DEFINIZIO-EREMUAK

1.- Aurkitu analitiko eta grafikoki funtzioaren definizio-eremua:

$$f(x,y)=\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}+\frac{\sqrt{x^2+y^2-2x}}{\sqrt{x^2+y^2+2x}}$$

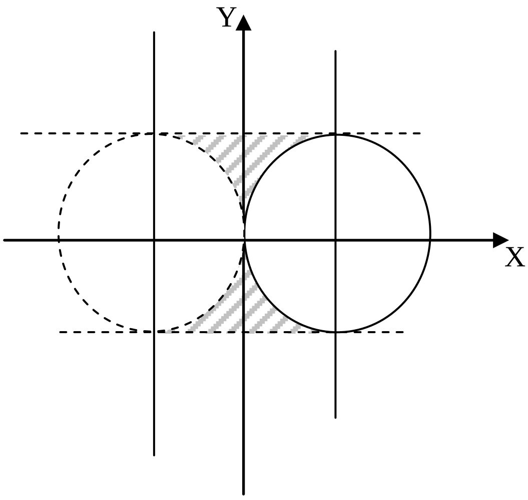
$$D=\left\{(x,y)\in\mathbb{R}^2\,/\,1-x^2\geq 0,1-y^2>0,\,x^2+y^2-2x\geq 0,\,x^2+y^2+2x>0\right\}$$

$1-x^2\geq 0 \quad\Leftrightarrow\quad -1\leq x\leq 1$

$1-y^2>0 \quad\Leftrightarrow\quad -1<y<1$

$x^2+y^2-2x\geq 0 \quad\Leftrightarrow\quad (x-1)^2+y^2\geq 1$

$x^2+y^2+2x>0 \quad\Leftrightarrow\quad (x+1)^2+y^2>1$



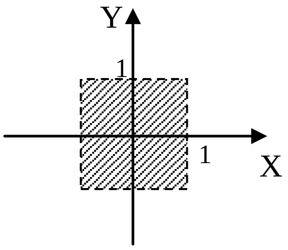
2.- Aurkitu analitiko eta grafikoki honako funtzio hauen definizio-eremua:

a) $f(x,y) = \mathrm{L}\left(1-x^2\right) - \mathrm{L}\left(1-y^2\right)$

b) $g(x,y) = \dfrac{\mathrm{L}\left(1-x^2\right)}{\mathrm{L}\left(1-y^2\right)}$

Berdinak dira *f* eta *g* funtzioak?

a) $D = \left\{(x,y) \in \mathbb{R}^2 / 1-x^2 > 0, 1-y^2 > 0\right\} = \left\{(x,y) \in \mathbb{R}^2 / -1 < x < 1, -1 < y < 1\right\}$

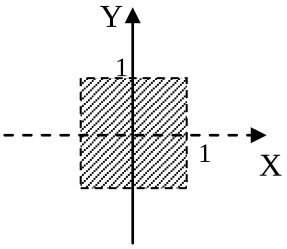


b) $D = \left\{(x,y) \in \mathbb{R}^2 / 1-x^2 > 0, 1-y^2 > 0, \mathrm{L}\left(1-y^2\right) \neq 0\right\}$

$1-x^2 > 0 \quad \Leftrightarrow \quad -1 < x < 1$

$1-y^2 > 0 \quad \Leftrightarrow \quad -1 < y < 1$

$\mathrm{L}(1-y^2) \neq 0 \quad \Leftrightarrow \quad 1-y^2 \neq 1 \quad \Leftrightarrow \quad y \neq 0$



f eta *g* ez dira funtzio berdinak.

3.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

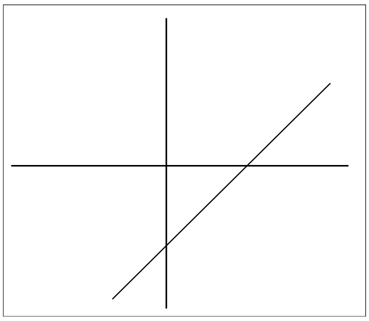
$$f(x,y)=\frac{\arcsin(x-y)}{\sqrt{4x^2+9y^2-1}}+\mathrm{L}\left(1-|x|\right)+\mathrm{L}\left(1-|y|\right)$$

$$D=\left\{(x,y)\in\mathbb{R}^2\,/-1\le x-y\le 1,\,4x^2+9y^2-1>0,\,1-|x|>0,\,1-|y|>0\right\}$$

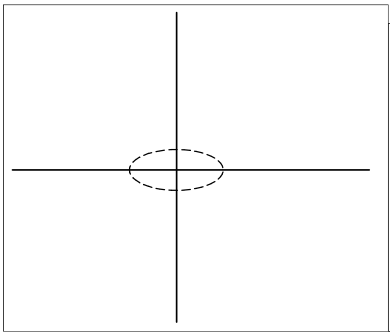
(1) $-1\le x-y\le 1\Leftrightarrow x-1\le y\le x+1$

(2) $4x^2+9y^2-1>0\Leftrightarrow 4x^2+9y^2>1\Leftrightarrow \dfrac{x^2}{1/4}+\dfrac{y^2}{1/9}>1$

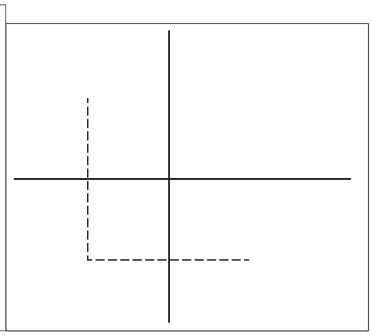
(3) $1-|x|>0,\,1-|y|>0\Leftrightarrow |x|<1,\,|y|<1\Leftrightarrow -1<x<1,\,-1<y<1$



