OHIKO DEIALDIA
LEHENENGO PARTZIALA

1. Kalkulatu hurrengo segidaren limitea:

$$\{a_n\} = \left\{\sqrt{5},\ \sqrt{5\sqrt{5}},\ \sqrt{5\sqrt{5\sqrt{5}}},\ \dots\right\}$$

(0.75 puntu)

Segidaren gai orokorra kalkulatzen hasiko gara:

$$\left.\begin{array}{l} a_1 = \sqrt{5} = 5^{1/2} \\ a_2 = \sqrt{5\sqrt{5}} = \left(5\cdot 5^{1/2}\right)^{1/2} = 5^{1/2}\cdot 5^{1/4} = 5^{1/2+1/4} \\ a_3 = \sqrt{5\sqrt{5\sqrt{5}}} = 5^{1/2}\cdot 5^{1/4}\cdot 5^{1/8} = 5^{1/2+1/4+1/8} \\ \vdots \end{array}\right\} \Rightarrow a_n = 5^{\frac{1}{2}+\frac{1}{2^2}+\dots+\frac{1}{2^n}}$$

Orduan, $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} 5^{\frac{1}{2}+\frac{1}{2^2}+\dots+\frac{1}{2^n}} = 5^{\lim\limits_{n\to\infty}\left(\frac{1}{2}+\frac{1}{2^2}+\dots+\frac{1}{2^n}\right)} = 5^{\sum_{n=1}^{\infty}\frac{1}{2^n}} \overset{(*)}{=} 5$

(*) $\sum\limits_{n=1}^{\infty}\frac{1}{2^n}$ serie geometrikoa da, $r = \frac{1}{2} \Rightarrow \sum\limits_{n=1}^{\infty}\frac{1}{2^n} = \frac{1/2}{1-1/2} = 1$

2. Kalkulatu: $\lim\limits_{n\to\infty}\sqrt[n]{\dfrac{3n\cdot(3n+1)\cdot\ldots\cdot(3n+n)\cdot 3^{3n}}{n^n\cdot 4^{4n}}}$

(1.25 puntu)

Zatidura-errodura irizpidea aplikatuko diogu:

$$\lim_{n\to\infty}\sqrt[n]{\frac{3n\cdot(3n+1)\cdot\ldots\cdot(3n+n)\cdot 3^{3n}}{n^n\cdot 4^{4n}}} = \lim_{n\to\infty}\frac{\dfrac{3n\cdot(3n+1)\cdot\ldots\cdot(3n+n)\cdot 3^{3n}}{n^n\cdot 4^{4n}}}{\dfrac{(3n-3)\cdot(3n-2)\cdot\ldots\cdot(4n-4)\cdot 3^{3n-3}}{(n-1)^{n-1}\cdot 4^{4n-4}}} =$$

$$= \lim_{n\to\infty}\frac{4n\cdot(4n-1)\cdot(4n-2)\cdot(4n-3)\cdot 3^3}{(3n-3)\cdot(3n-2)\cdot(3n-1)\cdot(n-1)\cdot 4^4}\cdot\left(\frac{n-1}{n}\right)^n = \lim_{n\to\infty}\frac{4^4\cdot 3^3}{3^3\cdot 4^4}\cdot\left(\frac{n-1}{n}\right)^n = \lim_{n\to\infty}\left(\frac{n-1}{n}\right)^n \overset{(*)}{=} \frac{1}{e}$$

(*) $\lim\limits_{n\to\infty}\left(\frac{n-1}{n}\right)^n = A \quad\Leftrightarrow\quad \mathrm{L}A = \lim\limits_{n\to\infty} n\cdot\left(\frac{n-1}{n}-1\right) = \lim\limits_{n\to\infty} n\cdot\frac{n-1-n}{n} = -1 \quad\Leftrightarrow\quad A = e^{-1}$

3. Aztertu hurrengo seriearen izaera eta, konbergentea bada, kalkulatu bere batura:

$$\sum_{n=1}^{\infty}\left(\frac{1}{2^{n/2}}+\frac{(-1)^n}{3^{n/2}}\right)$$

(puntu 1)

$$\sum_{n=1}^{\infty}\left(\frac{1}{2^{n/2}}+\frac{(-1)^n}{3^{n/2}}\right)=\sum_{n=1}^{\infty}\left(a_n+b_n\right)$$

non $\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{1}{2^{n/2}}=\frac{\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}=\frac{1}{\sqrt{2}-1}$ (serie geometriko konbergentea, $r=\frac{1}{\sqrt{2}}<1$)

eta, $\sum_{n=1}^{\infty}b_n=\sum_{n=1}^{\infty}\frac{(-1)^n}{3^{n/2}}=\frac{\frac{-1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}=\frac{-1}{\sqrt{3}+1}$ (serie geometriko konbergentea, $|r|=\left|\frac{-1}{\sqrt{3}}\right|<1$

