}\in[0,x_{AB}]$, since $x_{AB}=\operatorname{argmin}[\max\{t_f(|a-\hat{x}|),\ t_f(|(1-b)-\hat{x}|)\}]$. Thus, the problem for a consumer $x\in[0,\ x_{AB}]$ is:

$$\min_{\hat{x}}t_f(|(1-b)-\hat{x}|)+t_c(|\hat{x}-x|)$$

From the first-order condition:[19]

$$t_f{}'(|(1-b)-\hat{x}|)=t_c{}'(|\hat{x}-x|) \tag{11}$$

Under quadratic transportation costs, condition (11) implies that $t_f\ [(1-b)-\hat{x}]=t_c(\hat{x}-x)$ and therefore:

$$\hat{x}^*=\min\left\{\frac{(1-b)t_f+t_cx}{t_f+t_c},x_{AB}\right\}\quad\text{for }x\in[0,x_{AB}]$$

[19] Second-order conditions are satisfied since the transportation cost functions are convex.

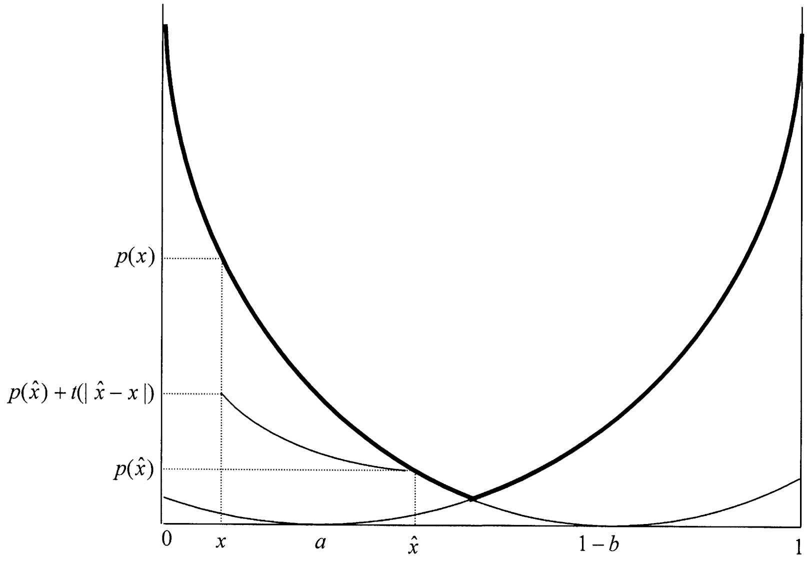

Fig. 1. Personal arbitrage and the tendency for consumers to demand at locations closer to the center of the market.

Hence, consumers located at $x \in \left[0, \frac{a(t_f + t_c) + (1-b)(t_c - t_f)}{2t_c}\right)$ buy the product at $\frac{(1-b)t_f + t_c x}{t_f + t_c}$, while consumers located at $x \in \left[\frac{a(t_f + t_c) + (1-b)(t_c - t_f)}{2t_c}, x_{AB}\right]$ buy at x_{AB}:

$$\hat{x}^* = \left\{ \begin{array}{ll} \frac{(1-b)t_f + t_c x}{t_f + t_c} & \text{for } x \in \left[0, \frac{a(t_f + t_c) + (1-b)(t_c - t_f)}{2t_c}\right) \\ x_{AB} & \text{for } x \in \left[\frac{a(t_f + t_c) + (1-b)(t_c - t_f)}{2t_c}, x_{AB}\right] \end{array} \right\} \tag{12}$$

Next, consider consumers located at $x \in [x_{AB}, 1]$. Using a similar reasoning it is easy to check that:

$$\hat{x}^* = \max\left\{\frac{vat_f + t_c x}{t_f + t_c}, x_{AB}\right\} \quad \text{for } x \in [x_{AB}, 1]$$

Therefore, consumers located at $x \in \left[x_{AB}, \frac{(1-b)(t_f + t_c) + a(t_c - t_f)}{2t_c}\right)$ buy at x_{AB}, and consumers located at $x \in \left[\frac{(1-b)(t_f + t_c) + a(t_c - t_f)}{2t_c}, 1\right]$ buy the product at $\frac{at_f + t_c x}{t_f + t_c}$:

$$\hat{x}^* = \left\{ \begin{array}{ll} x_{AB} & \text{for } x \in \left[x_{AB}, \dfrac{(1-b)(t_f+t_c)+a(t_c-t_f)}{2t_c} \right) \\ \dfrac{at_f+t_c x}{t_f+t_c} & \text{for } x \in \left[\dfrac{(1-b)(t_f+t_c)+a(t_c-t_f)}{2t_c}, 1 \right] \end{array} \right\} \tag{13}$$

We refer to this result as personal arbitrage because all consumers demand the product at a location different from the real one and incur transportation costs, even though the system used is delivered pricing. Note that personal arbitrage would be absent in equilibrium only when $t_c \to \infty$.