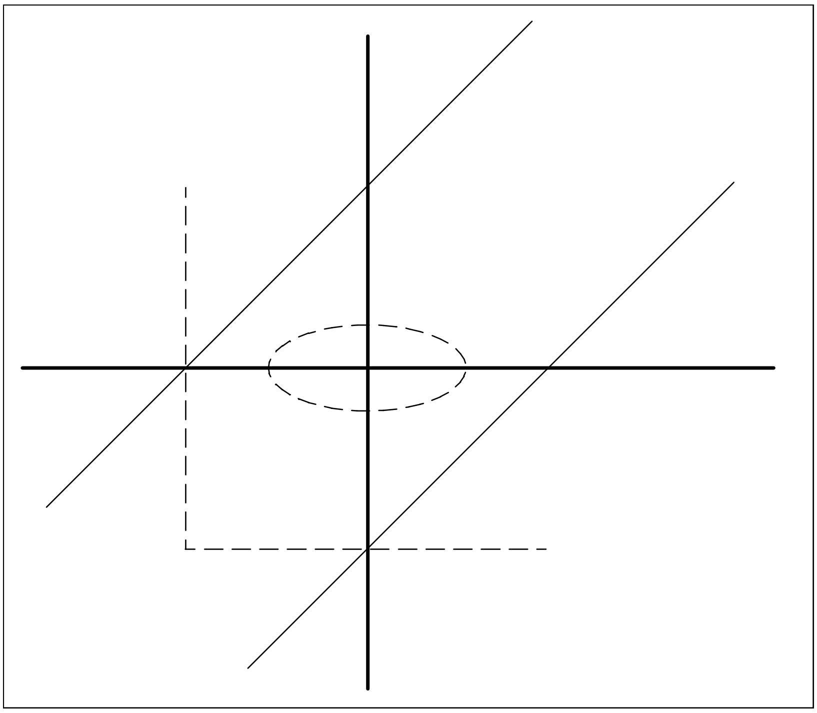
(1)



(2)



(3)



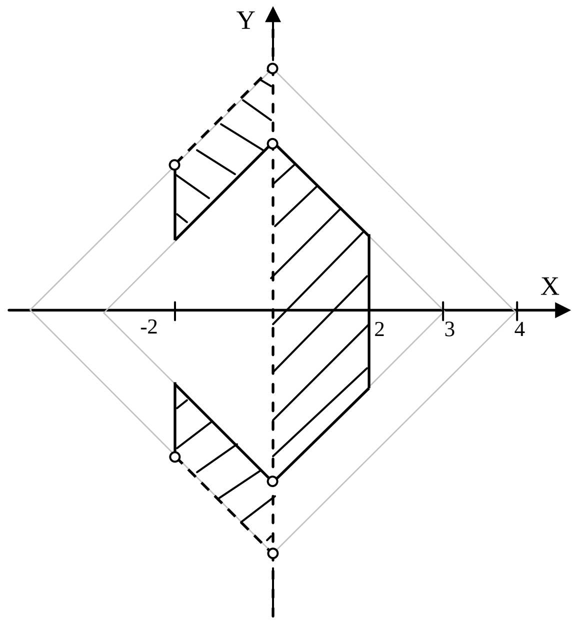
4.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua

$$f(x,y)=\frac{\sqrt{x\cdot \mathrm{L}(4-|x|-|y|)}}{\arcsin\left(\frac{x}{2}\right)}$$

$$D=\left\{(x,y)\in\mathbb{R}^2 \,/\, x\cdot \mathrm{L}(4-|x|-|y|)\geq 0,\ 4-|x|-|y|>0,\ \arcsin\left(\frac{x}{2}\right)\neq 0,\ -1\leq\frac{x}{2}\leq 1\right\}$$

- $-1\leq\frac{x}{2}\leq 1 \quad\Leftrightarrow\quad -2\leq x\leq 2$
- $\arcsin\left(\frac{x}{2}\right)\neq 0 \quad\Leftrightarrow\quad \frac{x}{2}\neq 0 \quad\Leftrightarrow\quad x\neq 0$
- $4-|x|-|y|>0 \quad\Leftrightarrow\quad |x|+|y|<4$
- $x\cdot \mathrm{L}(4-|x|-|y|)\geq 0 \quad\Rightarrow\quad \begin{cases} x\geq 0 \ \wedge\ \mathrm{L}(4-|x|-|y|)\geq 0 \ \Leftrightarrow\ x\geq 0 \ \wedge\ 4-|x|-|y|\geq 1 \\ \text{edo} \\ x\leq 0 \ \wedge\ \mathrm{L}(4-|x|-|y|)\leq 0 \ \Leftrightarrow\ x\leq 0 \ \wedge\ 4-|x|-|y|\leq 1 \end{cases}$

$$\Leftrightarrow\ \begin{cases} x\geq 0 \ \wedge\ |x|+|y|\leq 3 \\ \text{edo} \\ x\leq 0 \ \wedge\ |x|+|y|\geq 3 \end{cases}$$

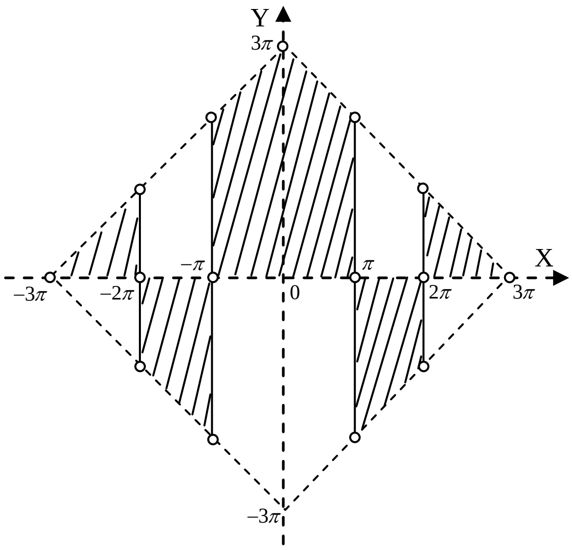


5.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua

$$f(x,y)=\sqrt{\frac{\sin x}{xy}}+\mathrm{L}(3\pi-|x|-|y|)$$

$$D=\left\{(x,y)\in\mathbb{R}\,/\,xy\neq 0,\ \frac{\sin x}{xy}\geq 0,\ 3\pi-|x|-|y|>0\right\}$$

- $xy\neq 0 \quad\Rightarrow\quad x\neq 0 \quad\wedge\quad y\neq 0$
- $\dfrac{\sin x}{xy}\geq 0 \quad\Rightarrow\quad \begin{cases}\sin x\geq 0 \ \wedge\ xy>0 \ \Rightarrow \begin{cases}x>0 & \wedge & y>0\\ x<0 & \wedge & y<0\end{cases}\\[2ex] \sin x\leq 0 \ \wedge\ xy<0 \ \Rightarrow \begin{cases}x>0 & \wedge & y<0\\ x<0 & \wedge & y>0\end{cases}\end{cases}$
- Eta $\sin x\geq 0 \quad\Leftrightarrow\quad x\in\bigcup_{k\in\mathbb{Z}}\left[2k\pi,(2k+1)\pi\right]$
- $3\pi-|x|-|y|>0 \quad\Leftrightarrow\quad |x|+|y|<3\pi$

