Ariketa 1	Ariketa 2	Ariketa 3	Ariketa 4	Ariketa 5	Guztira

Azterketaren iraupena: 2 ordu

IZEN-ABIZENAK:

TALDEA:

1.- Azter ezazu $I=\int_1^5 \frac{dx}{(\mathrm{L}x)^a} \quad \forall a>0$**, integral inpropioaren izaera.**

(2 puntu)

$I=\int_1^5 f(x)$ non $f(x)=\frac{1}{(\mathrm{L}x)^a}>0 \quad \forall x\in(1,5], \quad \forall a>0$

Eta $\lim_{x\to 1^+} f(x)=\infty \Rightarrow x=1$ puntu singularra da.

Integral eredua: $I=\int_1^5 \frac{dx}{(x-1)^m}$, $m>0 \left\langle \begin{matrix} \text{konbergentea } \forall m<1 \\ \text{dibergentea } \forall m\geq 1 \end{matrix}\right.$

Konparaziozko irizpidea aplikatuz:

$\lim_{x\to 1^+} \frac{f(x)}{1/(x-1)^m}=\lim_{x\to 1^+}\frac{(x-1)^m}{(\mathrm{L}x)^a}\sim \lim_{x\to 1^+}\frac{(x-1)^m}{(x-1)^a} \underset{(m=a)}{=} 1\in(0,\infty) \Rightarrow$ Bi integralek izaera bera daukate.

Beraz, I konbergentea da $\forall a<1$ eta dibergentea da $\forall a\geq 1$

2.- Baldin $\mathbb{R}$ multzoan $y = f(x)$ funtzio deribagarria eta bakoitia bada, $f\left(\frac{\pi}{2}\right)=1$ **eta honako baldintza hau egiaztatzen badu:**

$$\int_{x-\frac{\pi}{2}}^{2x+\frac{\pi}{2}} f(t)dt = f(2x) + f(x),$$

kalkula ezazu $f'(0)$**.**

(2 puntu)

$\int_{x-\frac{\pi}{2}}^{2x+\frac{\pi}{2}} f(t)dt = f(2x) + f(x)$ ekuazioan *x*-rekiko deribatuko dugu:

$$2f\left(2x+\frac{\pi}{2}\right) - f\left(x-\frac{\pi}{2}\right) = 2f'(2x) + f'(x)$$

$x = 0$ puntuan ordezkatuz:

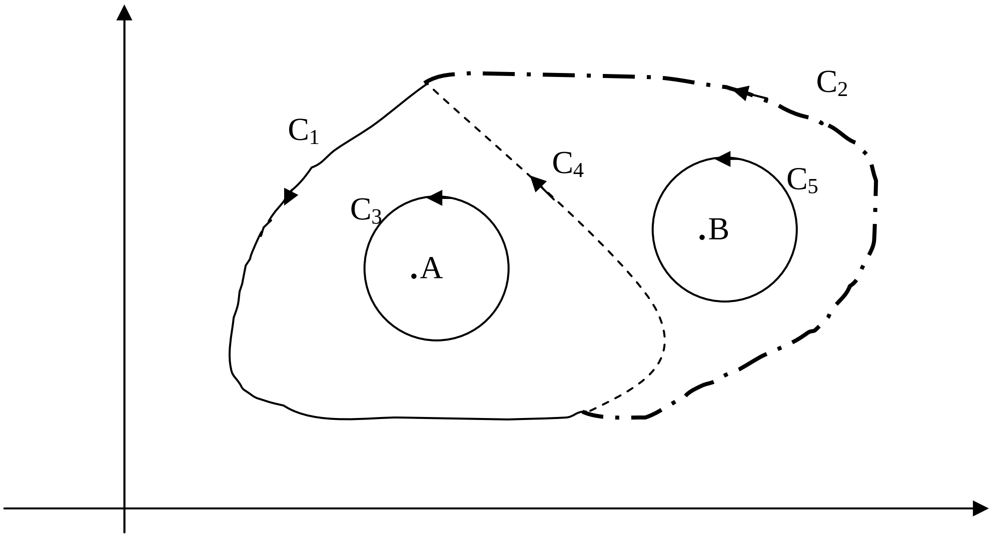
$$2f\left(\frac{\pi}{2}\right) - f\left(-\frac{\pi}{2}\right) = 2f'(0) + f'(0) \overset{(*)}{\Leftrightarrow} 3f\left(\frac{\pi}{2}\right) = 3f'(0) \Leftrightarrow f'(0) = f\left(\frac{\pi}{2}\right) = 1$$

(*) *f* bakoitia denez, $f\left(-\frac{\pi}{2}\right) = -f\left(\frac{\pi}{2}\right)$

3.- Izan bedi $\vec{F}(x,y) = P(x,y)\cdot\vec{i} + Q(x,y)\cdot\vec{j}$ jarraitua, lehenengo deribatu partzial jarraituekin eta $P'_y = Q'_x$ $D = \mathbb{R}^2 - \{A, B\}$ eremuan. Izan bitez grafikoan marrazturiko kurbak (adierazitako noranzkoetan ibilitakoak) eta $C = C_1 \cup C_2$.