Orduan, $\sum_{n=1}^{\infty}\left(\frac{1}{2^{n/2}}+\frac{(-1)^n}{3^{n/2}}\right)=\sum_{n=1}^{\infty}\left(a_n+b_n\right)$ konbergentea da, eta,

$$\sum_{n=1}^{\infty}\left(\frac{1}{2^{n/2}}+\frac{(-1)^n}{3^{n/2}}\right)=\sum_{n=1}^{\infty}\frac{1}{2^{n/2}}+\sum_{n=1}^{\infty}\frac{(-1)^n}{3^{n/2}}=\frac{1}{\sqrt{2}-1}-\frac{1}{\sqrt{3}+1}$$

4. Arrazoitu ea hurrengo baieztapenak egiazkoak ala gezurrezkoak diren eta, gezurrezkoak badira, eman kontrako adibide bat:

- **a) Serie konbergente guztiak absolutuki konbergenteak dira.**
- **b) Edozein serie konbergente, segida konbergente batetik dator.**
- **c) Gai ez-negatiboen serie bat aldi berean konbergentea eta ez absolutuki konbergentea izan daiteke.**
- **d) Baldin $\lim_{n\to\infty} a_n = l$ bada, orduan $\sum_{n=1}^{\infty} a_n$ seriea beti konbergentea da.**

(puntu 1)

a) Gezurrezkoa da. Adibidea: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ez da absolutuki konbergentea, baina bada konbergentea. Izan ere, baldintzaz konbergentea dela esaten da.

b) Egiazkoa da. $\sum_{n=1}^{\infty} a_n$ konbergentea bada, orduan $\lim_{n\to\infty} a_n = 0 \in \mathbb{R} \Rightarrow \{a_n\}$ konbergentea da.

c) Gezurrezkoa da. Gai ez-negatiboen seriea bada, orduan $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} |a_n|$, beraz konbergentea ba baldin eta solik baldin absolutuki konbergentea da.

d) Gezurrezkoa da. Adibidea: $\lim_{n\to\infty} \frac{1}{n} = 0 \in \mathbb{R} \Rightarrow \left\{\frac{1}{n}\right\}$ konbergentea da, eta, hala ere, $\sum_{n=1}^{\infty} \frac{1}{n}$ dibergentea da.

5. a) Aurkitu $f(x) = 5 + \arctan(5x)$ funtzioaren berretura-seriezko garapena, non balio duen adieraziz.

b) Kalkulatu $\arctan\left(\frac{5}{100}\right)$-ren balio hurbildua, errorea 10^{-2} baino txikiagoa izanik.

(2 puntu)

a) $f'(x) = \frac{5}{1+25x^2} \overset{(*)}{=} \sum_{n=0}^{\infty} 5\cdot\left(-25x^2\right)^n = \sum_{n=0}^{\infty} (-1)^n \cdot 5^{2n+1} \cdot x^{2n} \quad \forall x \in \left(-\frac{1}{5}, \frac{1}{5}\right)$

(*) f' serie geometrikoaren batura da, $r = -25x^2$, beraz konbergentea da $\Leftrightarrow |r| = 25x^2 < 1$ $\Leftrightarrow |x| < \frac{1}{5}$

Integratuz:

$$f(x) \overset{(**)}{=} 5 + \sum_{n=0}^{\infty} (-1)^n \cdot 5^{2n+1} \cdot \frac{x^{2n+1}}{2n+1} \quad \forall x \in \left(-\frac{1}{5}, \frac{1}{5}\right)$$

(**) $f(0) = 5$

Tarteko mugak aztertuz:

$x = \pm\frac{1}{5}$ puntuetan f jarraitua da.

Eta, garapena puntu horietan, hurrenez hurren, $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1}$ eta $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ serie konbergenteak dira (Leibniz-en teorema egiaztatzen da) beraz, batura jarraitua ere existitzen da.