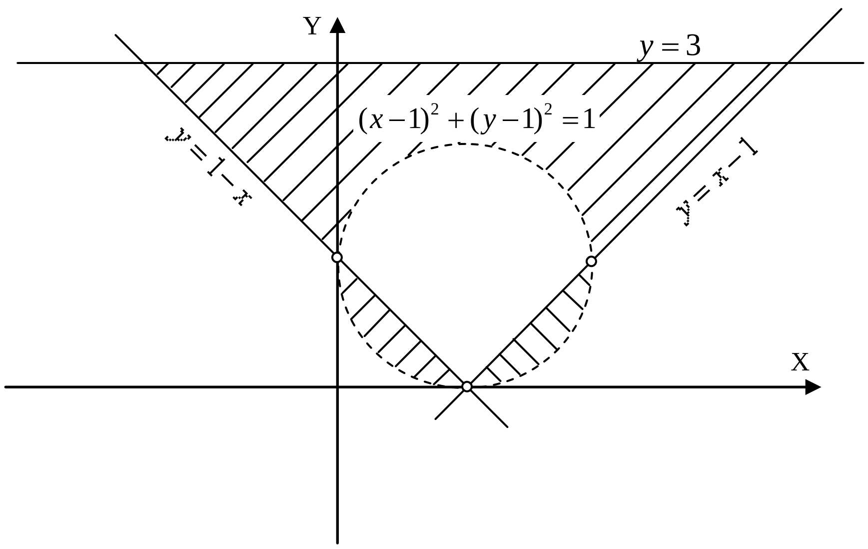


6.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\arcsin\left(\frac{2y}{3}-1\right)+\sqrt{\frac{|x-1|-y}{1-(x-1)^2-(y-1)^2}}$$

$$D=\left\{(x,y)\in\mathbb{R}^2 \,/ -1\le\frac{2y}{3}-1\le 1,\ \frac{|x-1|-y}{1-(x-1)^2-(y-1)^2}\ge 0 \wedge 1-(x-1)^2-(y-1)^2\ne 0\right\}$$

- $-1\le\frac{2y}{3}-1\le 1 \quad\Leftrightarrow\quad 0\le\frac{2y}{3}\le 2 \quad\Leftrightarrow\quad 0\le y\le 3$
- $1-(x-1)^2-(y-1)^2\ne 0 \quad\Leftrightarrow\quad (x-1)^2+(y-1)^2\ne 1$
- $$\frac{|x-1|-y}{1-(x-1)^2-(y-1)^2}\ge 0 \Rightarrow \begin{cases}\begin{cases}|x-1|-y\ge 0 \Rightarrow \begin{cases} y\le x-1 & \forall x\ge 1\\ y\le 1-x & \forall x<1\end{cases}\\ 1-(x-1)^2-(y-1)^2>0 \Leftrightarrow (x-1)^2+(y-1)^2<1\end{cases}\\ \text{edo}\\ \begin{cases}|x-1|-y\le 0 \Rightarrow \begin{cases} y\ge x-1 & \forall x\ge 1\\ y\ge 1-x & \forall x<1\end{cases}\\ 1-(x-1)^2-(y-1)^2<0 \Leftrightarrow (x-1)^2+(y-1)^2>1\end{cases}\end{cases}$$



7.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua

$$f(x,y) = \mathrm{L}\left(y - \mathrm{L}x\right) + \frac{\arcsin\left(|x| + |y| - 1\right)}{\sqrt{e^x - y}\cdot\sqrt{x^2 + y^2 - 1}}$$

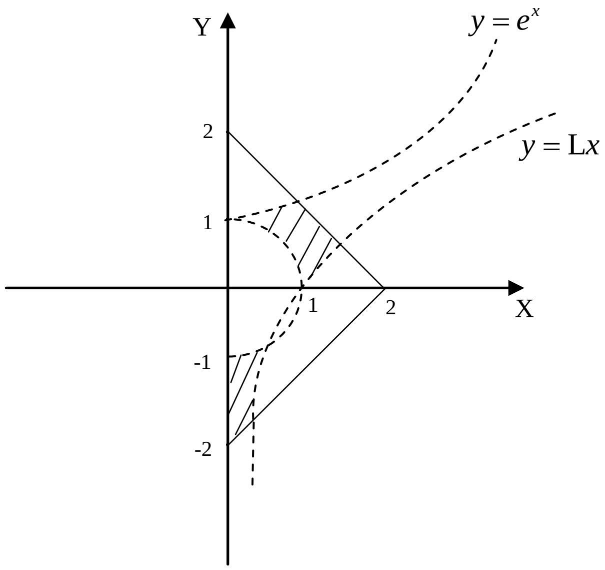
$$D = \left\{(x,y) \in \mathbb{R}^2 \,/\, x > 0, y - \mathrm{L}x > 0, -1 \le |x| + |y| - 1 \le 1, e^x - y > 0, x^2 + y^2 - 1 > 0\right\}$$

- $x > 0$

- $\left.\begin{array}{l} y - \mathrm{L}x > 0 \quad \Leftrightarrow \quad y > \mathrm{L}x \\ e^x - y > 0 \quad \Leftrightarrow \quad y < e^x \end{array}\right\} \Rightarrow \quad \mathrm{L}x < y < e^x$

- $-1 \le |x| + |y| - 1 \le 1 \quad \Leftrightarrow \quad |x| + |y| \le 2$

- $x^2 + y^2 - 1 > 0 \quad \Leftrightarrow \quad x^2 + y^2 > 1$



8.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\arcsin\left(\frac{2y}{3}-1\right)+\sqrt{\frac{|x|-y}{1-(x-1)^2-(y-1)^2}}$$

