FUNTZIO KONPOSATUAK

1.- Izan bitez bi funtzio jarraitu deribatu jarraituekin, $F:\mathbb{R}^2 \longrightarrow \mathbb{R}$ eta $f:\mathbb{R} \longrightarrow \mathbb{R}$, non $f(0)=0$ eta $f'(0)=-1$. Funtzio berria definituko dugu, $G:\mathbb{R}^2 \longrightarrow \mathbb{R}$, hurrengo erara:

$$G(x,y)=F\left(e^x+e^y,\ f(x+y)\right)$$

a) Zein da (2,0) puntuan *F*-ren lehenengo deribatu partzialen arteko erlazioa, *G* funtzioak puntu kritikoa izan dezan (0,0) puntuan?

b) Baldin *F*-ren lehenengo deribatu partzialen balioak (2,0) puntuan, hurrenez hurren, 1 eta -1 badira, zein norabidetan izaten da *G* funtzioaren aldakuntza maximoa (0,0) puntuan? Eta aldakuntza minimoa?

a) *G* funtzioak puntu kritikoa du (0,0) puntuan $\Leftrightarrow \begin{cases} G'_x(0,0)=0 \\ G'_y(0,0)=0 \end{cases}$

$$\begin{matrix} x \\ y \end{matrix}\Big\rangle G = F\Big\langle \begin{matrix} u\langle \begin{smallmatrix} x \\ y \end{smallmatrix} \\ f-t\langle \begin{smallmatrix} x \\ y \end{smallmatrix} \end{matrix}$$

non $u=e^x+e^y$ eta $t=x+y$.

$G'_x = F'_u \cdot u'_x + F'_f \cdot f' \cdot t'_x = F'_u \cdot e^x + F'_f \cdot f'$

$G'_y = F'_u \cdot u'_y + F'_f \cdot f' \cdot t'_y = F'_u \cdot e^y + F'_f \cdot f'$

Orduan:

$G'_x(0,0) = F'_u(2,0) + F'_f(2,0)\cdot f'(0) = F'_u(2,0) - F'_f(2,0) = 0 \Leftrightarrow F'_u(2,0) = F'_f(2,0)$

$G'_y(0,0) = F'_u(2,0) + F'_f(2,0)\cdot f'(0) = F'_u(2,0) - F'_f(2,0) = 0 \Leftrightarrow F'_u(2,0) = F'_f(2,0)$

b) *G* funtzioaren aldakuntza maximoa (0,0) puntuan gradientearen norabidean suertatzen da:

$\overrightarrow{\nabla G}(0,0) = G'_x(0,0)\cdot\vec{i} + G'_y(0,0)\cdot\vec{j} = \left(F'_u(2,0) - F'_f(2,0)\right)\cdot\vec{i} + \left(F'_u(2,0) - F'_f(2,0)\right)\cdot\vec{j} = 2\cdot\vec{i} + 2\cdot\vec{j}$

Eta aldakuntza minimoa gradientearekiko norabide elkartzutan izango da, (-1,1) alegia.

2.- $z=z(x,y)=7\cdot y^2\cdot f\left[g(x^2,xy)+h(y^2)\right]$ funtzioa emanik, non $g(0,0)=1$, $\overrightarrow{\nabla g}(0,0)=(1,-2)$, $h(1)=h'(1)=1$ eta $f(2)=2$, kalkulatu $z'_x(0,1)+\ z'_y(0,1)$.

$z(x,y)=7\cdot y^2\cdot f(u)$ non $u=g(v,w)+h(t)$ eta $\begin{cases} v=x^2 \\ w=xy \\ t=y^2 \end{cases}$

$$\begin{matrix} x \\ y \end{matrix}\Big\rangle z = 7y^2\cdot f - u\Big\langle \begin{matrix} g\langle \begin{smallmatrix} v-x \\ w\langle \begin{smallmatrix} x \\ y \end{smallmatrix} \end{smallmatrix} \\ h-t-y \end{matrix}$$

Emandako datuen arabera, $(x,y)=(0,1)\Rightarrow\begin{cases}(v,w)=(0,0)\\ t=1\end{cases}\Rightarrow u=g(0,0)+h(1)=2$