The following proposition summarizes the above results:

Proposition 1. *Given firms' locations, a and b*:

(i) *In equilibrium firms price according to*:

$$p_A(\hat{x}) = p_B(\hat{x}) = \max\{t_f(|a-\hat{x}|), t_f(|(1-b)-\hat{x}|)\} \quad \text{for all } \hat{x} \in \hat{X}$$

(ii) *Under quadratic transportation costs, the distribution of consumers' locations and the distribution of demanded varieties differ. The cumulative distribution function of consumers' locations is* $F(x)=x$, *for* $x \in [0,1]$, *while the cumulative distribution function of demanded varieties is*

$$\hat{F}^*(\hat{x}^*) = \left\{ \begin{array}{ll} \dfrac{t_f+t_c}{t_c}\hat{x}^* - (1-b)\dfrac{t_f}{t_c} & \text{if} (1-b)\dfrac{t_f}{t_f t_c} \le \hat{x}^* < x_{AB} \\ \dfrac{t_f+t_c}{t_c}\hat{x}^* - a\dfrac{t_f}{t_c} & \text{if } x_{AB} \le \hat{x}^* \le \dfrac{t_c+at_f}{t_f+t_c} \end{array} \right\} \tag{14}$$

We have shown that under quadratic transportation costs there is a difference between the distribution of consumer locations and the distribution of demanded varieties. Consumers do not demand their preferred product variety: they practice personal arbitrage and, as a consequence, there is concentration of demand on varieties located around the central place between the two firms. Consumers' preferences are uniformly distributed with unit density along [0,1]. However, from (12) and (13), the distribution of demanded varieties $\hat{x}^*$ has two parts: they are uniformly distributed with density $(t_f+t_c)/t_c$ on the interval $\left[(1-b)\dfrac{t_f}{t_f+t_c}, \dfrac{t_c+at_f}{t_f+t_c}\right]$, and there is a mass point at x_{AB} with weight $(1-b-a)\dfrac{t_f}{t_c}$. The smaller a, b and t_c the higher the concentration of demand around the center of the market. In the extreme case, when $t_c \le t_f$ and firms are located at the endpoints of the market, $a=b=0$, all consumers demand the same variety $\hat{x}=x_{AB}$ for all x, for which the delivered price is lowest. In this case, all consumers buy the product at x_{AB} and the result of minimal product diversity (from the point of view of consumer demand) holds: only one variety is sold in the market (see Fig. 2).

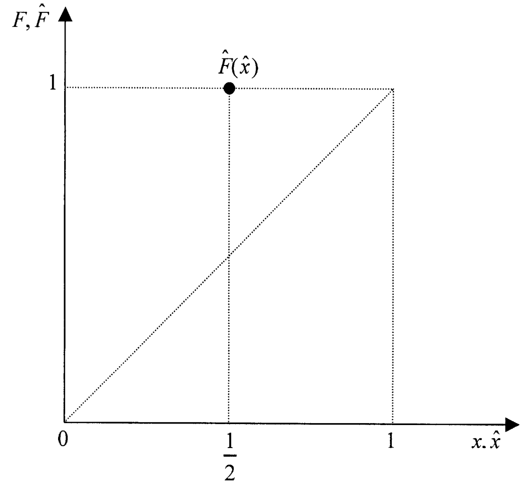

Fig. 2. Distribution function for demanded varieties, $\hat{F}(\hat{x})$. Minimal product diversity with locations fixed at a=b=0, and consumers more efficient than firms in transportation.

Personal arbitrage arises under quadratic transportation costs, and it causes partial agglomeration of consumers demand on the frontier between the two firms' market areas. Since market areas are determined by the locations chosen by firms, we need to solve the first stage of the game to take firms' incentives into consideration.

3.2. First stage: location decisions

In the first stage firms decide their locations by taking into account the effect on pricing policies, transportation costs and the behavior of consumers. The results are presented as Proposition 2.

Proposition 2. *The equilibrium firms' locations are*

$$a^* = b^* = \frac{t_c + 3t_f}{6t_f + 4t_c}.$$

Proof. Note that firms obtain zero profit from sales to consumers buying at x_{AB}. At such location, transportation costs are equal for both firms and, hence, the equilibrium price is equal to the common marginal cost. Therefore, at x_{AB} profits are zero regardless of any sharing rule we may consider. Firm A's profit may be written as:

$$\Pi_A(a,b) = \int_{\frac{(1-b)t_f}{t_f+t_c}}^{x_{AB}} \{t_f(|(1-b) - \hat{x}^*|) - t_f(|a - \hat{x}^*|)\} \mathrm{d}\hat{F}^*(\hat{x}^*) \tag{15}$$

where $\hat{x}^*$, $\hat{F}^*(\hat{x}^*)$ and x_{AB} are given by (12), (14) and (1), respectively. We can use (12) to write firm *A*'s profit in terms of the true consumer location:

$$\Pi_A(a,\ b) = \int_0^{\frac{a(t_f+t_c)+(1-b)(t_c-t_f)}{2t_c}} \left\{ t_f\left(\frac{t_c[(1-b)-x]}{t_f+t_c}\right) - t_f\left(a - \frac{(1-b)t_f + t_c x}{t_f+t_c}\right)^2 \right\} \mathrm{d}F(x) \tag{16}$$