Baldin $\oint_C \vec{F}\cdot d\vec{r} = 20$, $\oint_{C_3} \vec{F}\cdot d\vec{r} = 8$ eta $\int_{C_1} \vec{F}\cdot d\vec{r} = 5$ kalkula itzazu:

$$\int_{C_2} \vec{F}\cdot d\vec{r}, \quad \int_{C_4} \vec{F}\cdot d\vec{r} \quad \textbf{eta} \quad \oint_{C_5} \vec{F}\cdot d\vec{r}.$$



(1.5 puntu)

$$\oint_C \vec{F}\cdot d\vec{r} = \int_{C_1} \vec{F}\cdot d\vec{r} + \int_{C_2} \vec{F}\cdot d\vec{r} \quad \Leftrightarrow \quad \int_{C_2} \vec{F}\cdot d\vec{r} = 20 - 5 = 15$$

Eremu anizkoizki konexuetan bidearekiko independentziari buruzko emaitzak erabiliz:

$$\oint_C \vec{F}\cdot d\vec{r} = \oint_{C_3} \vec{F}\cdot d\vec{r} + \oint_{C_5} \vec{F}\cdot d\vec{r} \quad \Leftrightarrow \quad \oint_{C_5} \vec{F}\cdot d\vec{r} = 20 - 8 = 12$$

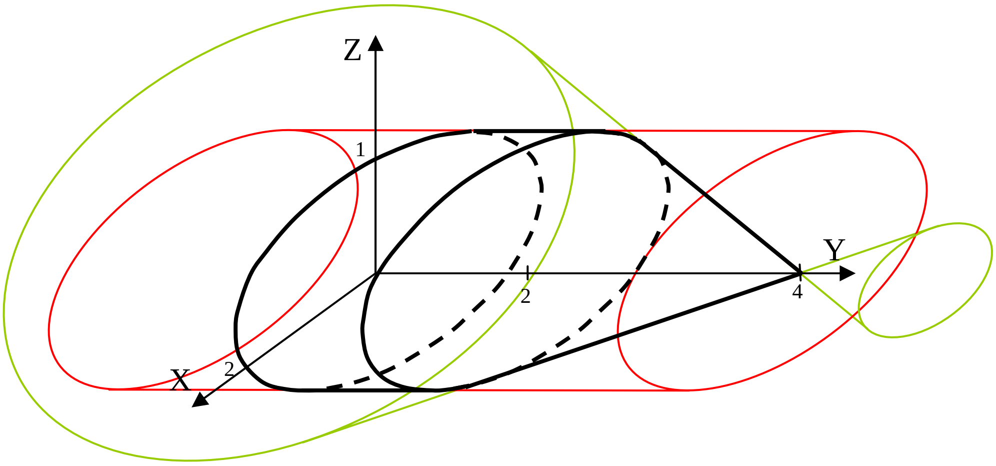
$$\int_{C_1} \vec{F}\cdot d\vec{r} + \int_{C_4} \vec{F}\cdot d\vec{r} = \oint_{C_3} \vec{F}\cdot d\vec{r} \quad \Leftrightarrow \quad \int_{C_4} \vec{F}\cdot d\vec{r} = 8 - 5 = 3$$

4.- Izan bitez $\begin{cases} S_1 \equiv x^2 + 4z^2 = 4 \\ S_2 \equiv y = 4 - \sqrt{x^2 + 4z^2} \\ S_3 \equiv y = 0 \end{cases}$ **gainazalak. Kalkulatu:**

a) gainazal hauek mugatzen duten bolumena (Oharra: Bolumenean $(0,0,0)$ **puntua dago).**
b) aurreko bolumenetik irtengo den $\vec{F} = (z+4)x \cdot \vec{i} - (z+4)y \cdot \vec{j} + xy \cdot \vec{k}$ **bektorearen fluxua.**
c) aurreko bolumena mugatzen duten gainazal bakoitzetik irtengo den fluxua.

