# Multimarket Competition and Welfare Effects of Price Discrimination 

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#### Abstract

The paper investigates the effects on welfare of price discrimination when a multimarket seller faces competition in one of its two markets. With respect to uniform pricing, price discrimination changes competition in such a way, that even with linear demands, price discrimination can be welfare-improving, both under price competition and quantity competition.


Key words: price discrimination, multimarket competition, welfare analysis. JEL Classification: L13, L41.

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## 1 Introduction

Theoretical literature on the welfare effects of third degree price discrimination has mainly focused on the case of final good monopolies. A well known result in this literature is that a move from uniform pricing to price discrimination reduces welfare if total output decreases. Robinson (1933) shows that if a monopolist faces two independent linear demand curves, the use of price discrimination will not affect industry output but reduce welfare. ${ }^{23}$ However, as Katz (1987) claims, monopoly is precisely a market structure where antidiscrimination legislations do not apply. For instance, Robinson-Patman Act concerns harm to competition, but in the case of a final good monopoly there is no competition among either sellers or buyers. ${ }^{4}$ Despite the empirical relevance and the importance for the competition policy, there are not many works analyzing the effects of price discrimination on competition and welfare in oligopolistic frameworks. Notable exceptions, where discriminating oligopolists are discussed, are the papers by Neven and Phlips (1985) and Holmes (1989). Neven and Phlips (1985) state that whenever demand has a different price elasticity in differents markets, oligopolists will tend to price discriminate exactly in the same way as the discriminating monopolist would. They consider a multimarket Cournot duopoly, with homogeneous product (and linear demands), and conclude that allowing duopolists to discriminate between submarkets leads to a welfare loss. Holmes (1989) also studies a discriminating duopoly, but firms produce differentiated products and compete in prices. He shows that price discrimination may increase as a market moves from monopoly to duopolistic competition and what determines which regime, uniform pricing or price discrimination, has a larger output is the sum of an adjusted-concavity condition and an elasticity-ratio condition.

[^1]In this note we consider a multimarket seller facing competition in one of its two markets, ${ }^{5}$ and show that its pricing policy, price discrimination or uniform pricing, meaningfully affects competition in the duopolistic market, both under strategic substitutes and strategic complements. Following the analysis of Varian (1985), (1989), we obtain upper and lower bounds on welfare change when a move is made by the multimarket firm from uniform pricing to price discrimination. These bounds on welfare change provide necessary and sufficient conditions for price discrimination to increase social welfare, and we apply them for the case of linear demands. We show, both under strategic substitutes and strategic complements, that price discrimination reduces welfare if the duopolistic market is weak, and that, if the duopolistic market is strong, it is satisfied the necessary condition for price discrimination to lead to a welfare improvement. ${ }^{6}$

This paper is organized as follows. Section 2 develops the basic model. Section 3 analyzes the welfare effects of price discrimination and discusses implications for antitrust policy. Section 4 offers concluding remarks.

## 2 Welfare effects of price discrimination

Our analysis is based on the general test for welfare improvement proposed by Varian (1985), (1989). Consider an aggregate utility function of the form $U\left(x_{1}, x_{I}, x_{E}\right)+y$, where $x_{1}$ is the consumption in market 1 (served by the multimarket firm, firm $I$ ), $x_{i}, i=I, E$ are the product varities consumed in market 2 (offered by firm $I$ and firm $E$, respectively) and $y$ is the money to be spent on other goods. We assume that $U($.$) is concave and differentiable. The inverse demand functions are given$ $\operatorname{by} P_{j}\left(x_{1}, x_{I}, x_{E}\right)=\frac{\partial U\left(x_{1}, x_{I}, x_{E}\right)}{\partial x_{j}}, j=1, I, E$. Consider two configurations of output, $\left(x_{1}^{0}, x_{I}^{0}, x_{E}^{0}\right)$ and $\left(x_{1}^{1}, x_{I}^{1}, x_{E}^{1}\right)$, with associated prices $\left(p_{1}^{0}, p_{I}^{0}, p_{E}^{0}\right)$ and $\left(p_{1}^{1}, p_{I}^{1}, p_{E}^{1}\right)$. By using the concavity of the aggregate utility function we obtain:

[^2]\[

$$
\begin{align*}
U\left(x_{1}^{1}, x_{I}^{1}, x_{E}^{1}\right) \leq & U\left(x_{1}^{0}, x_{I}^{0}, x_{E}^{0}\right)+\frac{\partial U\left(x_{1}^{0}, x_{I}^{0}, x_{E}^{0}\right)}{\partial x_{1}}\left(x_{1}^{1}-x_{1}^{0}\right) \\
& +\frac{\partial U\left(x_{1}^{0}, x_{I}^{0}, x_{E}^{0}\right)}{\partial x_{I}}\left(x_{I}^{1}-x_{I}^{0}\right)+\frac{\partial U\left(x_{1}^{0}, x_{I}^{0}, x_{E}^{0}\right)}{\partial x_{E}}\left(x_{E}^{1}-x_{E}^{0}\right) \tag{1}
\end{align*}
$$
\]

By rearranging and using the definition of inverse demand functions we have:

$$
\begin{equation*}
\triangle U \leq p_{1}^{0} \triangle x_{1}+p_{I}^{0} \triangle x_{I}+p_{E}^{0} \triangle x_{E} \tag{2}
\end{equation*}
$$

where $\triangle U=U\left(x_{1}^{1}, x_{I}^{1}, x_{E}^{1}\right)-U\left(x_{1}^{0}, x_{I}^{0}, x_{E}^{0}\right)$ and $\triangle x_{j}=x_{j}^{1}-x_{j}^{0}, j=1, I, E$. Using a similar argument, we get:

$$
\begin{equation*}
\triangle U \geq p_{1}^{1} \triangle x_{1}+p_{I}^{1} \triangle x_{I}+p_{E}^{1} \triangle x_{E} \tag{3}
\end{equation*}
$$

Under constant marginal cost, $\triangle C=c \triangle x_{1}+c \triangle x_{I}+c \triangle x_{E}$, and given that the change in social welfare is $\Delta W \geq \triangle U-\triangle C$, we obtain upper and lower bounds on welfare change:

$$
\begin{aligned}
\left(p_{1}^{0}-c\right) \triangle x_{1}+\left(p_{I}^{0}-c\right) \triangle x_{I}+\left(p_{E}^{0}-c\right) \Delta x_{E} \geq & \Delta W \geq\left(p_{1}^{1}-c\right) \triangle x_{1} \\
& +\left(p_{I}^{1}-c\right) \triangle x_{I}+\left(p_{E}^{1}-c\right) \triangle x_{E}(4)
\end{aligned}
$$

If ( $p^{u}, p^{u}, p_{E}^{u}$ ) are the prices when the multimarket seller engages in uniform pricing and $\left(p_{1}^{m}, p_{I}^{*}, p_{E}^{*}\right)$ the discriminatory prices, then:

$$
\begin{align*}
\left(p^{u}-c\right)\left(\triangle x_{1}+\triangle x_{I}\right)+\left(p_{E}^{u}-c\right) \triangle x_{E} \geq & \Delta W \geq\left(p_{1}^{m}-c\right) \triangle x_{1} \\
& +\left(p_{I}^{*}-c\right) \triangle x_{I}+\left(p_{E}^{*}-c\right) \triangle x_{E} \tag{5}
\end{align*}
$$

The upper bound $(U B)$ implies that a necessary condition for welfare to increase is that the sum of weighted output changes is positive, where the weights are the price cost margins under uniform pricing. ${ }^{7}$ The lower bound $(L B)$ gives a sufficient condition for welfare to increase under price discrimination, namely, that the sum of the weighted output changes is positive, where the weights are the equilibrium price-cost margins under price discrimination.

In order to stress the relevance of the above bounds on welfare, we shall next consider the following aggregate utility function:

[^3]$U\left(x_{1}, x_{I}, x_{E}\right)+y=\alpha_{1} x_{1}-\frac{1}{2} \beta_{1} x_{1}^{2}+\alpha\left(x_{I}+x_{E}\right)-\frac{1}{2}\left(\beta x_{I}^{2}+2 \gamma x_{I} x_{E}+\beta x_{E}^{2}\right)+y$
The maximization of the representative consumers in markets 1 and 2 yields the inverse and direct demands systems given by
\[

$$
\begin{align*}
p_{1}\left(x_{1}\right) & =\alpha_{1}-\beta_{1} x_{1} \\
p_{I}\left(x_{I}, x_{E}\right) & =\alpha-\beta x_{I}-\gamma x_{E}  \tag{6}\\
p_{E}\left(x_{I}, x_{E}\right) & =\alpha-\beta x_{E}-\gamma x_{I} \\
D_{1}\left(p_{1}\right) & =a_{1}-b_{1} p_{1} \\
D_{I}\left(p_{I}, p_{E}\right) & =a-b p_{I}+d p_{E}  \tag{7}\\
D_{E}\left(p_{I}, p_{E}\right) & =a-b p_{E}+d p_{I}
\end{align*}
$$
\]

where $a_{1}=\frac{\alpha_{1}}{\beta_{1}}>0, b_{1}=\frac{1}{\beta_{1}}>0, a=\frac{\alpha}{\beta+\gamma}>0, b=\frac{\beta}{\beta^{2}-\gamma^{2}}>\frac{\gamma}{\beta^{2}-\gamma^{2}}=d>0$. We assume that $a_{1}>b_{1} c$ and $a>(b-d) c$, where $c$ is the marginal cost common for the two firms. These assumptions ensure that under price discrimination all markets are served. ${ }^{8}$

### 2.1 Price competition

The changes of the output in market 1 and the output of firm $I$ and $E$ in market 2, which are due to a move from uniform pricing to price discrimination, under strategic complements (that is, price competition in market 2) are given by:

$$
\begin{equation*}
\triangle x_{1}=\frac{\left(4 b^{2}-d^{2}\right) A}{2 \Gamma(2 b-d)} ; \quad \triangle x_{I}=-\frac{\left(4 b^{2}-2 d^{2}\right) A}{2 \Gamma(2 b-d)} ; \quad \triangle x_{E}=\frac{2 b d A}{2 \Gamma(2 b-d)} \tag{8}
\end{equation*}
$$

where $A=2 a b_{1}-2 a_{1} b+a_{1} d+b_{1} d c$ and $\Gamma=\left[4 b\left(b+b_{1}\right)-d^{2}\right]$. The multimarket seller's total output change is:

$$
\begin{equation*}
\triangle x_{1}+\triangle x_{I}=\frac{d^{2} A}{2 \Gamma(2 b-d)} \tag{9}
\end{equation*}
$$

It is easy to check that the upper bound $(U B)$ and the lower bound $(L B)$ on welfare change are given by:

$$
\begin{equation*}
U B=\frac{d A}{2 \Gamma^{2}(2 b-d)}\left\{\left[4 b\left(b+b_{1}\right)+(4 b+d) d\right]+4 b d\left(a_{1}-b_{1} c\right)\right\} \tag{10}
\end{equation*}
$$

[^4]\[

$$
\begin{equation*}
L B=\frac{(2 b+d) A}{4 b_{1} \Gamma(2 b-d)^{2}}\left\{-(2 b-d) A+2 b_{1} d[a-(b-d) c]\right\} \tag{11}
\end{equation*}
$$

\]

The next proposition states the effects on social welfare of price discrimination with respect to uniform pricing.

Proposition 1.- Under price competition,
(i) if the duopolistic market is weak, price discrimination reduces social welfare.
(ii) if the duopolistic market is strong, it is satisfied the necessary condition for price discrimination to increase social welfare.

Proof. It is easy to check that if market 2 is weak then $A<0$. Thus, if market 2 is weak, the upper bound on welfare change is negative, see (10), and, consequently, price discrimination reduces welfare. Note that, from (8) and (9) $\triangle x_{1}+\triangle x_{I}<0$ and $\triangle x_{E}<0$, and therefore the two terms of the upper bound (see condition (5)) are negative. If market 2 is strong then the upper bound (10) is positive given that $\Delta x_{1}+\triangle x_{I}>0$ and $\triangle x_{E}>0$. Therefore, the necessary condition for price discrimination to increase welfare is satisfied.Q.E.D.