Eta $\overrightarrow{\nabla g}(0,0)=(1,-2)\quad\Rightarrow\quad\begin{cases}g'_v(0,0)=1\\ g'_w(0,0)=-2\end{cases}$

$$\left.\begin{aligned}z'_x&=7y^2\cdot f'\cdot u'_g\cdot\left(g'_v\cdot v'+g'_w\cdot w'_x\right)=7y^2\cdot f'\cdot\left(g'_v\cdot 2x+g'_w\cdot y\right)\\ z'_y&=14y\cdot f+7y^2\cdot f'\cdot\left(u'_g\cdot g'_w\cdot w'_y+u'_h\cdot h'\cdot t'\right)=14y\cdot f+7y^2\cdot f'\cdot\left(g'_w\cdot x+h'\cdot 2y\right)\end{aligned}\right\}\Rightarrow$$

$$\left.\begin{aligned}z'_x(0,1)&=7\cdot f'(2)\cdot g'_w(0,0)=7\cdot f'(2)\cdot(-2)=-14\cdot f'(2)\\ z'_y(0,1)&=14\cdot f(2)+7\cdot f'(2)\cdot h'(1)\cdot 2=28+14\cdot f'(2)\end{aligned}\right\}\Rightarrow$$

$$z'_x(0,1)+z'_y(0,1)=-14\cdot f'(2)+28+14\cdot f'(2)=28$$

3.- $w=xyz-f\left(\dfrac{x}{z},\dfrac{y}{z}\right)$ **funtzioa emanik, non** f **diferentziagarria den, aurkitu hurrengo adierazpenaren balioa:**

$$E=x\cdot w'_x+y\cdot w'_y+z\cdot w'_z$$

$w=xyz-f\left(u,v\right)$ non $u=\dfrac{x}{z}$ eta $v=\dfrac{y}{z}$.

$$\begin{matrix}x\nwarrow & & \\ y\leftarrow & w=xyz-f\Big\langle & \begin{matrix}u\Big\langle\begin{smallmatrix}x\\ z\end{smallmatrix}\\ v\Big\langle\begin{smallmatrix}y\\ z\end{smallmatrix}\end{matrix}\\ z\swarrow & & \end{matrix}$$

$$w'_x=yz-f'_u\cdot u'_x=yz-\frac{1}{z}\cdot f'_u$$

$$w'_y=xz-f'_v\cdot v'_y=xz-\frac{1}{z}\cdot f'_v$$

$$w'_z=xy-f'_u\cdot u'_z-f'_v\cdot v'_z=xy+\frac{x}{z^2}\cdot f'_u+\frac{y}{z^2}\cdot f'_v$$

Orduan, $E=x\cdot w'_x+y\cdot w'_y+z\cdot w'_z=3xyz-\dfrac{x}{z}\cdot f'_u-\dfrac{y}{z}\cdot f'_v+\dfrac{x}{z}\cdot f'_u+\dfrac{y}{z}\cdot f'_v=3xyz$

4.- Izan bitez *f* eta *g* funtzio diferentziagarriak non $f(x,y)=g(u,v,w)$**, eta**

$$\begin{cases} u = 2x + y \\ v = e^{2x+y} \\ w = \sin(2x+y) \end{cases}$$

. Baldin $\overrightarrow{\nabla g}(0,1,0)=(1,1,0)$ **:**

a) Kalkulatu $\overrightarrow{\nabla f}(0,0)$ **.**

b) Kalkulatu $\left.\dfrac{df}{d\vec{h}}\right|_{(0,0)}$ **non** $\vec{h}=(-1,2)$ **. Justifikatu emaitza.**

a) $\overrightarrow{\nabla f}(0,0)=\left(f'_x(0,0),\, f'_y(0,0)\right)$

$$f'_x = g'_u\cdot u'_x + g'_v\cdot v'_x + g'_w\cdot w'_x = 2\cdot g'_u + 2e^{2x+y}\cdot g'_v + 2\cos(2x+y)\cdot g'_w$$
$$f'_y = g'_u\cdot u'_y + g'_v\cdot v'_y + g'_w\cdot w'_y = g'_u + e^{2x+y}\cdot g'_v + \cos(2x+y)\cdot g'_w$$