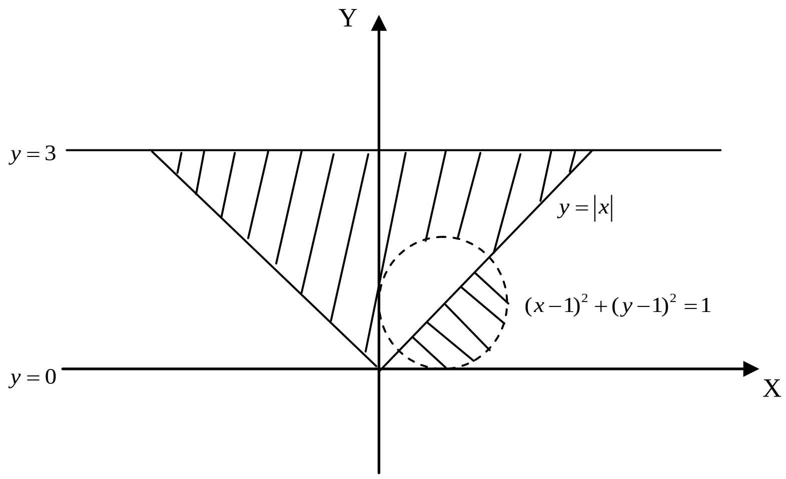
$$D=\left\{(x,y)\in\mathbb{R}^2\ /-1\le\frac{2y}{3}-1\le 1,\ \frac{|x|-y}{1-(x-1)^2-(y-1)^2}\ge 0,\ 1-(x-1)^2-(y-1)^2\ne 0\right\}$$

- $-1\le\frac{2y}{3}-1\le 1 \quad\Leftrightarrow\quad 0\le\frac{2y}{3}\le 2 \quad\Leftrightarrow\quad 0\le y\le 3$

- $1-(x-1)^2-(y-1)^2\ne 0 \quad\Leftrightarrow\quad (x-1)^2+(y-1)^2\ne 1$

- $\frac{|x|-y}{1-(x-1)^2-(y-1)^2}\ge 0 \quad\Rightarrow\quad \begin{cases}|x|-y\ge 0 \ \wedge\ 1-(x-1)^2-(y-1)^2>0\\ \qquad\qquad\vee\\ |x|-y\le 0 \ \wedge\ 1-(x-1)^2-(y-1)^2<0\end{cases}$

$$\Leftrightarrow\quad \begin{cases}y\le |x| \ \wedge\ (x-1)^2+(y-1)^2<1\\ \qquad\qquad\vee\\ y\ge |x| \ \wedge\ (x-1)^2+(y-1)^2>1\end{cases}$$



9.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\frac{\arcsin(|x|+|y-1|-2)}{\sqrt{y+x^2+2}}-\mathrm{L}(x^2+2-y)$$

$$D=\left\{(x,y)\in\mathbb{R}^2 / -1\le|x|+|y-1|-2\le 1,\ y+x^2+2>0,\ x^2+2-y>0\right\}$$

- $-1\le|x|+|y-1|-2\le 1 \quad\Leftrightarrow\quad 1\le|x|+|y-1|\le 3$

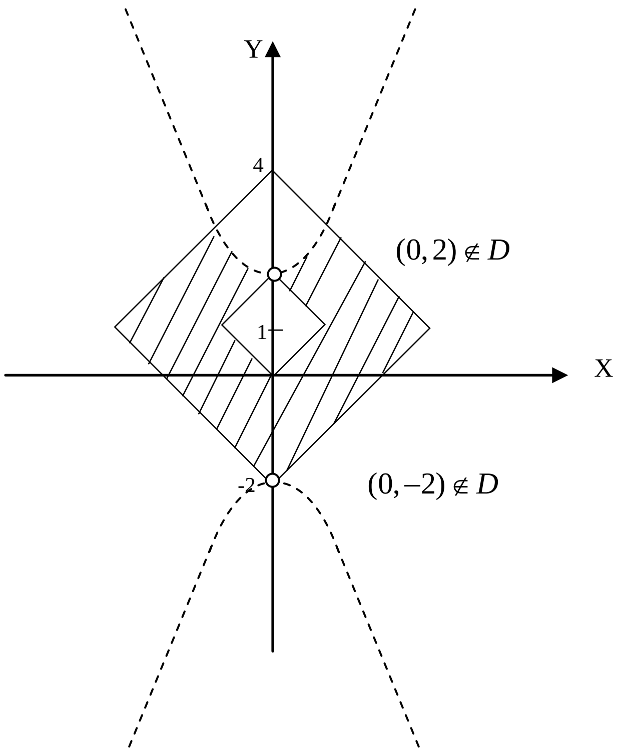
  Bi erronboren arteko eskualdea da.

- $y+x^2+2>0 \quad\Leftrightarrow\quad y>-x^2-2$

  $y=-x^2-2$ parabola da, erpina $(0,-2)$ puntuan daukana.

- $x^2+2-y>0 \quad\Leftrightarrow\quad y<x^2+2$

  $y=x^2+2$ parabola da, erpina $(0,2)$ puntuan daukana.

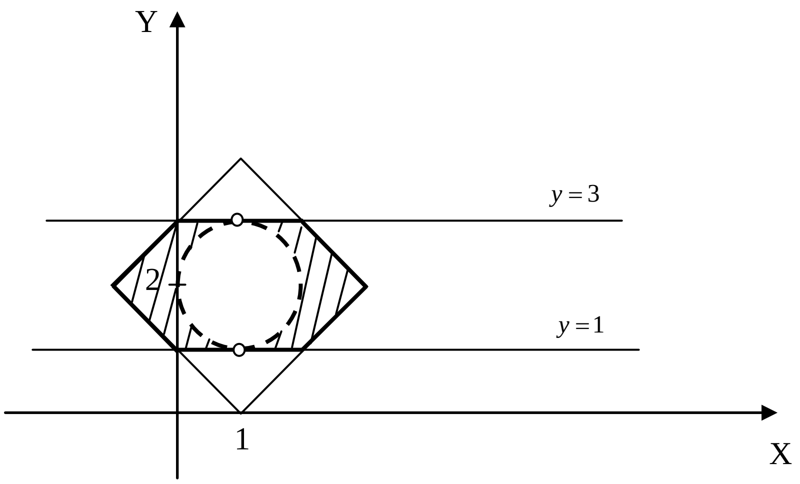


10.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y) = \arcsin(y-2) + \sqrt{2-|x-1|-|y-2|} \cdot \mathrm{L}\left((x-1)^2 + (y-2)^2 - 1\right)$$

$$D = \left\{(x,y) \in \mathbb{R}^2 \,/ -1 \le y-2 \le 1,\ 2-|x-1|-|y-2| \ge 0,\ (x-1)^2 + (y-2)^2 - 1 > 0\right\}$$

- $-1 \le y-2 \le 1 \quad \Leftrightarrow \quad 1 \le y \le 3$
- $2-|x-1|-|y-2| \ge 0 \quad \Leftrightarrow \quad |x-1|+|y-2| \le 2$
- $(x-1)^2 + (y-2)^2 - 1 > 0 \quad \Leftrightarrow \quad (x-1)^2 + (y-2)^2 > 1$