Beraz, $f(x) = 5 + \sum_{n=0}^{\infty} (-1)^n \cdot 5^{2n+1} \cdot \frac{x^{2n+1}}{2n+1} \quad \forall x \in \left[-\frac{1}{5}, \frac{1}{5}\right]$

b) $\arctan\left(\frac{5}{100}\right) = f\left(\frac{1}{100}\right) - 5 = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{5^{2n+1}}{100^{2n+1} \cdot (2n+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{20^{2n+1} \cdot (2n+1)}$

Serie alternatu honek Leibniz-en teorema egiaztatzen du, beraz:

$$|S - S_n| < |a_{n+1}| = \frac{1}{20^{2n+3} \cdot (2n+3)} \le \frac{1}{10^2} \Leftrightarrow 20^{2n+3} \cdot (2n+3) \ge 10^2$$

Eta, $n = 0$ baliorako betetzen da. Beraz, $\arctan\left(\frac{5}{100}\right) \simeq \frac{5}{100} = \frac{1}{20} = 0.05$

6. Aurki ezazu analitiko eta grafikoki funtzio honen definizio-eremua:

$$f(x,y)=\mathrm{L}\left(y-\mathrm{L}(x-1)\right)+\mathrm{L}\left(x-\mathrm{L}(y-1)\right)+\sqrt{(x-1)^2+(y-1)^2-1}$$

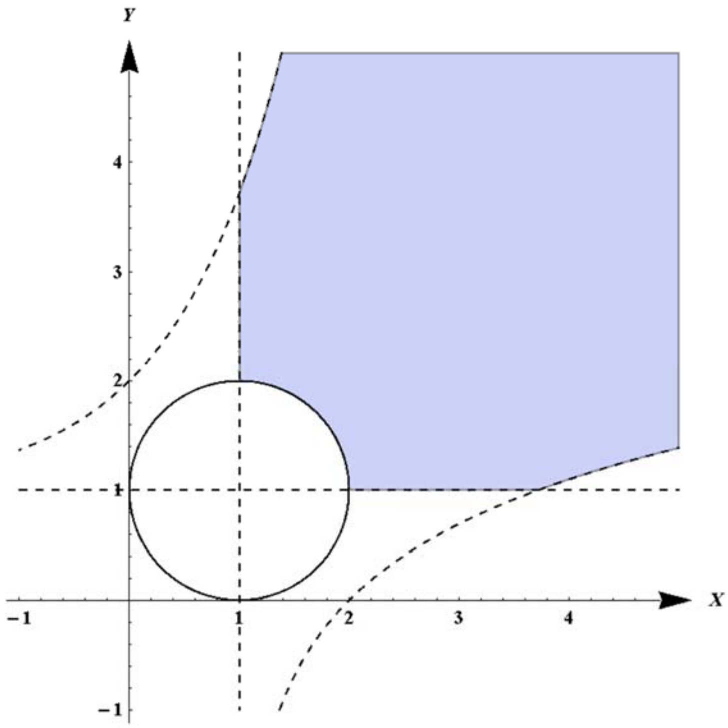
(1.5 puntu)

$D\left\{(x,y)\in\mathbb{R}^2 \,/\, x-1>0,\ y-\mathrm{L}(x-1)>0,\ y-1>0,\ x-\mathrm{L}(y-1)>0,\ (x-1)^2+(y-1)^2-1\geq 0\right\}$

$x>1,\ y>1$

$y>\mathrm{L}(x-1),\ x>\mathrm{L}(y-1)$

$(x-1)^2+(y-1)^2\geq 1$



7. Hurrengo funtzioa emanik, $f(x,y)=\begin{cases}1+\dfrac{x^3}{x^2+y^2} & \forall(x,y)\ /\ y\neq 0\\ A & \forall(x,y)\ /\ y=0\end{cases}$ **,** $A\in\mathbb{R}$**.**

a) Kalkulatu $f'_x(0,0)$ **eta** $f'_y(0,0)$

b) Aztertu *f*-ren diferentziagarritasuna (0,0) puntuan.

(1.75 puntu)

a) $f'_x(0,0)=\lim\limits_{h\to 0}\dfrac{f(h,0)-f(0,0)}{h}=\lim\limits_{h\to 0}\dfrac{A-A}{h}=0$

$$f'_y(0,0)=\lim_{k\to 0}\frac{f(0,k)-f(0,0)}{k}=\lim_{k\to 0}\frac{1-A}{k}\begin{cases}=0 & A=1\\ \nexists & A\neq 1\end{cases}$$

b) $\forall A\neq 1\ \nexists f'_y(0,0)\Rightarrow f$ ezin da diferentziagarria izan.