$$(x,y)=(0,0)\Rightarrow (u,v,w)=(0,1,0)$$

$$f'_x(0,0) = 2\cdot g'_u(0,1,0) + 2\cdot g'_v(0,1,0) + 2\cdot g'_w(0,1,0)$$
$$f'_y(0,0) = g'_u(0,1,0) + g'_v(0,1,0) + g'_w(0,1,0)$$

$$\overrightarrow{\nabla g}(0,1,0)=(1,1,0) \quad\Rightarrow\quad g'_u(0,1,0)=1 \quad g'_v(0,1,0)=1 \quad g'_w(0,1,0)=0 \quad\Rightarrow$$

$$\left.\begin{array}{l} f'_x(0,0)=2+2+0=4 \\ f'_y(0,0)=1+1+0=2 \end{array}\right\} \quad\Rightarrow\quad \overrightarrow{\nabla f}(0,0)=(4,2)$$

b) $\vec{h}=(-1,2) \quad\Rightarrow\quad \left|\vec{h}\right|=\sqrt{5} \quad\Rightarrow\quad \vec{h}=\left(\dfrac{-1}{\sqrt{5}},\dfrac{2}{\sqrt{5}}\right)$ unitarioa.

$$\left.\frac{df}{d\vec{h}}\right|_{(0,0)} = f'_x(0,0)\cdot\left(\frac{-1}{\sqrt{5}}\right) + f'_y(0,0)\cdot\frac{2}{\sqrt{5}} = 4\cdot\left(\frac{-1}{\sqrt{5}}\right) + 2\cdot\frac{2}{\sqrt{5}} = 0$$

$\vec{h}=(-1,2)$ -ren norabidean *f*-ren aldakuntza nulua da $\vec{h}=(-1,2)\perp\overrightarrow{\nabla f}(0,0)=(4,2)$ baitira.

5.- Izan bedi $f(x,y)=\sin\left(g(xy)-x\right)$ diferentziagarria, baita *g* funtzioa ere, non $g(1)=1$ eta $g'(1)=1$. Kalkulatu $P(x,y)=(1,1)$ puntuan *f*-ren deribatu direkzionala, OX ardatzaren noranzko positiboarekin 30º angelua osatzen duen norabidean.

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Kalkulatu behar dugu $\left.\dfrac{df}{d\vec{u}}\right|_{(1,1)} \overset{(*)}{=} f'_x(1,1)\cdot h_1 + f'_y(1,1)\cdot h_2$

non $\vec{u}=\left(h_1,h_2\right)=\left(\cos 30^{\circ},\sin 30^{\circ}\right)=\left(\dfrac{\sqrt{3}}{2},\dfrac{1}{2}\right)$

$$f'_x(x,y)=\left(y\cdot g'(xy)-1\right)\cdot\cos\left(g(xy)-x\right) \;\Rightarrow\; f'_x(1,1)=\left(g'(1)-1\right)\cdot\cos\left(g(1)-1\right)=0$$

$$f'_y(x,y)=x\cdot g'(xy)\cdot\cos\left(g(xy)-x\right) \;\Rightarrow\; f'_y(1,1)=g'(1)\cdot\cos\left(g(1)-1\right)=1$$

Beraz, $\left.\dfrac{df}{d\vec{u}}\right|_{(1,1)} = f'_x(1,1)\cdot\dfrac{\sqrt{3}}{2}+f'_y(1,1)\cdot\dfrac{1}{2}=\dfrac{1}{2}$

(*) *f* diferentziagarria baita.

<u>Oharra</u>:

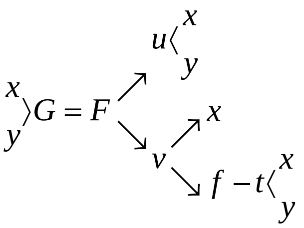
$$\left.\begin{matrix}x\\ y\end{matrix}\right\rangle f=\sin - u\left\langle\begin{matrix} g-v\left\langle\begin{matrix}x\\ y\end{matrix}\right. \\ x\end{matrix}\right.$$

non $u=g(xy)-x$ eta $v=xy$.