Using Liebnitz's Rule, the first-order condition of firm *A*'s maximization problem is:

$$\frac{\partial \Pi_A(a,b)}{\partial a} = -\int_0^{\frac{a(t_f+t_c)+(1-b)(t_c-t_f)}{2t_c}} 2t_f\left(a - \frac{(1-b)t_f + t_c x}{t_f+t_c}\right) \mathrm{d}x = 0 \tag{17}$$

. From (17) we obtain firm *A*'s reaction function,

$$a^*(b) = \frac{(1-b)(t_c+3t_f)}{3(t_f+t_c)}.$$

Using a similar reasoning, from the first-order condition of firm *B*'s maximization problem, we get

$$b^*(a) = \frac{(1-a)(t_c+3t_f)}{3(t_f+t_c)}.$$

Thus, the equilibrium locations are given by

$$a^* = b^* = \frac{t_c + 3t_f}{6t_f + 4t_c}. \quad \square$$

Note that when arbitrage is taken into consideration, in equilibrium firms locate closer to each other than in the standard case of spatial price discrimination where firms locate at the quartiles (see, for example, Lederer and Hurter, 1986). When $t_f = t_c$, then $a^* = b^* = \frac{2}{5}$; firms locate closer to the center if $t_f > t_c$: $a^* = b^* > \frac{2}{5}$, and further from the center if $t_f < t_c$: $a^* = b^* < \frac{2}{5}$. This is an unexpected result and shows the importance that consumers' arbitrage behavior (usually ignored) may have under convex transportation technologies. Not only do firms locate closer to the center with convex costs, but consumers also concentrate their demands around the center of the market. Fig. 3 shows the equilibrium demand distribution function.

4. Welfare and policy implications

In this section we analyze the welfare effects of personal arbitrage. We will see that this behavior creates inefficiencies in terms of excessive transportation costs. We evaluate welfare effects from two points of view: in Setion 4.1, we analyze efficient arbitrage by consumers and firms' optimal locations from a first best perspective; in Section 4.2, we study the second best optimal locations for firms.

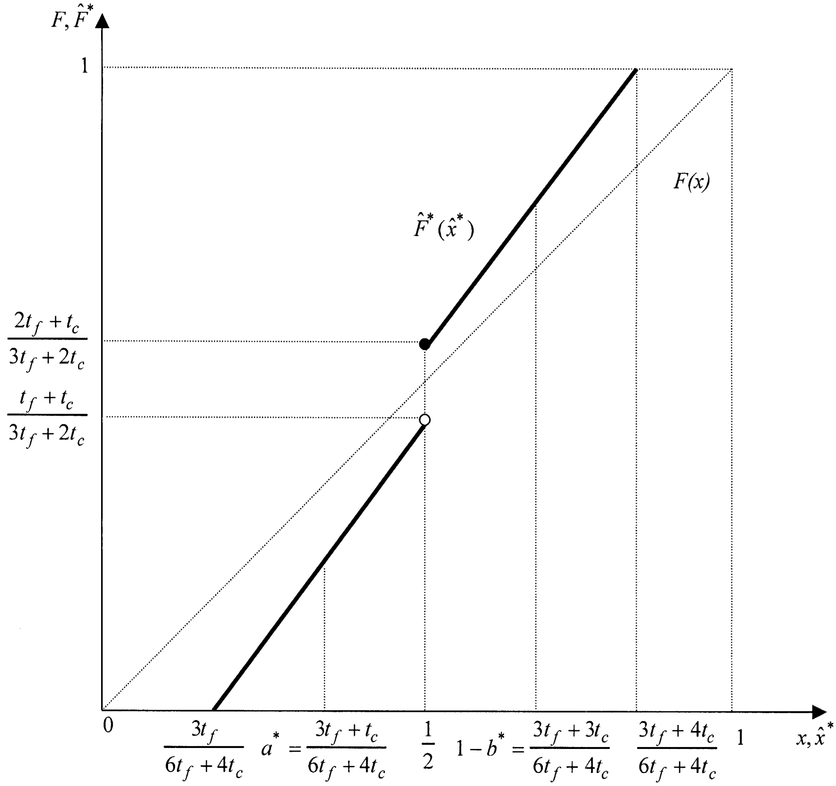

Fig. 3. Equilibrium demand distribution function, $F*(\hat{x}*)$. Note that $[vt_f/(3t_f + 2t_c)\times 100]$ per cent of consumers buy the product at the central place.

4.1. Efficient arbitrage and first best locations

First, we show that the first best locations under convex transportation costs are the same as with concave or linear transportation technologies, and this is so even when we consider the efficient level of personal arbitrage.