Eta, $A=1$ kasurako BBN aplikatuko dugu:

$$\lim_{(h,k)\to(0,0)}\frac{f(h,k)-f(0,0)-h\cdot f'_x(0,0)-k\cdot f'_y(0,0)}{\sqrt{h^2+k^2}}=\lim_{(h,k)\to(0,0)}\frac{1+\dfrac{h^3}{h^2+k^2}-1}{\sqrt{h^2+k^2}}=\lim_{(h,k)\to(0,0)}\frac{\dfrac{h^3}{h^2+k^2}}{\sqrt{h^2+k^2}}=$$

$=\lim\limits_{\substack{\rho\to 0^+\\ \forall\theta}}\dfrac{\rho^3\cdot\cos^3\theta}{\rho^3}=\lim\limits_{\substack{\rho\to 0^+\\ \forall\theta}}\cos^3\theta\Rightarrow\nexists\Rightarrow f$ ez da diferentziagarria (0,0) puntuan.

8. Kalkulatu: $\displaystyle\int_0^3\frac{dx}{x^2-1}$

(0.75 puntu)

$\displaystyle\int_0^3\frac{dx}{x^2-1}=\int_0^3 f(x)dx$ non $f(x)=\dfrac{1}{x^2-1}\in\mathbb{R}\quad\forall x\in[0,3]-\{1\}$

Eta, $\lim\limits_{x\to 1^{\pm}}f(x)=\pm\infty\Rightarrow x=1$ puntu singularra da. Beraz, integrala bitan bananduko dugu:

$$I=\int_0^3 f(x)dx=\int_0^1 f(x)dx+\int_1^3 f(x)dx=I_1+I_2$$

Eta, I konbergentea da $\Leftrightarrow$ I_1 eta I_2 konbergenteak dira.

$$\frac{1}{x^2-1}=\frac{A}{x-1}+\frac{B}{x+1}=\frac{(A+B)x+A-B}{x^2-1}\Rightarrow\begin{cases}A+B=0\\ A-B=1\end{cases}\Rightarrow A=\frac{1}{2}\text{ eta } B=-\frac{1}{2}$$

$$\int\frac{dx}{x^2-1}=\frac{1}{2}\int\frac{dx}{x-1}-\frac{1}{2}\int\frac{dx}{x+1}=\frac{1}{2}\mathrm{L}|x-1|-\frac{1}{2}\mathrm{L}|x+1|+k=\frac{1}{2}\mathrm{L}\left|\frac{x-1}{x+1}\right|+k$$

Orduan, $I_1=\displaystyle\int_0^1\frac{dx}{x^2-1}=\frac{1}{2}\mathrm{L}\left|\frac{x-1}{x+1}\right|_0^1=-\infty\Rightarrow I_1$ dibergentea da. Beraz, I dibergentea da.

BIGARREN PARTZIALA

1. Kalkulatu *a*–ren eta *b*–ren balioak $f(x,y)=e^{2ax+by}\cdot\cos(x+y)$ **funtzioaren deribatu direkzional maximoa (0,0) puntuan $3\sqrt{2}$ izateko, lehenengo koadrantearen erdikariaren norabidean.**

(puntu 1)

f-ren deribatu direkzional maximoa gradientearen modulua da, eta, gradientearen norabidean ematen da, kasu honetan lehenengo koadrantearen erdikariaren norabidean, (1,1) hain zuzen ere.

$$\left.\begin{array}{l} f'_x(x,y)=2a\cdot e^{2ax+by}\cdot\cos(x+y)-e^{2ax+by}\cdot\sin(x+y)\Rightarrow f'_x(0,0)=2a \\ f'_y(x,y)=b\cdot e^{2ax+by}\cdot\cos(x+y)-e^{2ax+by}\cdot\sin(x+y)\Rightarrow f'_y(0,0)=b \end{array}\right\}\Rightarrow \overrightarrow{\nabla f}(0,0)=(2a,b)$$