11.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

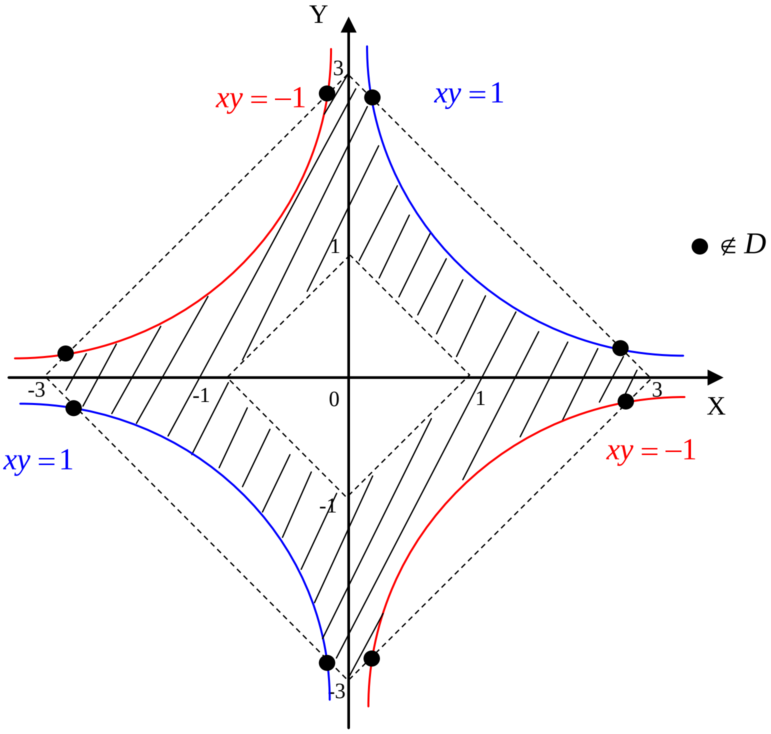
$$f(x,y)=\sqrt{1-x^2\cdot y^2}+\mathrm{L}\left(|x|+|y|-1\right)+\mathrm{L}\left(3-|x|-|y|\right)$$

$$D=\left\{(x,y)\in\mathbb{R}^2\,/\,1-x^2\cdot y^2\geq 0,\ \ |x|+|y|-1>0,\ \ 3-|x|-|y|>0\right\}$$

$$1-x^2\cdot y^2\geq 0 \quad\Leftrightarrow\quad x^2\cdot y^2\leq 1 \quad\Leftrightarrow\quad |xy|\leq 1 \quad\Leftrightarrow\quad -1\leq xy\leq 1$$

$$|x|+|y|-1>0 \quad\Leftrightarrow\quad |x|+|y|>1$$

$$3-|x|-|y|>0 \quad\Leftrightarrow\quad |x|+|y|<3$$

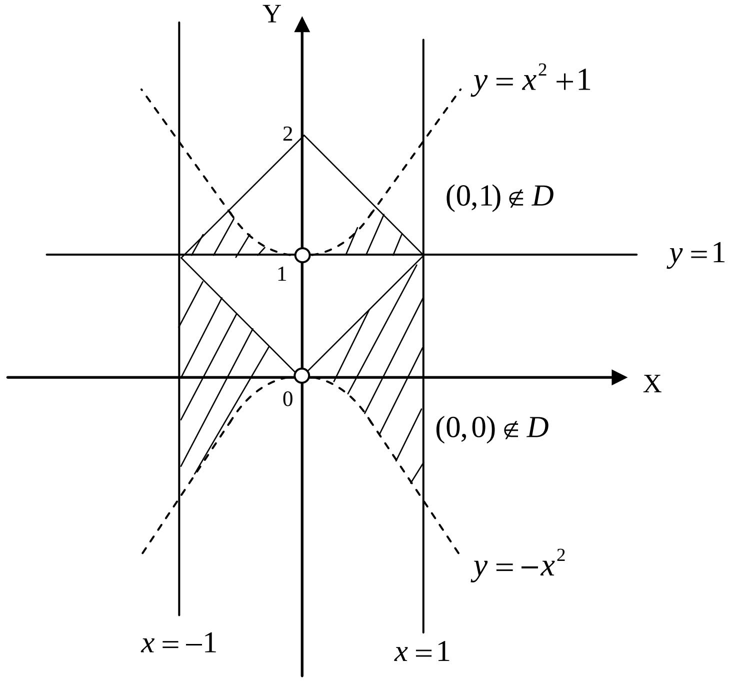


12.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\arcsin x\cdot\sqrt{(1-y)\cdot(|x|+|y-1|-1)}+\frac{L(x^2+1-y)}{\sqrt{y+x^2}}$$

$$D=\left\{(x,y)\in\mathbb{R}^2 / -1\le x\le 1,\ (1-y)\cdot(|x|+|y-1|-1)\ge 0,\ x^2+1-y>0,\ y+x^2>0\right\}$$

- $-1\le x\le 1$
- $(1-y)\cdot(|x|+|y-1|-1)\ge 0 \quad\Rightarrow\quad \begin{cases}\begin{cases}1-y\ge 0\\ \text{eta}\\ |x|+|y-1|-1\ge 0\end{cases}\\ \text{edo}\\ \begin{cases}1-y\le 0\\ \text{eta}\\ |x|+|y-1|-1\le 0\end{cases}\end{cases} \quad\Leftrightarrow\quad \begin{cases}\begin{cases}y\le 1\\ \text{eta}\\ |x|+|y-1|\ge 1\end{cases}\\ \text{edo}\\ \begin{cases}y\ge 1\\ \text{eta}\\ |x|+|y-1|\le 1\end{cases}\end{cases}$
- $x^2+1-y>0 \quad\Leftrightarrow\quad y<x^2+1$
- $y+x^2>0 \quad\Leftrightarrow\quad y>-x^2$

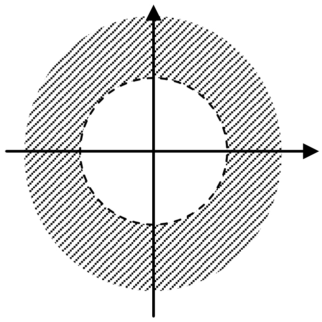


13.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\frac{\mathrm{L}(x^2+y^2-3)}{\sqrt{xy-1}}+|y|\sqrt{\frac{x^2-y^2}{x^2+y^2}}$$