6.- Izan bitez *F* eta *f* bi funtzio jarraitu deribatu jarraituekin, non $f(2)=1$ **eta *F*-ren deribatu partzialak positiboak diren. *G* funtzio berria definituko dugu, hurrengo erara:**

$$G(x,y)=F\left(\mathrm{L}(x)\cdot\mathrm{L}(y)\,,\,x\cdot f(x+y)\right)$$

Kalkulatu (1,1) puntuan *G*-ren deribatu partzialak. Zein da handiagoa?



non $u=\mathrm{L}(x)\cdot\mathrm{L}(y)$, $v=x\cdot f(t)$ eta $t=x+y$. Orduan:

$$G'_x=F'_u\cdot u'_x+F'_v\cdot\left(v'_x+v'_f\cdot f'\cdot t'_x\right)=F'_u\cdot\frac{\mathrm{L}y}{x}+F'_v\cdot\left(f+x\cdot f'\right)\quad\Rightarrow$$

$$\Rightarrow\quad G'_x(1{,}1)=F'_v(0,\,f(2))\cdot\left(f(2)+f'(2)\right)=F'_v(0{,}1)+F'_v(0{,}1)\cdot f'(2)$$

$$G'_y=F'_u\cdot u'_y+F'_v\cdot v'_f\cdot f'\cdot t'_y=F'_u\cdot\frac{\mathrm{L}x}{y}+F'_v\cdot x\cdot f'\quad\Rightarrow$$

$$\Rightarrow\quad G'_y(1{,}1)=F'_v(0,\,f(2))\cdot f'(2)=F'_v(0{,}1)\cdot f'(2)$$

Eta $G'_x(1{,}1)-G'_y(1{,}1)=F'_v(0{,}1)>0\quad\Leftrightarrow\quad G'_x(1{,}1)>G'_y(1{,}1)$

7.- $w=x\cdot f[g(y)]$ **funtzioa emanik, frogatu** $x\cdot(w'_x+w'_y)-x^2\cdot w''_{xy}=w$

$$\left.\begin{matrix}x\\y\end{matrix}\right\rangle w=x\cdot f-g-y$$

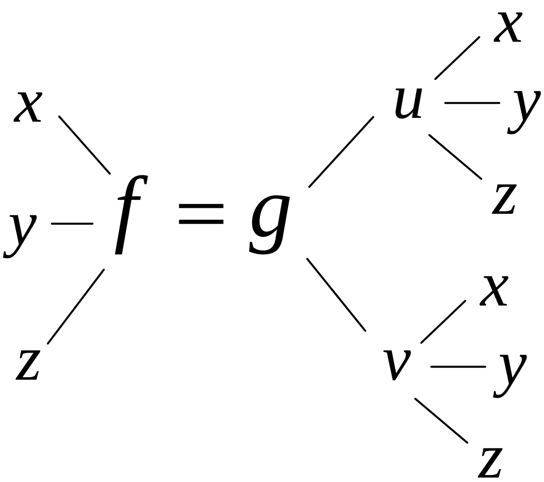
$$\left.\begin{aligned}&w'_x=f[g(y)]\quad\Rightarrow\quad w''_{xy}=f'[g(y)]\cdot g'(y)\\&w'_y=x\cdot f'[g(y)]\cdot g'(y)\end{aligned}\right\}\quad\Rightarrow$$

$$\Rightarrow\quad x\cdot(w'_x+w'_y)-x^2\cdot w''_{xy}=x\cdot\left(f+x\cdot f'\cdot g'\right)-x^2\cdot f'\cdot g'=x\cdot f=w$$

8.- Izan bitez *f* eta *g* funtzio diferentziagarriak non $f(x,y,z)=g(v,w)$ **eta** $\begin{cases} v=x+y+2z \\ w=e^{2x+y+z} \end{cases}$ **. Baldin** $\overrightarrow{\nabla g}(0,1)=(1,-1)$ **:**

a) Kalkulatu $\overrightarrow{\nabla f}(0,0,0)$ **.**

b) Kalkulatu $\left.\dfrac{df}{d\vec{u}}\right|_{(0,0,0)}$ **non** $\vec{u}=(1,2,1)$ **. Justifikatu emaitza.**



a) $\overrightarrow{\nabla f}(0,0,0)=\left(f'_x(0,0,0),\ f'_y(0,0,0),\ f'_z(0,0,0)\right)$

$$f'_x=g'_u\cdot u'_x+g'_v\cdot v'_x=g'_u+2e^{2x+y+z}\cdot g'_v$$
$$f'_y=g'_u\cdot u'_y+g'_v\cdot v'_y=g'_u+e^{2x+y+z}\cdot g'_v$$
$$f'_z=g'_u\cdot u'_z+g'_v\cdot v'_z=2\cdot g'_u+e^{2x+y+z}\cdot g'_v$$