Beraz, hasieran ezarritako baldintzak kontuan hartuz:

$$\left.\begin{array}{l} \left|\overrightarrow{\nabla f}(0,0)\right|=\sqrt{4a^2+b^2}=3\sqrt{2} \\ f'_x(0,0)=2a=f'_y(0,0)=b \end{array}\right\}\Rightarrow \sqrt{8a^2}=3\sqrt{2}\Leftrightarrow 2\sqrt{2}\,|a|=3\sqrt{2}\Leftrightarrow a=\pm\frac{3}{2}\Leftrightarrow b=\pm 3$$

2. a) Aztertu ea $\left\{\begin{array}{l} xe^{u+v}+2uv-1=0 \\ ye^{u-v}-\dfrac{u}{1+v}-2x=0 \end{array}\right.$ **ekuazio-sistemak, *P*(*x*,*y*,*u*,*v*)=(1,2,0,0) puntuaren ingurunean,** $u=u(x,y)$ **eta** $v=v(x,y)$ **bi funtzio diferentziagarriak inplizituki definitzen dituen**

b) Aurkitu $du(1,2)$ **eta** $dv(1,2)$ **.**

(1.5 puntu)

a) $\left\{\begin{array}{l} F(x,y,u,v)=xe^{u+v}+2uv-1=0 \\ G(x,y,u,v)=ye^{u-v}-\dfrac{u}{1+v}-2x=0 \end{array}\right.$ ekuazio-sistemari funtzio inplizituaren teorema aplikatuko diogu:

i. $\left\{\begin{array}{l} F(P)=0 \\ G(P)=0 \end{array}\right.$

ii. $\left\{\begin{array}{llll} F'_x=e^{u+v} & F'_y=0 & F'_u=xe^{u+v}+2v & F'_v=xe^{u+v}+2u \\ G'_x=-2 & G'_y=e^{u-v} & G'_u=ye^{u-v}-\dfrac{1}{1+v} & G'_v=-ye^{u-v}-\dfrac{u}{(1+v)^2} \end{array}\right.$ jarraituak dira *P* puntuaren ingurunean non $v\neq -1$

iii. $\left|\dfrac{D(F,G)}{D(u,v)}\right|_P=\left|\begin{array}{cc} F'_u & F'_v \\ G'_u & G'_v \end{array}\right|_P=\left|\begin{array}{cc} 1 & 1 \\ 1 & -2 \end{array}\right|=-3\neq 0$

Orduan, $\exists! \begin{cases} u = u(x,y) \\ v = v(x,y) \end{cases}$, diferentziagarriak *P* puntuaren ingurunean, non $\begin{cases} u(1,2) = 0 \\ v(1,2) = 0 \end{cases}$

b) Orain, $\begin{cases} F(x,y,u(x,y),v(x,y)) = 0 \\ G(x,y,u(x,y),v(x,y)) = 0 \end{cases}$ sisteman *x*-rekiko eta *y*-rekiko deribatuko dugu.

x-rekiko:

$$\begin{cases} e^{u+v} + \left(xe^{u+v} + 2v\right) \cdot u'_x + \left(xe^{u+v} + 2u\right) \cdot v'_x = 0 \\ -2 + \left(ye^{u-v} - \dfrac{1}{1+v} \right) \cdot u'_x + \left(-ye^{u-v} - \dfrac{u}{(1+v)^2} \right) \cdot v'_x = 0 \end{cases}$$

Eta, *P* puntuan ordezkatuz:

$$\begin{cases} 1 + u'_x(1,2) + v'_x(1,2) = 0 \\ -2 + u'_x(1,2) - 2v'_x(1,2) = 0 \end{cases} \Leftrightarrow \quad 3 + 3v'_x(1,2) = 0 \quad \Leftrightarrow \quad v'_x(1,2) = -1 \quad \Rightarrow \quad u'_x(1,2) = 0$$

y-rekiko:

$$\begin{cases} \left(xe^{u+v} + 2v\right) \cdot u'_y + \left(xe^{u+v} + 2u\right) \cdot v'_y = 0 \\ e^{u-v} + \left(ye^{u-v} - \dfrac{1}{1+v} \right) \cdot u'_y + \left(-ye^{u-v} - \dfrac{u}{(1+v)^2} \right) \cdot v'_y = 0 \end{cases}$$

Eta, *P* puntuan ordezkatuz:

$$\begin{cases} u'_y(1,2) + v'_y(1,2) = 0 \\ 1 + u'_y(1,2) - 2v'_y(1,2) = 0 \end{cases} \Leftrightarrow \quad -1 + 3v'_y(1,2) = 0 \quad \Leftrightarrow \quad v'_y(1,2) = \frac{1}{3} \quad \Rightarrow \quad u'_y(1,2) = -\frac{1}{3}$$