(2.5 puntu)

a)



Zilindrikoetan planteatzen badugu:

$$\begin{cases} z = \rho\cos\theta \\ x = 2\rho\sin\theta \\ y = y \end{cases} \quad |J| = 2\rho \;\Rightarrow\; \begin{cases} S_1 \equiv \rho = 1 \\ S_2 \equiv y = 4 - 2\rho \\ S_3 \equiv y = 0 \end{cases} \Rightarrow\; 0 \le \theta \le 2\pi,\; 0 \le \rho \le 1,\; 0 \le y \le 4 - 2\rho$$

Orduan:

$$\text{Bolumena} = \int_0^{2\pi}\int_0^1\int_0^{4-2\rho} 2\rho\, dy\, d\rho\, d\theta = 4\pi\int_0^1 \rho(4-2\rho)d\rho = 8\pi\left(1 - \frac{1}{3}\right) = \frac{16\pi}{3}$$

b) Izan bedi bolumenaren muga S gainazal itxia ($S = S_1 \cup S_2 \cup S_3$). Orduan:

$$\Phi_S \overset{GAUSS}{=} \iiint_V div\left(\vec{F}\right) dxdydz \overset{(div(\vec{F})=0)}{=} 0$$

c) $\Phi_S = \Phi_{S_1} + \Phi_{S_2} + \Phi_{S_3} = 0$

$$\Phi_{S_3} = \iint_{S_3} \vec{F} d\vec{S} = \iint_{S_3} \left((z+4)x\, dydz - (z+4)y\, dzdx + xy\, dxdy\right) \underset{(S_3 \equiv y=0 \Rightarrow dy=0)}{=} 0$$

Izan bedi orain $S_5 = S_2 \cup S_4$ gainazal itxia, non $S_4 \equiv y = 2 \quad \forall (x,z) \in R_{xz}$ eta $R_{xz} \equiv x^2 + 4z^2 \le 4$.

Aurreko atalean frogatu dugun bezala, $\Phi_{S_5} = \Phi_{S_2} + \Phi_{S_4} = 0 \quad \Leftrightarrow \quad \Phi_{S_2} = -\Phi_{S_4}$.

Eta $\Phi_{S_4} = \iint_{S_4} \vec{F} d\vec{S} \underset{(S_4 \equiv y=2 \Rightarrow dy=0)}{=} \pm \iint_{R_{xz}} -2(z+4) dzdx \underset{(\beta=\pi)}{=} 2\iint_{R_{xz}} (z+4) dzdx$

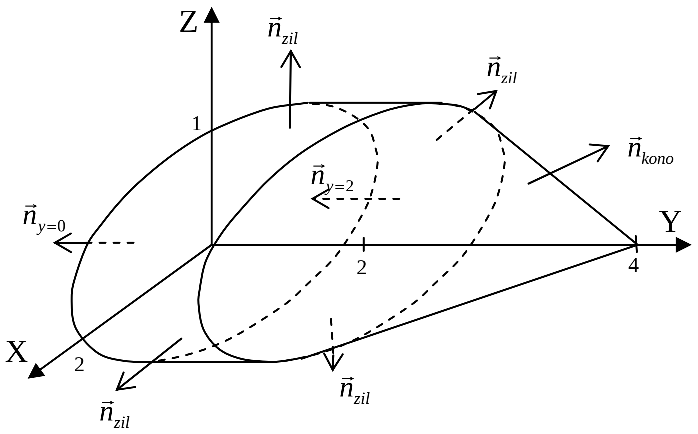
Polarretan: $\begin{cases} z = \rho\cos\theta \\ x = 2\rho\sin\theta \end{cases} \quad |J| = 2\rho \quad \Rightarrow \quad 0 \le \theta \le 2\pi,\ 0 \le \rho \le 1$. Orduan:

$$\Phi_{S_4} = 2\int_0^{2\pi}\int_0^1 2\rho(\rho\cos\theta + 4) d\rho\, d\theta = 4\int_0^{2\pi}\left(\frac{\cos\theta}{3} + 2\right) d\theta = 4\left(\frac{\sin\theta}{3} + 2\theta\right)_0^{2\pi} = 16\pi$$

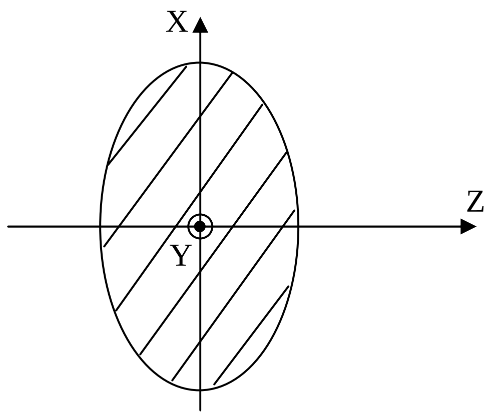
Beraz, $\Phi_{S_2} = -\Phi_{S_4} = -16\pi$ eta $\Phi_{S_1} = \Phi_S - \Phi_{S_2} - \Phi_{S_3} = 16\pi$

Oharra:

1. marrazki honetan fluxuak kalkulatzean aukeratutako bektore normalen noranzkoak adierazita daude:



2. marrazkian $R_{xz} \equiv x^2 + 4z^2 \le 4$ eskualdea duzue.