$(x,y,z)=(0,0,0)\Rightarrow(u,v)=(0,1)$

$\overrightarrow{\nabla g}(0,1)=(1,-1)\quad\Rightarrow\quad g'_u(0,1)=1\quad g'_v(0,1)=-1\quad\Rightarrow$

$$\left.\begin{array}{l} f'_x(0,0,0)=1-2=-1 \\ f'_y(0,0,0)=1-1=0 \\ f'_z(0,0,0)=2-1=1 \end{array}\right\}\quad\Rightarrow\quad\overrightarrow{\nabla f}(0,0,0)=\left(-1,0,1\right)$$

b) $\vec{u}=(1,2,1)\quad\Rightarrow\quad|\vec{u}|=\sqrt{6}\quad\Rightarrow\quad\vec{u}=\left(\dfrac{1}{\sqrt{6}},\dfrac{2}{\sqrt{6}},\dfrac{1}{\sqrt{6}}\right)$ unitarioa da.

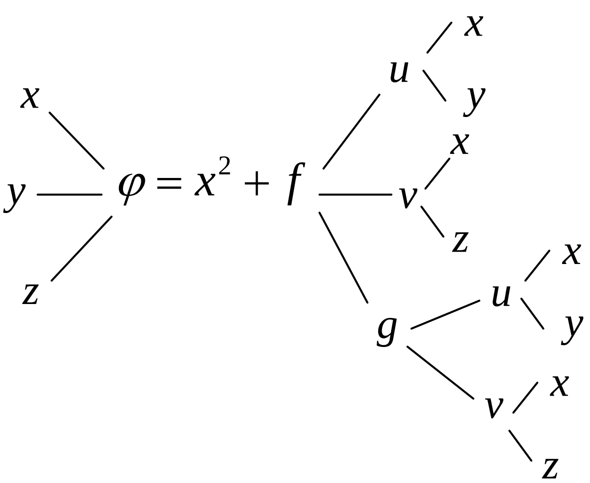
$$\left.\frac{df}{d\vec{u}}\right|_{(0,0)}=f'_x(0,0,0)\cdot\frac{1}{\sqrt{6}}+f'_y(0,0,0)\cdot\frac{2}{\sqrt{6}}+f'_z(0,0,0)\cdot\frac{1}{\sqrt{6}}=-\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{6}}=0$$

$\vec{u}=(1,2,1)$ -ren norabidean *f*-ren aldakuntza nulua da $\vec{u}=(1,2,1)\perp\overrightarrow{\nabla f}(0,0,0)=(-1,0,1)$ baitira.

9.- Izan bedi $\varphi(x,y,z)=x^2+f(xy,-xz,g(xz,xy))$ non φ, f eta g funtzio diferentziagarriak diren. Kalkulatu hurrengo adierazpenaren balioa:

$$E\equiv\varphi'_x(1,1,1)-\varphi'_y(1,1,1)-\varphi'_z(1,1,1)$$

$$\varphi(x,y,z)=x^2+f(u,-v,g(v,u)) \text{ non } \begin{cases} u=xy \\ v=xz \end{cases}$$



$$\left.\begin{aligned}
\varphi'_x &= 2x+f'_u\cdot u'_x-f'_v\cdot v'_x+f'_g\cdot\left(g'_v\cdot v'_x+g'_u\cdot u'_x\right)=2x+y\cdot f'_u-z\cdot f'_v+f'_g\cdot\left(z\cdot g'_v+y\cdot g'_u\right)\\
\varphi'_y &= f'_u\cdot u'_y+f'_g\cdot g'_u\cdot u'_y=x\cdot f'_u+f'_g\cdot x\cdot g'_u\\
\varphi'_z &= -f'_v\cdot v'_z+f'_g\cdot g'_v\cdot v'_z=-x\cdot f'_v+f'_g\cdot x\cdot g'_v
\end{aligned}\right\}\Rightarrow$$