Beraz, $du(1,2) = -\dfrac{1}{3}dy$ eta $dv(1,2) = -dx + \dfrac{1}{3}dy$

3. Kalkulatu $f(x,y)=x^3-3x^2+3xy-3y^2$ funtzioaren mutur erlatiboak.

(1.5 puntu)

Puntu kritikoak kalkulatuko ditugu:

$$\begin{cases} f'_x = 3x^2-6x+3y=0 \\ f'_y = 3x-6y=0 \quad \Leftrightarrow \quad y=\dfrac{x}{2} \end{cases} \Rightarrow \quad 3x^2-6x+\frac{3x}{2}=0 \quad \Leftrightarrow \quad x^2-\frac{3x}{2}=0 \quad \Rightarrow \quad \begin{cases} x=0 \Rightarrow y=0 \\ x=\dfrac{3}{2} \Rightarrow y=\dfrac{3}{4} \end{cases}$$

Bi puntu kritiko atera zaizkigu: $A=(0,0)$ eta $A=\left(\dfrac{3}{2},\dfrac{3}{4}\right)$

Eta, sailkatzeko, bigarren diferentzialaren zeinua aztertuko dugu (Sylvester-en irizpidea erabiliz):

$$\begin{cases} f''_{x^2}=6x-6 \\ f''_{xy}=3 \\ f''_{y^2}=-6 \end{cases} \Rightarrow \begin{cases} f''_{x^2}(A)=-6 \\ f''_{xy}(A)=3 \\ f''_{y^2}(A)=-6 \end{cases} \text{eta} \begin{cases} f''_{x^2}(B)=3 \\ f''_{xy}(B)=3 \\ f''_{y^2}(B)=-6 \end{cases}$$

Orduan:

$$Hf(A)=\begin{vmatrix} -6 & 3 \\ 3 & -6 \end{vmatrix} \Rightarrow \begin{cases} \Delta_1=-6<0 \\ \Delta_2=27>0 \end{cases} \Rightarrow \quad A \text{ maximo erlatiboa da.}$$

$$Hf(B)=\begin{vmatrix} 3 & 3 \\ 3 & -6 \end{vmatrix} \Rightarrow \begin{cases} \Delta_1=3>0 \\ \Delta_2=-27<0 \end{cases} \Rightarrow \quad B \text{ zeladura-puntua da.}$$

4. $F(a)=\displaystyle\int_0^{a^2} \arctan\left(\frac{x}{a}\right)dx$ **funtzioa emanik, kalkulatu** $F'(a)$ **deribazio parametrikoa erabiliz.**

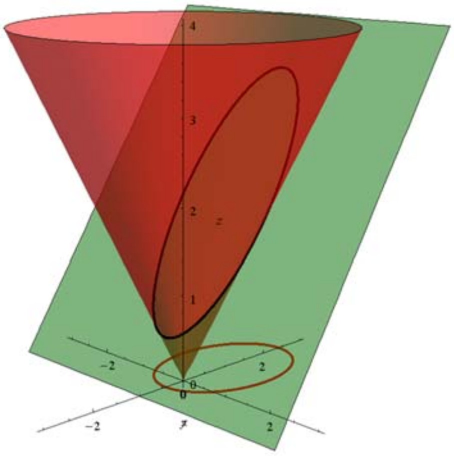
(puntu 1)

$$F'(a)=\int_0^{a^2} \frac{-\dfrac{x}{a^2}}{1+\dfrac{x^2}{a^2}}dx+2a\cdot\arctan\left(\frac{a^2}{a}\right)=-\frac{1}{2}\mathrm{L}\left(1+\frac{x^2}{a^2}\right)\Bigg|_0^{a^2}+2a\cdot\arctan(a)=$$

$$=-\frac{1}{2}\mathrm{L}\left(1+a^2\right)+2a\cdot\arctan(a)$$

5. Kalkulatu $\int_C \left((y-1)dx + z^2dy + ydz\right)$ C kurban zehar, non $C \equiv \begin{cases} x^2+y^2 = \dfrac{z^2}{2} \\ z = y+1 \end{cases}$.