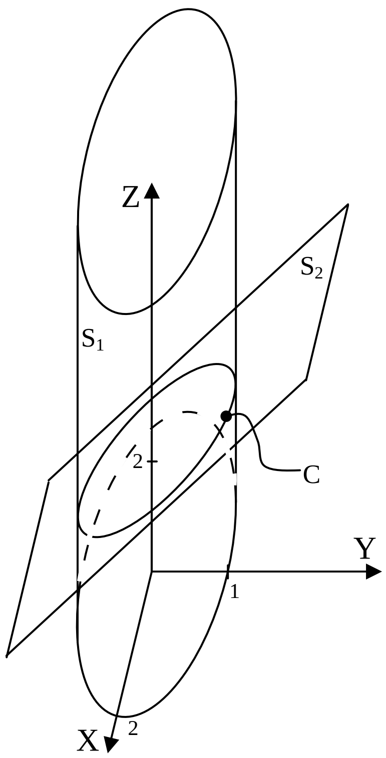


5.- a) Kalkulatu $\vec{F}=(z-y)\cdot\vec{i}+x\cdot\vec{j}+xy\cdot\vec{k}$ **bektorearen zirkulazioa** $\begin{cases} S_1\equiv x^2+4y^2=4 \\ S_2\equiv z=y+2\end{cases}$ **gainazalen arteko ebakidura-kurban zehar.**

b) Kalkulatu $\overrightarrow{rot}(\vec{F})$ **bektorearen fluxua** S_1 **gainazalak mugaturiko** S_2 **gainazalaren zatian zehar.**

(2 puntu)

a)



Bi eratan:

(a.1) Zuzenean lerro-integrala ebatziz:

$$\oint_C \vec{F}d\vec{r}=\oint_C((z-y)\,dx+x\,dy+xy\,dz)$$

non $c=S_1\cap S_2\equiv\begin{cases}x=2\cos t\\ y=\sin t\\ z=\sin t+2\end{cases}$, $t\in[0,2\pi]$

Orduan,

$$\oint_C \vec{F}d\vec{r}=\int_0^{2\pi}(-4\sin t+2\cos^2 t+2\cos^2 t\cdot\sin t)dt=$$

$$=\left[4\cos t+t+\frac{\sin(2t)}{2}-\frac{2\cos^3 t}{3}\right]_0^{2\pi}=2\pi$$

(a.2) Stokes-en teorema erabiliz:

$$\oint_C \vec{F}d\vec{r}=\iint_{S_2}\overrightarrow{rot}(\vec{F})d\vec{S}=\iint_{S_2}\left((x-0)dydz+(1-y)dzdx+(1+1)dxdy\right)$$

$S_2\equiv z=y+2 \quad\Rightarrow\quad \vec{N}=(0,-1,1)$. Orduan:

$$\iint_{S_2}\left(x\,dydz+(1-y)\,dzdx+2\,dxdy\right)=\pm\iint_{R_{xy}}(y-1+2)\,dxdy\overset{(*)}{=}$$

(*) $R_{xy}\equiv x^2+4y^2\le 4$. Polarretan $\begin{cases}x=2\rho\cos\theta\\ y=\rho\sin\theta\end{cases}$ $|J|=2\rho \quad\Rightarrow\quad 0\le\theta\le 2\pi,\ 0\le\rho\le 1$

$$\overset{(*)}{=}\int_0^{2\pi}\int_0^1 2\rho(\rho\sin\theta+1)\,d\rho\,d\theta=\int_0^{2\pi}\left(\frac{2\sin\theta}{3}+1\right)d\theta=\left[-\frac{2\cos\theta}{3}+\theta\right]_0^{2\pi}=2\pi$$

b) $\Phi_{S_2}\left(\overrightarrow{rot}(\vec{F})\right)=\iint_{S_2}\overrightarrow{rot}(\vec{F})d\vec{S}\overset{STOKES}{=}\oint_C \vec{F}\,d\vec{r}\overset{((a)\text{ atala})}{=}2\pi$