### 2.2 Quantity competition

The output changes due to a move from uniform pricing to price discrimination, under quantity competition in market 2 , are given by:

$$
\begin{gather*}
\triangle x_{1}=\frac{\left(4 \beta^{2}-\gamma^{2}\right) C}{2 \Phi(2 \beta+\gamma)} ; \quad \triangle x_{I}=-\frac{4 \beta^{2} C}{2 \Phi(2 \beta+\gamma)} ; \quad \triangle x_{E}=\frac{2 \beta \gamma C}{2 \Phi(2 \beta+\gamma)}  \tag{12}\\
\triangle x_{1}+\triangle x_{I}=-\frac{\gamma^{2} C}{2 \Phi(2 \beta+\gamma)} \tag{13}
\end{gather*}
$$

where $C=2 \alpha \beta-2 \alpha_{1} \beta-\alpha_{1} \gamma+\gamma c$ and $\Phi=4 \beta^{2}\left(\beta+\beta_{1}\right)-\gamma^{2}\left(2 \beta+\beta_{1}\right)$. It is easy to check that if $C<0(C>0)$ market 2 is weak (strong). The upper bound $(U B)$ and the lower bound on welfare change can be written :

$$
\begin{gather*}
U B=\frac{\beta \gamma C}{2 \Phi^{2}(2 \beta+\gamma)}\left\{4 \beta\left(\beta+\beta_{1}\right)(\beta-\gamma)(\alpha-c)+\gamma^{2}\left[(\alpha-c) \beta_{1}+\left(\alpha_{1}-c\right) \gamma\right]\right\}  \tag{14}\\
L B=\frac{(2 \beta-\gamma) C}{4 \Phi(2 \beta+\gamma)^{2}}\left\{-2 \beta C+\left(\alpha_{1}-c\right) \gamma(2 \beta-\gamma)\right\} \tag{15}
\end{gather*}
$$

Proposition 2.- Under quantity competition,
(i) if the duopolistic market is weak, price discrimination reduces social welfare.
(ii) if the duopolistic market is strong, it is satisfied the necessary condition for price discrimination to increase social welfare.

### 2.3 Market opening under price discrimination

In the above analysis we have assumed that both markets are served under uniform pricing; that is, the multimarket seller sells in both markets. The following proposition analyzes the effects on social welfare when price discrimination makes the multimarket seller open the weak market.

Proposition 3.- When the multimarket seller only serves the weak market under price discrimination:
(i) If the duopolistic market is weak price discrimination might increase welfare given that uniform pricing leads to a monopolization of the weak market by the rival firm.
(ii) If the duopolistic market is strong, price discrimination yields a Pareto improvement by opening the weak market.

Proof. (i) The lower bound on welfare change is $\left(p_{I}^{*}-c\right) \triangle x_{I}+\left(p_{E}^{*}-c\right) \triangle x_{E}$ given that $\triangle x_{1}=0$. As the multimarket firm does not serve market 2 under uniform pricing, then $\triangle x_{I}=x_{I}^{*}$ and $\triangle x_{E}=x_{E}^{*}-x_{2}^{m}$ because firm $E$ is a monopolist in market 2 under uniform pricing. The effect on social welfare is in general ambiguous. When firms sell perfect substitutes it is straightforward to check that price discrimination increases social welfare. If the duopolistic (Bertrand or Cournot) equilibrium were symmetric under price discrimination (this is not strictly necessary), $p_{I}^{*}=p_{E}^{*}$, the lower bound might be written as $\left(p_{I}^{*}-c\right)\left(x_{I}^{*}+x_{E}^{*}-x_{2}^{m}\right)$, which is positive given that a duopoly produces more than a monopoly.
(ii) When the duopolistic market is strong, and if the weak market is no served under uniform pricing, then the lower bound is positive: $\left(p_{1}^{*}-c\right) \triangle x_{1}>0$ given that $\triangle x_{I}=\triangle x_{E}=0$ and $\triangle x_{1}=x_{1}^{m}$. In fact, price discrimination yields a Pareto improvement because it benefits consumers in the weak market, benefits the multimarket seller, but does not harm either consumers or the rival in market 2. Q.E.D.

### 2.4 On the "meeting competition defence"

The previous analysis serves to illustrate some perverse effects arising from the R-P Act. Assume that the multimarket firm engages in price discrimination, and imagine that the Federal Trade Commission initiates a case against this firm under section 2 of the R-P Act (which says that it is unlawful "to discriminate in price between different purchases of commodities of like grade and quality"). The Act permits the multimarket firm to rebut
the presumption of illegality by showing that its discriminatory price was quoted "in good faith to meet an equally low price of a competitor". As the following proposition states, for the case of linear demands, this defence may allow price discrimination to occur in situations in which it would reduce welfare.