Denote by $\hat{x}^e$ the efficient location, defined as the location where consumer x should be served in order to minimize the total transportation cost. For $x\in[0,x_{AB}]$, $\hat{x}^e$ is the solution to:

$$\min_{\hat{x}}\{t_f(|a-\hat{x}|)+t_c(|\hat{x}-x|)\}$$

Note that if $x\in[0,x_{AB}]$ then $\hat{x}^e\in[0,x_{AB}]$ since $t_f(|a-x_{AB}|)=t_f(|(1-b)-x_{AB}|)$. From the first-order condition:

$$t_f'(|a-\hat{x}^e|)=t_c'(|\hat{x}^e-x|) \tag{18}$$

Hence, under quadratic transportation costs, it must be the case that $t_f(a-\hat{x}^e)=t_c(\hat{x}^e-x)$ and then:

$$\hat{x}^e=\frac{at_f+t_cx}{t_f+t_c}\quad \text{for } x\in[0,x_{AB}] \tag{19}$$

Consumers located at $x\in[0,x_{AB}]$ should buy the good at $\frac{at_f+t_cx}{t_f+t_c}$. Notice that, to minimize total freight costs, consumers located at 0 should purchase the product at $\frac{at_f}{t_f+t_c}$, consumers at a should buy at a, and consumers at x_{AB} should buy the product at $\frac{a(2t_f+t_c)+(1-b)t_c}{2(t_f+t_c)}$.

Next, consider consumers located at $x\in[x_{AB},1]$. Using a similar reasoning it is easy to check that:

$$\hat{x}^e=\frac{(1-b)t_f+t_cx}{t_f+t_c}\quad \text{for } x\in[x_{AB},1] \tag{20}$$

Therefore, the efficient demand distribution function is:

$$\hat{F}^e(\hat{x}^e)=\left\{\begin{array}{ll}\frac{t_f+t_c}{t_c}\hat{x}^e-a\frac{t_f}{t_c} & \text{if } \frac{at_f}{t_f+t_c}\le\hat{x}^e\le\frac{a(2t_f+t_c)+(1-b)t_c}{2(t_f+t_c)}\\ \frac{t_f+t_c}{t_c}\hat{x}^e-(1-b)\frac{t_f}{t_c} & \text{if } \frac{(1-b)(2t_f+t_c)+at_c}{2(t_f+t_c)}\le\hat{x}^e\le\frac{t_c+(1-b)t_f}{t_f+t_c}\end{array}\right\} \tag{21}$$

The next proposition states the first best locations for firms given efficient personal arbitrage by consumers.

Proposition 3. *First best locations are $a^e=b^e=\frac{1}{4}$.*

Proof. See Appendix B.1.

With convex transportation costs, in the social welfare optimum consumers would not get their favorite product but something between that and the variety that the firm is best fitted to produce. The efficient locations are $a^e=b^e=\frac{1}{4}$. Note that equilibrium locations are efficient only in the limit: $\lim a^*_{t_c\to\infty}=\frac{1}{4}$ and $\lim a^*_{t_f\to 0}=\frac{1}{4}$.

Fig. 4 shows the efficient demand distribution function, $F^e(x^e)$. Note that efficient arbitrage requires that consumers at locations $x\in[0,\frac{1}{2}]$ order delivery uniformly (with density $(t_f+t_c)/t_c$) around firm A's location and consumers at locations $x\in[\frac{1}{2},1]$ order delivery uniformly (with density $(t_f+t_c)/t_c$) around firm B's location. Comparing Figs. 3 and 4 we can see that in equilibrium the demand distribution is too concentrated compared to the efficient distribution. In other words, even though there is some arbitrage in the first best, in equilibrium there is inefficient arbitrage. Efficiency requires consumers and firms to share transportation costs optimally. From Propositions 2 and 3, in equilibrium firms set their basic variety inefficiently close to each other. This is not, however, the only source of inefficiency, as the following Corollary points out.