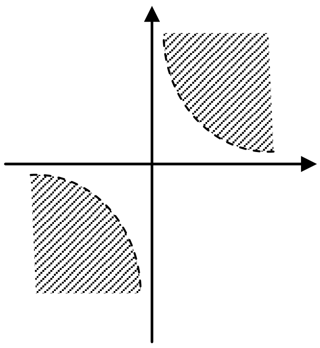
$$D=\left\{(x,y)\in\mathbb{R}^2 \,/\, x^2+y^2-3>0,\ xy-1>0,\ \frac{x^2-y^2}{x^2+y^2}\geq 0,\ x^2+y^2\neq 0\right\}$$

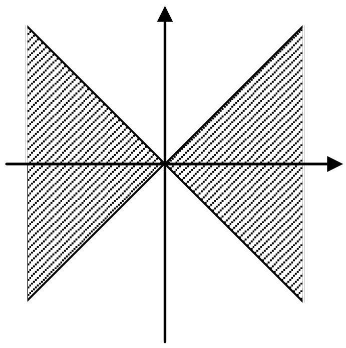
$x^2+y^2-3>0 \Leftrightarrow x^2+y^2>3$



$xy-1>0 \Leftrightarrow xy>1$

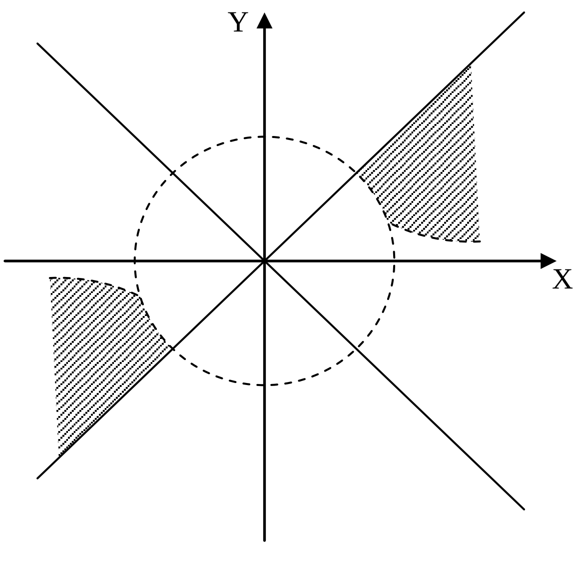
$\frac{x^2-y^2}{x^2+y^2}\geq 0 \Leftrightarrow x^2-y^2\geq 0 \Leftrightarrow x^2\geq y^2 \Leftrightarrow |x|\geq|y|$





$x^2+y^2\neq 0 \Leftrightarrow (x,y)\neq(0,0)$

Eta zati guztien arteko ebakidura, honako definizio-eremua emango digu:



14.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

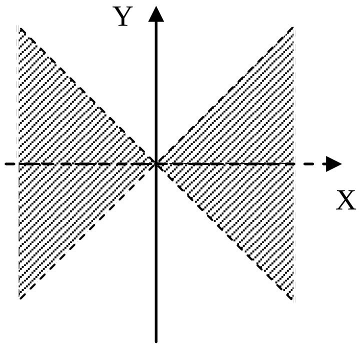
$$f(x,y)=\frac{1}{\mathrm{L}\left(\dfrac{x^2+y^2}{x^2-y^2}\right)}+\sqrt{1-x^2-y^2}$$

$$D=\left\{(x,y)\in\mathbb{R}^2 \,/\, \mathrm{L}\left(\frac{x^2+y^2}{x^2-y^2}\right)\neq 0, \frac{x^2+y^2}{x^2-y^2}>0, x^2-y^2\neq 0, 1-x^2-y^2\geq 0\right\}$$

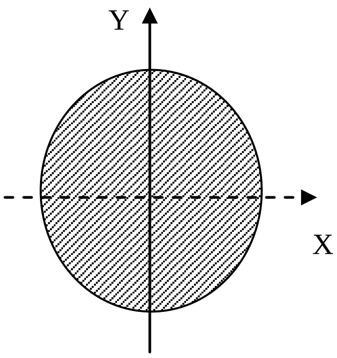
$$\mathrm{L}\left(\frac{x^2+y^2}{x^2-y^2}\right)\neq 0 \Leftrightarrow \frac{x^2+y^2}{x^2-y^2}\neq 1 \Leftrightarrow x^2+y^2\neq x^2-y^2 \Leftrightarrow y\neq 0$$

$$\frac{x^2+y^2}{x^2-y^2}>0 \Rightarrow \begin{cases} (x,y)\neq(0,0) \\ x^2-y^2>0 \Leftrightarrow |x|>|y| \end{cases}$$

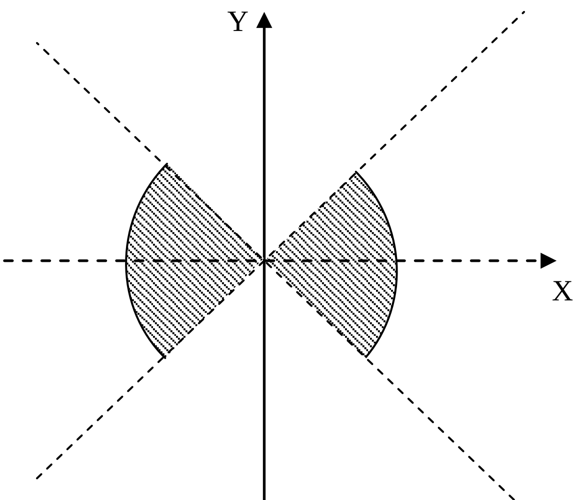
$$x^2-y^2\neq 0 \Leftrightarrow |x|\neq|y|$$



$$1-x^2-y^2\geq 0 \Leftrightarrow x^2+y^2\leq 1$$



Eta zati guztien ebakidura honako eremu hau ematen digu:



15.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

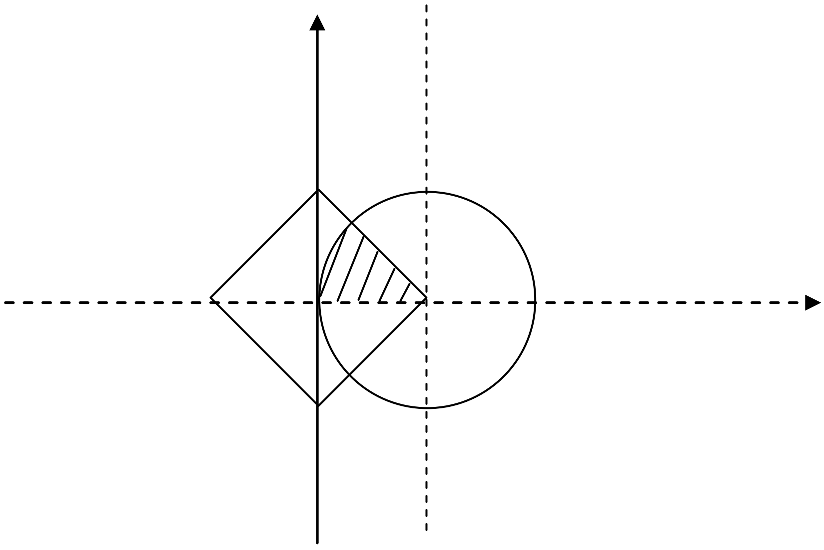
$$f(x,y)=\mathrm{L}(y\cdot(1-x))+\arcsin\left(|x|+|y|\right)+\sqrt{1-e^{x^2+y^2-2x}}$$

$$D=\left\{(x,y)\in\mathbb{R}^2\,/\,y\cdot(1-x)>0,\ -1\le|x|+|y|\le 1,\ 1-e^{x^2+y^2-2x}\ge 0\right\}$$

- $y\cdot(1-x)>0\Rightarrow\begin{cases} y>0 \ \wedge\ 1-x>0 \ \Leftrightarrow\ y>0 \ \wedge\ x<1 \\ y<0 \ \wedge\ 1-x<0 \ \Leftrightarrow\ y<0 \ \wedge\ x>1 \end{cases}$

- $-1\le|x|+|y|\le 1 \ \Leftrightarrow\ |x|+|y|\le 1$

- $1-e^{x^2+y^2-2x}\ge 0 \ \Leftrightarrow\ e^{x^2+y^2-2x}\le 1 \ \Leftrightarrow\ x^2+y^2-2x\le 0 \ \Leftrightarrow\ (x-1)^2+y^2\le 1$

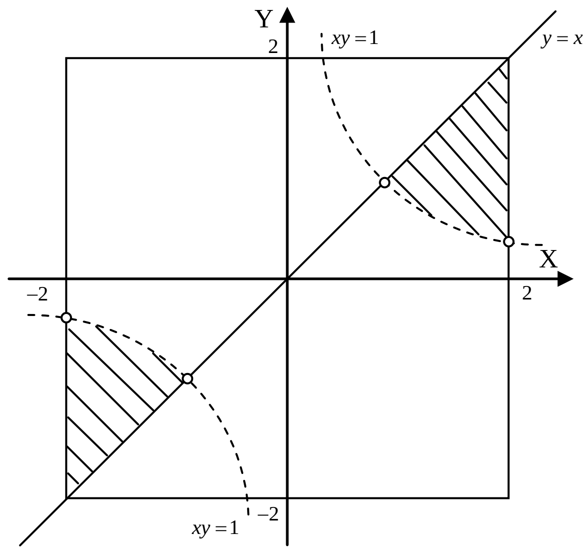


16.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y) = \mathrm{L}(xy-1) + \arcsin\left(\frac{x}{2}\right) + \arccos\left(\frac{y}{2}\right) + \sqrt{x^2 - xy}$$

$$D = \left\{(x,y) \in \mathbb{R}^2 \,/\, xy - 1 > 0,\ -1 \le \frac{x}{2} \le 1, -1 \le \frac{y}{2} \le 1,\ x^2 - xy \ge 0\right\}$$

- $xy - 1 > 0 \quad \Leftrightarrow \quad xy > 1$
- $-1 \le \frac{x}{2} \le 1 \quad \Leftrightarrow \quad -2 \le x \le 2$
- $-1 \le \frac{y}{2} \le 1 \quad \Leftrightarrow \quad -2 \le y \le 2$
- $x^2 - xy = x(x-y) \ge 0 \quad \Rightarrow \quad \begin{cases} x \ge 0 & \wedge \quad x \ge y \\ x \le 0 & \wedge \quad x \le y \end{cases}$



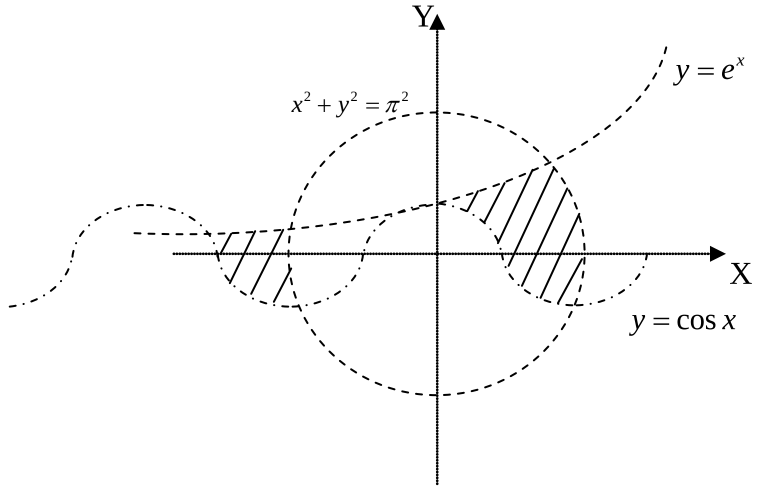
17.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\frac{\mathrm{L}(x\cdot(\pi^2-x^2-y^2))}{\sqrt{e^x-y}\cdot\sqrt{y-\cos x}}\cdot\arctan\left(\frac{x}{y}\right)$$

$$D=\left\{(x,y)\in\mathbb{R}^2\,/\,x\cdot(\pi^2-x^2-y^2)>0,\ e^x-y>0,\ y-\cos x>0,\ y\neq 0\right\}$$

$$\left.\begin{array}{lll} e^x-y>0 & \Leftrightarrow & y<e^x \\ y-\cos x>0 & \Leftrightarrow & y>\cos x \end{array}\right\}\Rightarrow \quad \cos x<y<e^x$$

$$x\cdot(\pi^2-x^2-y^2)>0 \quad\Rightarrow\quad \left\{\begin{array}{lll} x>0 & \wedge & \pi^2-x^2-y^2>0 \\ & \vee\Leftrightarrow & \\ x<0 & \wedge & \pi^2-x^2-y^2<0 \end{array}\right. \quad \vee \quad \left\{\begin{array}{lll} x>0 & \wedge & x^2+y^2<\pi^2 \\ & & \\ x<0 & \wedge & x^2+y^2>\pi^2 \end{array}\right.$$



