Functional Analysis Program

The Functional Analysis is an important branch of the Mathematics developed with the purpose to cover theoretical needs of Partial Differential Equations and Mathematical Analysis. The Functional Analysis is related to problems arising in Partial Differential Equations, Measure Theory and other branches of Mathematics.

We do not encourage the registration in the course to students with less than a B2 English level. To take the course we recommend to have first taken the courses: Calculus I, Calculus II, Complex Analysis, Linear Algebra and Geometry I, Linear Algebra and Geometry II, Differential Equations, Measure and Integration and Partial Differential Equations.

2. COMPETENCIES

CM04- To understand the concepts of Banach and Hilbert spaces and to learn to classify the standard examples. In particular, spaces of sequences and functions.
CM05- To learn to use properly the specific techniques for bounded operators over normed and Hilbert spaces.
CM06- To understand how to use the main properties of compact operators.
CM07- To learn to explain the fundamental results in the theory with accuracy and proper formalism.
CM08- To apply the spectral analysis of compact self-adjoint operators to the resolution of integral equations.

LEARNING OUTCOMES

To learn to recognize the fundamental properties of normed spaces and of the transformations between them. To be acquainted with the statement of the Hahn-Banach theorem and its corollaries. To understand the notions of dot product and Hilbert space. To apply the spectral theorem to the resolution of integral equations and Sturm-Liouville problems.

3. THEORETICAL AND PRACTICAL CONTENTS

OBJECTIVES

The objectives of the course are the study of the main properties of bounded operators between Banach and Hilbert spaces, the basic results associated to different types of convergences in normed spaces and the spectral theorem and some of its applications.

PROGRAMME

1. BANACH AND HILBERT SPACES: Banach spaces, finite dimensional normed spaces, examples of Banach spaces, Hilbert spaces, best approximation, projection theorem, dual of a Hilbert space, Riesz-Fréchet theorem, variational problems, the Dirichlet principle, bases in Hilbert spaces, orthogonality.

2. HAHN-BANACH THEOREM AND ITS CONSEQUENCES: Hahn-Banach theorem, the extension property. Topological dual of clasical spaces. Weak topology and reflexive spaces.

3. SPECTRAL THEOREM: Spectral theorem for self-adjoint compact operators: examples of bounded operators on Hilbert spaces, inversion of operators, spectrum, adjoint of operators on a Hilbert space, compact operators, some applications of the spectral theorem.

4. BAIRE THEOREM AND ITS COROLLARIES: open mapping theorem, uniform boundedness theorem and closed graph theorem.

4. METHODOLOGY

The standard one: lectures, problem sessions and personal homework carried out by the students with the help of the lecturer.

Evaluation system: Mixture

See orientations 100%

ORDINARY CALL

The evaluation system is Final Evaluation: a final written examination with questions related to the theory and problems. Students will turn in on the day of the final examination part of the collection set of problems assigned during the course, according to how it is explained during the lectures. See the web page http://www.ehu.es/luis.escauriaza/ for more information.

Written examination: not less than 85% of the total score.

Homework evaluation: not more than 15% of the total score.

The final grade will be *No presentado* whenever the written examination is not turned in.

EXTRAORDINARY CALL

Final written examination with questions related to the theory and problems. Students will turn in on the day of the examination part of the collection set of problems assigned during the course, according to how it is explained in the lectures. See the web page http://www.ehu.es/luis.escauriaza.

Written examination: not less than 85% of the total score.

Homework evaluation: not more than 15% of the total score.

The final grade will be *No presentado* when the written examination is not turned in.

BASIC BIBLIOGRAPHY

In the lectures and problem sessions we shall mainly use the books:

B. Cascales, J.M. Mira, J. Orihuela, M. Raja. Análisis Funcional. Textos Universitarios. RSME

W. Rudin. Real and Complex Analysis. MacGrow-Hill Company.

H. Brezis. Functional Analysis, Sobolev Spaces and Partial Differential Equations. Springer.

K. Saxe. Beginning Functional Analysis. Springer

WEB PAGES OF INTEREST

http://www.ehu.eus/luis.escauriaza/

http://www.matematica.ciens.ucv.ve/labfg/anfun/afb-t.pdf

OBSERVATIONS

It is strongly recommended to attend the lectures and problem sessions. The problems solved during the problem sessions complement and contain important parts of the theory explained in the lectures. To pretend to pass the course without working personally the assigned problems amounts to wasting your time.