$$\left.\begin{aligned}
\varphi'_x(1,1,1) &= 2+f'_u(1,-1,g(1,1))-f'_v(1,-1,g(1,1))+f'_g(1,-1,g(1,1))\cdot\left(g'_v(1,1)+g'_u(1,1)\right)\\
\varphi'_y(1,1,1) &= f'_u(1,-1,g(1,1))+f'_g(1,-1,g(1,1))\cdot g'_u(1,1)\\
\varphi'_z(1,1,1) &= -f'_v(1,-1,g(1,1))+f'_g(1,-1,g(1,1))\cdot g'_v(1,1)
\end{aligned}\right\}\Rightarrow$$

$$\begin{aligned}
E&\equiv\varphi'_x(1,1,1)-\varphi'_y(1,1,1)-\varphi'_z(1,1,1)=\\
&=2+f'_u(1,-1,g(1,1))-f'_v(1,-1,g(1,1))+f'_g(1,-1,g(1,1))\cdot\left(g'_v(1,1)+g'_u(1,1)\right)\\
&\quad-f'_u(1,-1,g(1,1))-f'_g(1,-1,g(1,1))\cdot g'_u(1,1)\\
&\quad+f'_v(1,-1,g(1,1))-f'_g(1,-1,g(1,1))\cdot g'_v(1,1)=2
\end{aligned}$$

10.- Irakasle batek badaki gaindituen kopurua (*w*) bi aldagairen mendean dagoela, irakaslea klasera ez den joaten orduak (*u*) eta zelan azaltzen duen (*v*):

$$w = v\cdot f(u\cdot v)$$

f **diferentziagarria izanik. Horrez gain, badaki *u* eta *v* beste bi aldagairen mende daudela, Athleticek zelan jokatzen duen (*x*) eta Interneten ibilitako orduak (*y*):**

$$\begin{cases} u = -2x + 2y \\ v = 4x \end{cases}$$

Zein da gaindituen kopuruaren aldakuntza maximoa baldin $x = \frac{1}{2}$**,** $y = 1$**,** $f(2) = 1$ **eta** $f'(2) = 1$**?**

Emandako funtzioen arteko erlazioak honako hauek dira:

$$\left.\begin{matrix} x \\ y\end{matrix}\right\rangle w = v\cdot f\left\langle \begin{matrix} v-x \\ f-t\left\langle \begin{matrix} u\left\langle\begin{matrix} x\\ y\end{matrix}\right. \\ v-x\end{matrix}\right.\end{matrix}\right. \qquad \text{non} \qquad t=u\cdot v$$

Eta kalkulatu behar duguna $\left|\vec{\nabla} w\left(\frac{1}{2},1\right)\right| = \sqrt{\left[w'_x\left(\frac{1}{2},1\right)\right]^2 + \left[w'_y\left(\frac{1}{2},1\right)\right]^2}$.

$$w'_x = v'.f + v\cdot f'\cdot(v\cdot u'_x + u\cdot v') \Rightarrow w'_x\left(\frac{1}{2},1\right) = 4f(2) + 2f'(2)\cdot(-2\cdot 2 + 4\cdot 1) = 4$$

$$w'_y = v\cdot f'\cdot v\cdot u'_y = v^2\cdot f'\cdot u'_y \Rightarrow w'_y\left(\frac{1}{2},1\right) = 2^2\cdot f'(2)\cdot 2 = 8$$

Orduan, $\left|\vec{\nabla} w\left(\frac{1}{2},1\right)\right| = \sqrt{4^2+8^2} = \sqrt{80} = 4\sqrt{5}$

11.- $z = z(x,y) = x^y + f(y^x)$ **funtzioa emanik, kalkulatu hurrengo adierazpenaren balioa:**

$$E = x\cdot\frac{\partial z}{\partial x} - y\cdot \mathrm{L}y\cdot\frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} = y\cdot x^{y-1} + y^x\cdot f'\cdot \mathrm{L}y$$

$$\frac{\partial z}{\partial y} = x^y\cdot \mathrm{L}x + x\cdot y^{x-1}\cdot f'$$