(1.5 puntu)



$$C \equiv \begin{cases} x^2+y^2 = \dfrac{z^2}{2} \\ z = y+1 \end{cases} \Rightarrow \quad x^2+y^2 = \frac{(y+1)^2}{2} \quad \Leftrightarrow$$

$$\Leftrightarrow x^2+y^2 = \frac{y^2+2y+1}{2} \Leftrightarrow 2x^2+y^2-2y-1=0 \Leftrightarrow$$

$$\Leftrightarrow \quad 2x^2+(y-1)^2 = 2 \quad \Leftrightarrow \quad x^2+\frac{(y-1)^2}{2}=1 \quad \Rightarrow$$

$$\Rightarrow \quad C \equiv \begin{cases} x^2+\dfrac{(y-1)^2}{2}=1 \\ z = y+1 \end{cases}$$

Bi eratan egin daiteke.

1) Lerro-integrala kalkulatuz. Horretarako, C kurba parametrizatuko dugu:

$$C \equiv \begin{cases} x = \cos t \\ y = 1+\sqrt{2}\sin t \quad 0 \le t \le 2\pi \\ z = 2+\sqrt{2}\sin t \end{cases}$$

Orduan:

$$\int_C \left((y-1)dx + z^2dy + ydz\right) = \int_0^{2\pi} \left(-\sqrt{2}\sin^2 t + \sqrt{2}\cos t \cdot \left(2+\sqrt{2}\sin t\right)^2 + \sqrt{2}\cos t \cdot \left(1+\sqrt{2}\sin t\right)\right)dt =$$

$$= \int_0^{2\pi} \left(\frac{\sqrt{2}\left(\cos(2t)-1\right)}{2} + 4\sqrt{2}\cos t + 8\sin t\cdot\cos t + 2\sqrt{2}\sin^2 t\cdot\cos t + \sqrt{2}\cos t\cdot\left(1+\sqrt{2}\sin t\right)\right)dt =$$

$$= \left[\frac{\sqrt{2}\sin(2t)}{2} - \frac{\sqrt{2}}{2}t + 4\sqrt{2}\sin t + 4\sin^2 t + \frac{2\sqrt{2}}{3}\sin^3 t + \sqrt{2}\sin t + \sin^2 t\right]_0^{2\pi} = -\pi\sqrt{2}$$

2) *C* kurba itxia eta leuna da, eta integratu nahi dugun funtzio bektoriala eta bere lehenengo deribatu partzialak jarraituak dira beraz, Stokes-en teorema erabil daiteke:

$$\int_C \left((y-1)dx + z^2dy + ydz\right) = \oint_C \vec{F}\cdot d\vec{r} = \iint_S \overrightarrow{rot\left(\vec{F}\right)}\cdot d\vec{S} = \pm\iint_{R_{xy}} \left(\overrightarrow{rot\left(\vec{F}\right)}\cdot\vec{N}\right)dxdy$$

non $S \equiv z = y+1 \quad \forall (x,y)\in R_{xy} \equiv x^2+\dfrac{(y-1)^2}{2} \le 1$ (*C* kurbak mugatzen duena)

$\overrightarrow{rot}\left(\vec{F}\right)(1-2z,0,-1)$, $\vec{N}=(0,-1,1)$ eta $\gamma<\dfrac{\pi}{2}$

Orduan, $\int_C \left((y-1)dx+z^2dy+ydz\right)=+\iint_{R_{xy}} -dxdy=-\text{Azalera}\left(R_{xy}\right)=-\pi\sqrt{2}$

<u>Oharra</u>: Azken integrala kalkulatu behar badugu, polarretan:

$$\begin{cases} x=\rho\cos\theta \\ y=1+\sqrt{2}\rho\sin\theta \end{cases} \quad |J|=\sqrt{2}\rho \quad \Rightarrow \quad R_{xy}\equiv\begin{cases} 0\le\theta\le 2\pi \\ 0\le\rho\le 1 \end{cases}$$

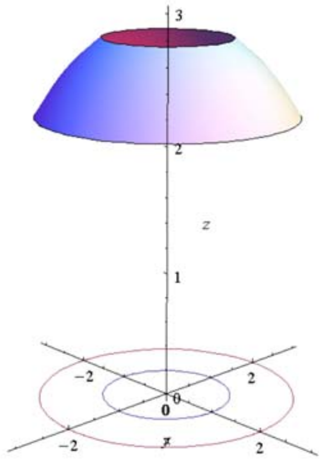
6. Izan bedi *S* $x^2+y^2+z^2=9$ esferaren zatia non $1\le x^2+y^2\le 4$ eta $z\ge 0$.