Proposition 4.- (i) If the duopolistic market is weak, the "meeting competition" defence (if it were successful) allows price discrimination precisely when it reduces welfare. (ii) If the duopolistic market is strong, though price discrimination can be welfare improving, the " meeting competition defence" cannot be invoked.

This defence could be used successfully (in an economic sense) if the duopolistic market were weak but not if it were strong. However, if linearity of demand is not a bad approximation, we might expect the impact of price discrimination on welfare to be negative when the duopolistic market is weak (see propositions 1 and 2). Therefore, the banning of price discrimination would imply a welfare improvement. When the duopolistic market is strong the "meeting competition defence" is unsuccessful but to allow price discrimination, precisely in this case, can increase welfare. Note that the above conclusions depend on both markets being served under uniform pricing (see proposition 3 ).

## 3 Concluding remarks

The existence of a competitor in one market makes price discrimination by a multimarket firm welfare improving in settings, linear demands, where price discrimination would reduce welfare if the multimarket seller were a monopolist in both markets. The banning of price discrimination is particularly harmful when it leads to some markets not being served by the multimarket firm: not only may it lead to the closure of markets but also to a market monopolization by the rival. The paper also illustrates some perverse effects arising from the R-P Act.

Results do not depend on the type of competition in the duopolistic market. We have shown that results under quantity competition are similar to those under price competition.

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[^0]:    ${ }^{1}$ Financial support from Ministerio de Ciencia y Tecnología and FEDER (BEC2003-04430) is gratefully acknowledged.

[^1]:    ${ }^{2}$ Schmalensee (1981) proves this conjecture assuming nonlinear demand curves, perfectly separated markets and constant marginal cost. Varian (1985) extends the result by allowing imperfect arbitrage and by allowing marginal cost to be constant or increasing. Using a revealed-preference argument, Schwartz (1990) generalizes the result to the case in which marginal cost is decreasing.
    ${ }^{3}$ It is important to pointed out that the latter result depends on the assumption that all markets are served under both pricing regimes. Some authors have shown that when there are two potential markets price discrimination may lead, by opening markets, to a Pareto welfare improvement. Hausman and MacKie-Mason (1988) show that if the marginal cost is constant of falling, then price discrimination results in a Pareto improvement if it serves to open new markets. Even when price discrimination does not open new markets Hausman and MacKie-Mason (1988) and Nahata et al. (1990) have shown that price discrimination can result in a Pareto improvement by lowering prices in all markets.
    ${ }^{4}$ On the other hand, some recent empirical works have analyzed price dispersion in oligopolistic markets. Evidence of price discrimination is found by Shephard (1991) for the (Massachusetts) retail gasoline market, by Borenstein and Rose (1994) for the U.S. airline industry, and by Verboven (1996) for the European car market. In these works, neither cost differences or peak-load pricing seem to be the most plausible explanations for the observed price differences.

[^2]:    ${ }^{5}$ Note that this is a fit setting giving that a common feature of most cases under antidiscrimination litigation is that competition varies across markets. See Hausman and MacKie-Mason (1988) for an interesting case in which Du Pont's adversary (Akzo) charged that third-degree price discrimination practices were a misuse of a patent. See other cases in Scherer and Ross (1990) or Varian (1989).
    ${ }^{6}$ We follow Robinson's (1933) terminology, and call one market the "strong" ("weak") market if the discriminatory price in that market is at least as great as (not greater than) the uniform pricing. Many works in the literature on price discrimination have used Robinson's terminology. See, for example, Schmalensee (1981) and Holmes (1989).

[^3]:    ${ }^{7}$ Note that if firm I and E sold independent products, we would obtain the traditional result according to which an increase in total industry output is a necessary condition for price discrimination to be welfare improving.

[^4]:    ${ }^{8}$ Bertrand and Cournot reaction functions are well behaved and that there exist unique Bertrand and Cournot equilibria under both pricing policies. Furthermore, the profit function of the multimarket seller in market 1 is concave and the monopoly output (and price) is well defined.