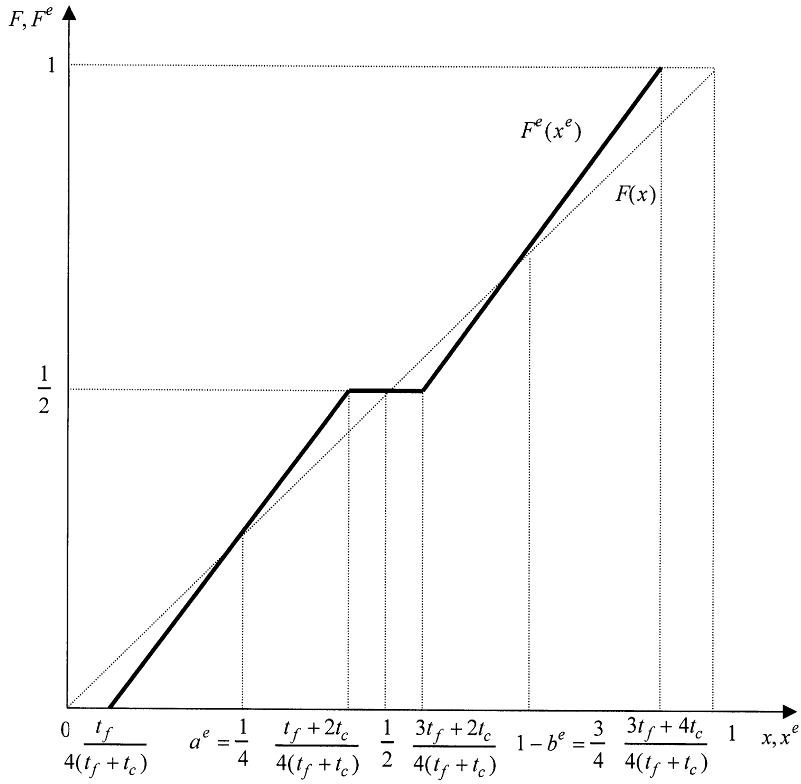

Fig. 4. Efficient demand distribution function. $F(x)$: distribution function of preferences. $F^e(x^e)$: first-best demand distribution function.

Corollary 1. *Under convex transportation costs even if firms had fixed locations at the first best,* $a^e = b^e = \frac{1}{4}$, *in equilibrium there would be a welfare loss due to inefficient arbitrage.*

From a product differentiation point of view it is interesting to compare equilibrium product diversity with the efficient level. If we consider the length of the support of the cumulative distribution function as a measure of product diversity (for instance, if the demand distribution function had a support [0,1] then 100% of product varieties would be demanded), we have

Corollary 2. *Equilibrium product diversity is lower than the efficient level.*

From a social welfare point of view, the efficient level of product diversity depends on the comparison between transportation costs for consumers and firms. In particular, the more efficient consumers are in transportation the lower the efficient level of product diversity is. At the limit, when $t_c \rightarrow 0$ efficiency requires only two varieties, *a* and *b*. Only when $t_c \rightarrow \infty$ would maximal product diversity be optimal. However, the market outcome implies too low product diversity. In equilibrium, a $[4t_c/(6t_f + 4t_c) \cdot 100]$ per cent of the potential product varieties will be served. However, efficiency requires higher product diversity, namely

$$\left[\frac{4t_c}{4(t_f+t_c)}\cdot 100\right]\%.$$

We have shown that the welfare effects are unambiguously negative. First, firms locate inefficiently close to each other and, secondly, even if locations are fixed at the efficient levels equilibrium pricing policies give rise to inefficient arbitrage and transportation costs are not minimized. The welfare loss associated with inefficient arbitrage is due to excessive transportation costs. In terms of product differentiation, firms that could produce specialized products sell standard varieties to consumers who would like more specialized products. From a social point of view, it is optimal for consumers and firms to share transportation costs. In other words, it is necessary for consumers to practice efficient arbitrage, but arbitrage behavior depends on firms' pricing policies and it will not be optimal. To induce consumers towards efficient arbitrage each firm should price at its own transportation cost: $p(x)=\min\{t_f\,(|a-x|),\ t_f(|(1-b)-x|)\}$.

4.2. Second best locations

We obtain second best locations for firms by minimizing the total transportation cost given equilibrium pricing policies and condition (14), the equilibrium distribution function of consumer demand.

Proposition 4. *Second best locations are*

$$\tilde{a}=\tilde{b}=\frac{5(t_f)^2-(t_c)^2-4t_ft_c+t_c\sqrt{(t_c)^2+11(t_f)^2+8t_ft_c}}{10(t_f)^2}$$

Proof. See Appendix B.2

It is easy to check that $\tilde{a}=\tilde{b}>a^*=b^*$. Only at the limit, equilibrium locations are equal to the second-best locations: $\lim a^*_{t_c\to 0}=\frac{1}{2}=\lim \tilde{a}_{t_c\to 0}$ and $\lim a^*_{t_f\to\infty}=\frac{1}{2}=\lim \tilde{a}_{t_f\to\infty}$. Fig. 5 shows the second best demand distribution function (the dotted line), $\tilde{F}(\tilde{x})$ (that is, the equilibrium demand distribution function, condition (14), evaluated at the second best locations). Comparing $\tilde{F}(\tilde{x})$ and $F^*(\hat{x}^*)$ (the distribution function (14) evaluated at the equilibrium locations) we can see that in equilibrium the demand distribution is too concentrated compared to the (second best) efficient distribution. For example, when $t_f=t_c$ (then $a^*=b^*=\frac{2}{5}<\frac{\sqrt{5}}{5}=\tilde{a}=\tilde{b}$) in equilibrium 20% of consumers buy the product at the center of the market while in the second best outcome this percentage drops to 10.5%.

5. Concluding remarks

We have shown that under convex transportation costs, consumers may find it in their interest to demand not their most preferred varieties but more standard varieties. As a consequence of this personal arbitrage behavior, product diversity decreases, giving rise to a demand distribution that is not only more concentrated than the distribution of preferences but also more concentrated than the efficient distribution.