a) Kalkulatu *S*-ren azalera.

b) Kalkulatu $\iint_S \dfrac{1}{z}dS$

(1.75 puntu)

a)



$$Azalera(S)=\iint_S dS=\iint_{R_{xy}}\left|\vec{N}\right|dxdy=\iint_{R_{xy}}\frac{3}{\sqrt{9-x^2-y^2}}dxdy$$

non

$$S\equiv z=\sqrt{9-x^2-y^2} \quad \forall(x,y)\in R_{xy}\equiv 1\le x^2+y^2\le 4$$

$$\vec{N}=\left(\frac{x}{\sqrt{9-x^2-y^2}},\frac{y}{\sqrt{9-x^2-y^2}},1\right) \Rightarrow$$

$$\Rightarrow \quad \left|\vec{N}\right|=\frac{3}{\sqrt{9-x^2-y^2}}$$

Eta, polarretan: $\begin{cases} x=\rho\cos\theta \\ y=\rho\sin\theta \end{cases} \quad |J|=\rho \quad \Rightarrow \quad R_{xy}\equiv\begin{cases} 0\le\theta\le 2\pi \\ 1\le\rho\le 2 \end{cases}$

$$Azalera(S)=\int_0^{2\pi}\int_1^2 \frac{3\rho}{\sqrt{9-\rho^2}}d\rho\, d\theta=2\pi\left[-3\sqrt{9-\rho^2}\right]_1^2=6\pi\left(\sqrt{8}-\sqrt{5}\right)$$

b) $$\iint_S \frac{1}{z}dS=\iint_{R_{xy}}\frac{1}{\sqrt{9-x^2-y^2}}\left|\vec{N}\right|dxdy=\iint_{R_{xy}}\frac{3}{9-x^2-y^2}dxdy=\int_0^{2\pi}\int_1^2\frac{3\rho}{9-\rho^2}d\rho\, d\theta=$$

$$=2\pi\left[-\frac{3}{2}\text{L}\left(9-\rho^2\right)\right]_1^2=3\pi\left(\text{L}(8)-\text{L}(5)\right)$$

7. Izan bedi $\vec{F}(x,y,z)=(x,y,2z)$ bektorea:

a) Kalkulatu $\vec{F}$ -ren fluxua $V \equiv 0 \le z \le 4-2x^2-2y^2$ solidoa mugatzen duen *S* gainazal itxian zehar.

b) Kalkulatu $\vec{F}$ -ren fluxua *S* osatzen duen paraboloidearen zatian zehar.

(1.75 puntu)

a) $\vec{F}$ eta bere lehenengo deribatu partzialak jarraituak dira, eta S gainazal zatika leuna eta itxia da beraz, Gauss-en teorema erabil daiteke:

$$\iint_S \vec{F}\,d\vec{S} = \iiint_V div\left(\vec{F}\right) dxdydz = 4\iiint_V dxdydz$$

Eta, zilindrikoetan: $\begin{cases} x=\rho\cos\theta \\ y=\rho\sin\theta \\ z=z \end{cases} \quad |J|=\rho \quad \Rightarrow \quad R_{xy} \equiv \begin{cases} 0\le\theta\le 2\pi \\ 0\le\rho\le\sqrt{2} \\ 0\le z\le 4-2\rho^2 \end{cases}$

$$\iint_S \vec{F}\,d\vec{S} = 4\int_0^{2\pi}\int_0^{\sqrt{2}}\int_0^{4-2\rho^2} \rho\,dz\,d\rho\,d\theta = 8\pi\int_0^{\sqrt{2}} \rho\left(4-2\rho^2\right)d\rho = 8\pi\left[2\rho^2-\frac{\rho^4}{2}\right]_0^{\sqrt{2}} = 16\pi$$

b) $\iint_{S_1} \vec{F}\,d\vec{S} = \pm\iint_{R_{xy}} \left(\vec{F}\cdot\vec{N}\right) dxdy$

$S_1 \equiv z=4-2x^2-2y^2 \quad \forall(x,y)\in R_{xy} \equiv x^2+y^2\le 2 \quad \Rightarrow \quad \vec{N}=(4x,4y,1)$ eta $\gamma < \frac{\pi}{2}$

Orduan, $\iint_{S_1} \vec{F}\,d\vec{S} = +\iint_{R_{xy}} 8dxdy = 8\cdot \text{Azalera}\left(R_{xy}\right) = 16\pi$