KALKULUA – MINTEGIETAKO 1. KONTROLA

IZEN-ABIZENAK:

TALDEA: 31

1.- Osatu hurrengo taula limite bakoitzaren emaitza adieraziz (existitzen ez bada, idatz ezazu $\nexists$):

$\lim_{x\to 0}\left(\frac{2}{3}\right)^{1/x} \quad \nexists$	$\lim_{x\to\infty}\left(\frac{2}{3}\right)^{1/x} = 1$	$\lim_{x\to-\infty}\left(\frac{2}{3}\right)^{1/x} = 1$	$\lim_{x\to\infty}\left(\frac{2}{3}\right)^{x} = 0$
$\lim_{x\to-\infty} 2^{1/x} = 1$	$\lim_{x\to\infty} 2^{1/x} = 1$	$\lim_{x\to 0} 2^{1/x} \quad \nexists$	$\lim_{x\to-\infty} 2^{x} = 0$
$\lim_{x\to\infty} 2^{x} = \infty$	$\lim_{x\to\infty} 2^{-x} = 0$	$\lim_{x\to 0} 2^{-x} = 1$	$\lim_{x\to-\infty} 2^{-x} = \infty$
$\lim_{x\to\infty}\cos\left(\frac{1}{x}\right) = 1$	$\lim_{x\to\infty}\sin\left(\frac{1}{x}\right) = 0$	$\lim_{x\to 0}\sin\left(\frac{1}{x^2}\right) \quad \nexists$	$\lim_{x\to\infty}\tan\left(\frac{1}{x}\right) = 0$
$\lim_{x\to\frac{\pi}{2}^+} \tan x = -\infty$	$\lim_{x\to-\infty} \arctan x = -\frac{\pi}{2}$	$\lim_{x\to 0} \arctan x = 0$	$\lim_{x\to 2}\frac{\mathrm{L}(x-1)}{x-2} = 1$
$\lim_{x\to\infty}\frac{x^{10}}{3^x} = 0$	$\lim_{x\to\infty}\frac{x}{(\mathrm{L}x)^5} = \infty$	$\lim_{x\to 0}\frac{\mathrm{L}\left(3x^2+1\right)}{3x^2} = 1$	$\lim_{x\to\infty}\frac{\mathrm{L}\left(3x^2+1\right)}{3x^2} = 0$
$\lim_{x\to 0}\frac{\mathrm{L}\left(3x^2+1\right)}{\mathrm{L}\left(x^2\right)} = 0$	$\lim_{x\to\infty}\frac{\mathrm{L}\left(3x^2+1\right)}{\mathrm{L}\left(x^2\right)} = 1$	$\lim_{x\to\pi}\frac{\sin(x-\pi)}{x-\pi} = 1$	$\lim_{x\to 0}\frac{\sin(x-\pi)}{x-\pi} = 0$

(1.5 puntu)

2.- Kalkula ezazu $\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)\cdot \mathrm{L}(1+3x)\cdot\arctan(5x)}{\cos(x+\pi)\cdot\tan^2(2x)\cdot\left(e^{3x}-1\right)}$ **(Puntu 1)**

$$\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)\cdot \mathrm{L}(1+3x)\cdot\arctan(5x)}{\cos(x+\pi)\cdot\tan^2(2x)\cdot\left(e^{3x}-1\right)} = \lim_{x\to 0}\frac{\frac{x}{2}\cdot 3x\cdot 5x}{\cos(\pi)\cdot(2x)^2\cdot \mathrm{L}\left(e^{3x}\right)} = \lim_{x\to 0}\frac{15x^3}{-2\cdot 4x^2\cdot 3x} = -\frac{5}{8}$$

3.- Aurki itzazu hurrengo funtzioen definizio-eremuak: **(1.5 puntu)**

a) $f(x)=\dfrac{1}{\mathrm{L}\left(|x|+1\right)}$

$$D=\left\{x\in\mathbb{R}\,/\,\mathrm{L}\left(|x|+1\right)\neq 0,\,|x|+1>0\right\}$$

$\mathrm{L}\left(|x|+1\right)\neq 0 \quad\Leftrightarrow\quad |x|+1\neq 1 \quad\Leftrightarrow\quad |x|\neq 0 \quad\Leftrightarrow\quad x\neq 0$

$|x|+1>0 \quad \forall x\in\mathbb{R}$

Beraz, $D=\mathbb{R}-\{0\}$

b) $f(x)=\arcsin\left(|x|-2\right)$

$$D=\left\{x\in\mathbb{R}\,/\,-1\leq |x|-2\leq 1\right\}$$

$$-1\leq |x|-2\leq 1 \quad\Leftrightarrow\quad 1\leq |x|\leq 3 \quad\Rightarrow\quad \begin{cases} |x|\geq 1 \quad\Leftrightarrow\quad \begin{cases} x\geq 1 \\ x\leq -1 \end{cases} \quad\Leftrightarrow\quad x\in\left(-\infty,-1\right]\cup\left[1,\infty\right) \\ |x|\leq 3 \quad\Leftrightarrow\quad -3\leq x\leq 3 \quad\Leftrightarrow\quad x\in\left[-3,3\right] \end{cases}$$

Beraz, $D=\left[-3,-1\right]\cup\left[1,3\right]$

c) $f(x)=\sqrt{e^{1/x}-1}$

$$D=\left\{x\in\mathbb{R}\,/\,e^{1/x}-1\geq 0,\,x\neq 0\right\}$$

$e^{1/x}-1\geq 0 \quad\Leftrightarrow\quad e^{1/x}\geq 1 \quad\Leftrightarrow\quad \dfrac{1}{x}\geq 0 \quad\Leftrightarrow\quad x\geq 0$

Eta $x\neq 0$

Beraz, $D=(0,\infty)$

OHARRAK

Azterketa zuzentzean ikusitako zenbait “bitxikeria”:

$$\lim_{x\to-\infty} 2^x = -\infty$$

$$\lim_{x\to 0}\sin\left(\frac{1}{x^2}\right)=\infty$$

$$\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)\cdot \mathrm{L}\left(1+3x\right)\cdot \arctan\left(5x\right)}{\cos\left(x+\pi\right)\cdot \tan^2\left(2x\right)\cdot\left(e^{3x}-1\right)}=\lim_{x\to 0}\frac{15x^3}{-8x^2\cdot\left(e^{3x}-1\right)}\overset{\text{L´H}}{=}\cdots$$ (L´H aplikatu sinplifikatu barik)

$$\left|x\right|\geq 1 \quad\Leftrightarrow\quad x\geq \pm 1$$

$$\left|x\right|\geq 1 \quad\Leftrightarrow\quad 1\leq x\leq -1$$

$$\left|x\right|\leq 3 \quad\Leftrightarrow\quad x\leq \pm 3$$

$$1\leq\left|x\right|\leq 3 \quad\Leftrightarrow\quad x\in\left[-1,-3\right]\cup\left[1,3\right]$$

$$e^{1/x}-1\geq 0 \quad\Leftrightarrow\quad e^{1/x}\geq 1 \quad\Leftrightarrow\quad \frac{1}{x}\geq \mathrm{L}1 \quad\Leftrightarrow\quad x\leq\frac{1}{\mathrm{L}1}$$