Our results could be generalized in several directions. The assumption of inelastic demand could be relaxed without altering the result of the tendency to agglomeration under convex transportation costs. The timing of the game could be changed. Timing reflects commitment and it is difficult to judge who has more commitment power, the firm setting prices or the consumers announcing location, so that in our model those decisions are simultaneous. If consumers took their decision before firms, the outcome of the game would be the same, but if we allowed firms to move first and make price offers conditioned on announced location, formally this would amount to a revelation game where consumers declare their type. If firms could commit themselves to the price schedule, then in general the optimal policy is not to price at the rival's marginal cost at each location in the local market, since the firm could do better by inducing consumers

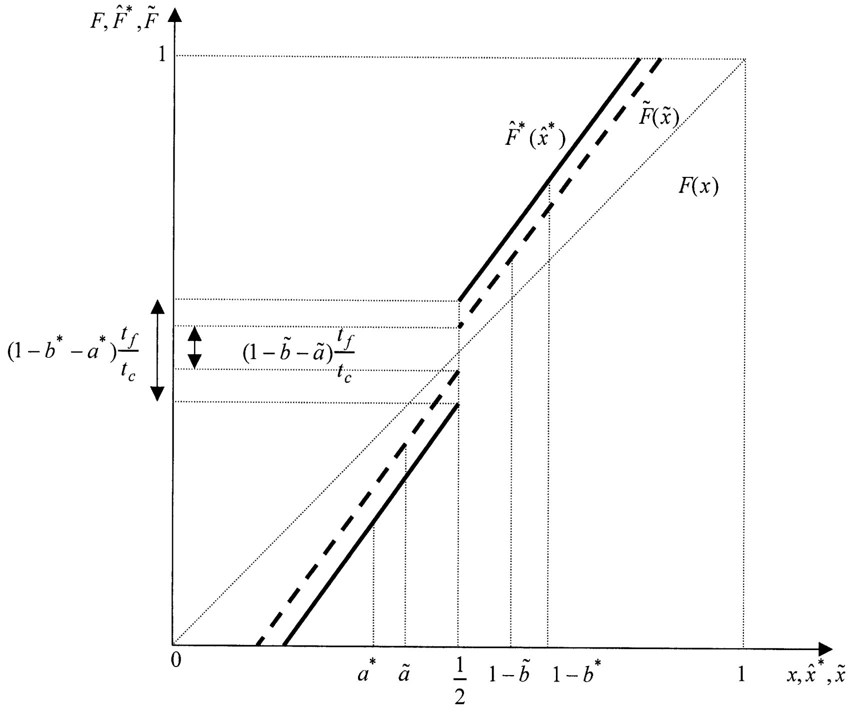

Fig. 5. Second-best demand distribution function. $F(x)$: distribution function of preferences. $F^*(\hat{x}^*)$: equilibrium demand distribution function. $\tilde{F}(\tilde{x})$: second-best demand distribution function.

to demand varieties closer to the base variety. Thus, the game would be one of preference revelation with competition between firms. Unfortunately, a location-price equilibrium for that duopoly model is not easy to characterize and the question is left for further research.

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Appendix A. Convex and nonconvex transportation technologies

In this appendix we show that our results do not depend on the assumption of quadratic transportation costs for consumers and firms, and some of them not even on convex transportation costs. The first observation is that personal arbitrage is an equilibrium phenomenon whenever consumers' transportation costs are such that $t_c'(0)=0$, and firms' transportation technology such that $t_f'(d)>0$ for $d>0$ and $t_f'(0)\geq 0$ for $d=0$. To see why this is so, consider the optimization problem for a consumer at $x\in[0,x_{AB}]$:

$$\min_{\hat{x}} p(\hat{x}) + t_c(|\hat{x}-x|)$$

In equilibrium $p(\hat{x}) = t_f(|(1-b)-\hat{x}|)$ for $\hat{x}\in[0,x_{AB}]$. The first-order condition is:

$$t_f'(|(1-b)-\hat{x}^*|) - t_c'(|\hat{x}^*-x|) = 0 \tag{A.1}$$

Assume that t_f and t_c are such that the objective function is convex and (A.1) characterizes the solution for the consumer. If $\hat{x}^*=x$ then $t_c'(|\hat{x}^*-x|)=0$ since by assumption $t_c'(0)=0$ which implies $t_f'(|(1-b)-\hat{x}^*|)=0$ but that is not possible when $t_f'(d)>0$ for any $d>0$.