Orduan,

$$E = x\cdot\frac{\partial z}{\partial x} - y\cdot \mathrm{L}y\cdot\frac{\partial z}{\partial y} = x\cdot\left(y\cdot x^{y-1} + y^x\cdot f'\cdot \mathrm{L}y\right) - y\cdot \mathrm{L}y\cdot\left(x^y\cdot \mathrm{L}x + x\cdot y^{x-1}\cdot f'\right) =$$

$$= y\cdot x^y + x\cdot y^x\cdot f'\cdot \mathrm{L}y - y\cdot x^y\cdot \mathrm{L}y\cdot \mathrm{L}x - x\cdot y^x\cdot f'\cdot \mathrm{L}y = y\cdot x^y(1 - \mathrm{L}y\cdot \mathrm{L}x)$$

12.- Biraketa-konoaren enborraren bolumena honako adierazpen honek ematen digu:

$$V = f(x,y,z) = \frac{1}{3}\pi z(x^2 + xy + y^2)$$

non *x* goiko estalkiaren erradioa, *y* oinarriaren erradioa eta *z* altuera diren.

Baldin goiko estalkiaren erradioa 2 unitate segundoko abiaduraz txikitzen bada, oinarriaren erradioa 3 unitate segundoko abiaduraz hazten bada eta altuera 4 unitate segundoko abiaduraz txikitzen bada, kalkulatu zenbateko abiaduraz aldatzen den aurreko bolumena, $x = 10,\ y = 12,\ z = 18$ **unean.**

$V = V(t) = f(x,y,z) = f(x(t), y(t), z(t))$ eta kalkulatu behar duguna $V'(t)$ da.

$V'(t) = f'_x \cdot x'(t) + f'_y \cdot y'(t) + f'_z \cdot z'(t)$ non:

$x'(t) = -2 \qquad y'(t) = 3 \qquad z'(t) = -4$

$f'_x = \frac{1}{3}\pi z(2x + y) \quad \Rightarrow \quad f'_x(10,12,18) = 192\pi$

$f'_y = \frac{1}{3}\pi z(x + 2y) \quad \Rightarrow \quad f'_y(10,12,18) = 204\pi$

$f'_z = \frac{1}{3}\pi(x^2 + xy + y^2) \quad \Rightarrow \quad f'_z(10,12,18) = \frac{364}{3}\pi$

Orduan, $x = 10,\ y = 12,\ z = 18$ unean:

$$V'(t) = -2 \cdot 192\pi + 3 \cdot 204\pi - 4 \cdot \frac{364}{3}\pi = \frac{\pi}{3}(-1152 + 1836 - 1456) = -\frac{772}{3}\pi$$

13.- $w = z \cdot \varphi(e^z + e^t)$ **funtzioa emanik, non** $z = z(x \cdot y)$ **eta** $t = \sin x \cdot \cos y$**, eta** $z(0) = 1$ **eta** $\varphi'(e+1) = 1$ **direla ezagutuz, egiaztatu funtzio horren gradientea** $(x,y) = (0,0)$ **puntuan (1,0) bektorea dela.**

Honako hau da frogatu behar duguna:

$$\vec{\nabla} w(0,0) = w'_x(0,0) \cdot \vec{i} + w'_y(0,0) \cdot \vec{j} = \vec{i} \Leftrightarrow \begin{cases} w'_x(0,0) = 1 \\ w'_y(0,0) = 0 \end{cases}$$

$w'_x = y \cdot z' \cdot \varphi + z \cdot \varphi' \cdot \left(e^z \cdot y \cdot z' + e^t \cdot \cos x \cdot \cos y\right) \Rightarrow w'_x(0,0) = z(0) \cdot \varphi'\left(e^{z(0)} + e^0\right) = \varphi'(e+1) = 1$

$w'_y = x \cdot z' \cdot \varphi + z \cdot \varphi' \cdot \left(e^z \cdot x \cdot z' - e^t \cdot \sin x \cdot \sin y\right) \Rightarrow w'_y(0,0) = 0$