We conclude this appendix by showing that personal arbitrage may be an equilibrium phenomenon with nonconvex transportation technologies. Consider asymmetric transportation costs, linear in distance, for consumers and firms: $t_f d$ is the transportation cost for firms and the transportation cost for consumers is given by

$$t_c(d) = \left\{ \begin{array}{ll} t_c d & \text{for } d \leq \tilde{d} \\ \\ \infty & \text{for } d > \tilde{d} \end{array} \right\}.$$

Obviously, if consumers are more efficient in transportation than firms, $t_c < t_f$ and $\tilde{d}=1$, all consumers will engage in an extreme arbitrage behavior by purchasing at the center of the market. However, for personal arbitrage to arise in equilibrium it is sufficient that consumers be slightly more efficient than firms in very short-distance travels (i.e., $t_c < t_f$ and any $\tilde{d} > 0$ no matter how small).[20]

Appendix B

B.1. First best locations

Given the efficient demand distribution function, the total transportation cost is:

$$\begin{aligned} TC(a,b) = & \int_{\frac{at_f}{t_f+t_c}}^{\frac{a(2t_f+t_c)+(1-b)t_c}{2(t_f+t_c)}} \{t_f(a-\hat{x}^e)^2 + t_c(\hat{x}^e - x)^2\}\mathrm{d}\hat{F}^e(\hat{x}^e) \\ & + \int_{\frac{(1-b)(2t_f+t_c)+at_c}{2(t_f+t_c)}}^{\frac{t_c+(1-b)t_f}{t_f+t_c}} \{t_f(1-b-\hat{x}^e)^2 + t_c(\hat{x}^e - x)^2\}\mathrm{d}\hat{F}^e(\hat{x}^e) \end{aligned} \tag{A.2}$$

From (19) and (20), we can express the total transportation cost in terms of true consumer locations:

$$\begin{aligned} TC(a,b) = & \int_0^{\frac{(1-b)+a}{2}} \left\{ t_f \left[(a-x)\frac{t_c}{t_c+t_f} \right]^2 + t_c \left[(a-x)\frac{t_f}{t_c+t_f} \right]^2 \right\} \mathrm{d}F(x) \\ & + \int_{\frac{(1-b)+a}{2}}^{1} \left\{ t_f \left[(1-b-x)\frac{t_c}{t_c+t_f} \right]^2 + t_c \left[(1-b-x)\frac{t_f}{t_c+t_f} \right]^2 \right\} \mathrm{d}F(x) \end{aligned} \tag{A.3}$$

It is easy to check, from the first-order conditions of the transportation cost minimization problem, that: $a^e(b) = \frac{1}{3}(1-b)$ and $b^e(a) = \frac{1}{3}(1-a)$. Therefore, the first best locations are $a^e = b^e = \frac{1}{4}$. □

B.2. Second-best locations

The second best optimal locations for firms will be obtained by minimizing the total transportation cost given condition (14), the (equilibrium) cumulative distribution function of announced location. In equilibrium, consumer x demands the variety $\hat{x}*$(see conditions (12) and (13)) given by

$$\hat{x}* = \left\{ \begin{array}{ll} \dfrac{(1-b)t_f + t_c x}{t_f + t_c} & \text{for } x \in \left[0, \dfrac{a(t_f+t_c) + (1-b)(t_c - t_f)}{2t_c}\right) \\ x_{AB} & \text{for } x \in \left[\dfrac{a(t_f+t_c)+(1-b)(t_c-t_f)}{2t_c}, \dfrac{(1-b)(t_f+t_c)+a(t_c-t_f)}{2t_c}\right] \\ \dfrac{at_f + t_c x}{t_f + t_c} & \text{for } x \in \left(\dfrac{(1-b)(t_f+t_c)+a(t_c-t_f)}{2t_c}, 1\right] \end{array} \right\}$$

[20] Similar results are obtained if firms have concave transportation costs and consumers are more efficient in short hauls.

Given equilibrium pricing policies and the associated inefficient arbitrage by consumers, the total transportation cost can be written as

$$\begin{aligned}\tilde{T}\tilde{C}(a,b) = {} & \int_0^{\frac{a(t_f+t_c)+(1-b)(t_c-t_f)}{2t_c}} \left\{ t_f\left(a - \frac{(1-b)t_f + t_c x]}{(t_f+t_c)}\right)^2 + t_c\left(\frac{(1-b)t_f - t_f x}{(t_f+t_c)}\right)^2 \right\}\mathrm{d}x \\ & + \int_{\frac{a(t_f+t_c)+(1-b)(t_c-t_f)}{2t_c}}^{\frac{(1-b)(t_f+t_c)+a(t_c-t_f)}{2t_c}} \left\{ t_f\left(\frac{1-b-a}{2}\right)^2 + t_c\left(\frac{1-b+a}{2} - x\right)^2 \right\}\mathrm{d}x \\ & + \int_{\frac{(1-b)(t_f+t_c)+a(t_c-t_f)}{2t_c}}^{1} \left\{ t_f\left(1 - b - \frac{a t_f + t_c x]}{(t_f+t_c)}\right) + t_c\left(\frac{a t_f - t_f x}{(t_f+t_c)}\right)^2 \right\}\mathrm{d}x\end{aligned}$$

From the first-order conditions of the transportation cost minimization problem we obtain that the second-best locations are

$$\tilde{a} = \tilde{b} = \frac{5(t_f)^2 - (t_c)^2 - 4t_f t_c + t_c\sqrt{(t_c)^2 + 11(t_f)^2 + 8t_f t_c}}{10(t_f)^2}$$

References

Aguirre, I., Martín, A.M., 2001. On the strategic choice of spatial price policy: the role of the pricing game rules. Economics Bulletin 12 (2), 1–7.

Aguirre, I., Espinosa, M.P., Macho, I., 1998. Strategic entry deterrence through spatial price discrimination. Regional Science and Urban Economics 28, 297–314.

Anderson, S.P., Neven, D., 1991. Cournot competition yields spatial agglomeration. International Economic Review 32, 793–808.

Anderson, S.P., De Palma, A., Thisse, J.-F., 1992. In: Discrete Choice Theory of Product Differentiation. MIT Press, Cambridge, MA.

Barros, P.P., Martinez-Giralt, X., 1996. The impact of the 'Chunnel' on the local production activities. Mimeo, Universidad Autónoma de Barcelona.

D'Aspremont, C., Gabszewicz, J.J., Thisse, J.-F., 1979. On Hotelling's stability in competition. Econometrica 47, 1145–1150.

De Fraja, G., Norman, G., 1993. Product differentiation, pricing policy and equilibrium. Journal of Regional Science 33, 343–363.

De Palma, A., Ginsburgh, V., Papageorgiou, Y.Y., Thisse, J.-F., 1985. The principle of minimum differentiation holds under sufficient heterogeneity. Econometrica 53, 767–781.

Eaton, C., Schmitt, N., 1994. Flexible manufacturing and market structure. American Economic Review 84, 875–888.

Eaton, C., Wooders, M., 1985. Sophisticated entry in a model of spatial competition. Rand Journal of Economics 16, 282–297.

Economides, N., 1986. Minimal and maximal product differentiation in Hotelling's duopoly. Economics Letters 21, 67–71.

Friedman, J.W., Thisse, J.-F., 1993. Partial collusion fosters minimum product differentiation. Rand Journal of Economics 24, 631–645.

Fujita, M., Thisse, J.-F., 1986. Spatial competition with a land market: Hotelling and Von Thunen unified. Review of Economic Studies LIII, 819–841.

Gabszewicz, J.J., Thisse, J.-F., 1986. On the nature of competition with differentiated products. Economic Journal 96, 160–172.

García, M.A., Georgantzís, N., Orts, V., 1996. Oligopsonistic spatial competition with price-discriminating sellers. Mimeo, Universitat Jaume I.

Green, P., Srinivasan, V., 1978. Conjoint analysis in consumer research: issues and outlook. Journal of Consumer Research 5, 103–123.

Greenhut, M.L., Norman, G., Hung, C.-S., 1987. In: The Economics of Imperfect Competition: A Spatial Approach. Cambridge University Press, Cambridge.

Gronberg, T., Meyer, J., 1981. Transport inefficiency and the choice of spatial pricing mode. Journal of Regional Science 21, 541–549.

Hamilton, J.H., Thisse, J.-F., 1997. Nonlinear pricing in spatial oligopoly. Economic Design 2, 379–397.

Hoover, E., 1937. Spatial price discrimination. Review of Economic Studies 4, 182–191.

Lederer, P., Hurter, A., 1986. Competition of firms: discriminatory pricing and location. Econometrica 54, 623–640.

Lewis, W.A., 1945. Competition in Retail Trade. Economica (November), pp. 202–234.

Machlup, F., 1949. In: The Basing-point System. Blakinston, Philadelphia, PA.

MacLeod, W.B., Norman, G., Thisse, J.-F., 1988. Price discrimination and equilibrium in monopolistic competition. International Journal of Industrial Organization 6, 429–446.

Martinez-Giralt, X., Usategui, J.M., 1997. Iceberg transport costs I: Monopoly. Discussion Papers, W.P. 399.97, Universidad Autónoma de Barcelona.

Neven, D., 1985. Two stage (perfect) equilibrium in Hotelling's model. Journal of Industrial Economics 33, 317–325.

Novshek, W., Sonnenschein, H., 1979. Marginal consumers and neoclassical demand theory. Journal of Political Economy 87, 317–325.

Phlips, L., 1983. The Economics of Price Discrimination. Cambridge University Press, Cambridge.

Pires, C.P., Sarkar, S., 2000. Delivered nonlinear pricing by duopolists. Regional Science and Urban Economics 30, 429–456.

Rhee, B.-D., De Palma, A., Fornell, C., Thisse, J.-F., 1992. Restoring the principle of minimum differentiation in product positioning. Journal of Economics and Management Strategy 1, 475–505.

Samuelson, P.A., 1954. The transfer problem and transport costs II: Analysis of effects of trade impediments. Economic Journal LXIV, 264–289.

Scherer, F.M., 1970. Industrial Market Structure and Economic Performance. Rand McNally.

Stole, L.A., 1995. Nonlinear pricing and oligopoly. Journal of Economics and Management Strategy 4, 529–562.

Thisse, J.-F., Vives, X., 1988. On the strategic choice of spatial price policy. American Economic Review 78, 122–137.

Tirole, J., 1988. The Theory of Industrial Organization. MIT Press, Cambridge, MA.

Zhang, Z.J., 1995. Price matching policy and the principle of minimum differentiation. Journal of Industrial Economics XLIII, 287–299.