18.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

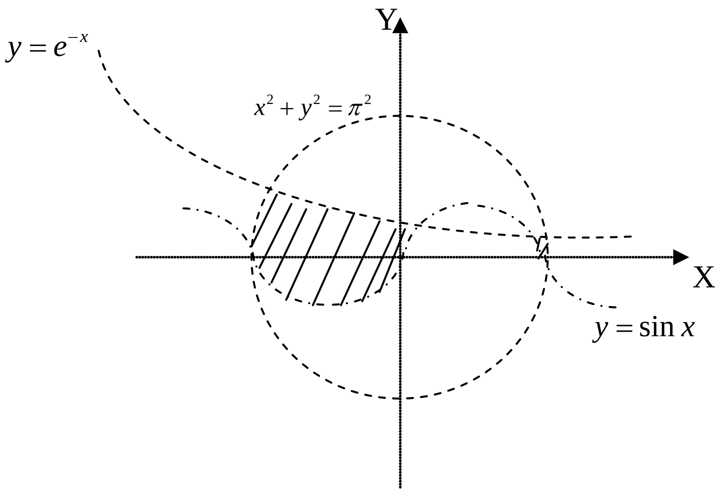
$$f(x,y)=\frac{L(y-\sin x)}{xy\sqrt{e^{-x}-y}}+\arctan\left(\sqrt{\pi^2-x^2-y^2}\right)$$

$$D=\left\{(x,y)\in\mathbb{R}^2 \,/\, y-\sin x>0,\ e^{-x}-y>0,\ xy\neq 0,\ \pi^2-x^2-y^2\geq 0\right\}$$

$$\left.\begin{array}{l} e^{-x}-y>0 \quad\Leftrightarrow\quad y<e^{-x} \\ y-\sin x>0 \quad\Leftrightarrow\quad y>\sin x \end{array}\right\}\Rightarrow\quad \sin x<y<e^{-x}$$

$$\pi^2-x^2-y^2\geq 0 \quad\Leftrightarrow\quad x^2+y^2\leq\pi^2$$

$$xy\neq 0 \quad\Rightarrow\quad x\neq 0,\quad y\neq 0$$



19.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

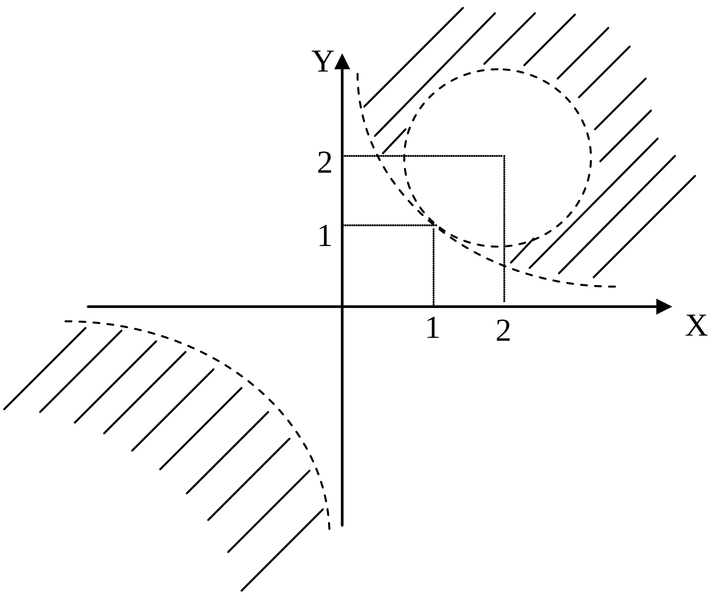
$$f(x,y)=\mathrm{L}\left[(x^2+y^2-4x-4y+6)\cdot(xy-1)\right]$$

$$D=\left\{(x,y)\in\mathbb{R}^2\,/(x^2+y^2-4x-4y+6)\cdot(xy-1)>0\right\}$$

$$(x^2+y^2-4x-4y+6)\cdot(xy-1)>0 \quad\Rightarrow\quad \begin{cases} x^2+y^2-4x-4y+6>0 \quad\wedge\quad xy-1>0 \\ \vee \\ x^2+y^2-4x-4y+6<0 \quad\wedge\quad xy-1<0 \end{cases} \quad\Leftrightarrow$$

(*) $x^2+y^2-4x-4y+6=0 \quad\Leftrightarrow\quad (x-2)^2+(y-2)^2=2$

$$\overset{(*)}{\Leftrightarrow}\quad \begin{cases} (x-2)^2+(y-2)^2>2 \quad\wedge\quad xy>1 \\ (x-2)^2+(y-2)^2<2 \quad\wedge\quad xy<1 \quad\Leftrightarrow\quad \varnothing \end{cases}$$



20.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

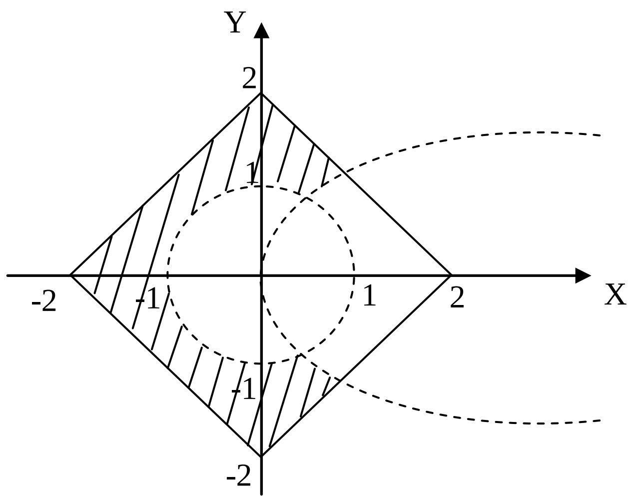
$$f(x,y)=\frac{\mathrm{L}\left(y^2-x\right)}{\sqrt{x^2+y^2-1}}+\arcsin\left(|x|+|y|-1\right)$$

$$D=\left\{(x,y)\in\mathbb{R}^2 \,/\, y^2-x>0,\ x^2+y^2-1>0,\ -1\le|x|+|y|-1\le1\right\}$$

$y^2-x>0 \quad\Leftrightarrow\quad x<y^2$

$x^2+y^2-1>0 \quad\Leftrightarrow\quad x^2+y^2>1$

$-1\le|x|+|y|-1\le1 \quad\Leftrightarrow\quad |x|+|y|\le 2$

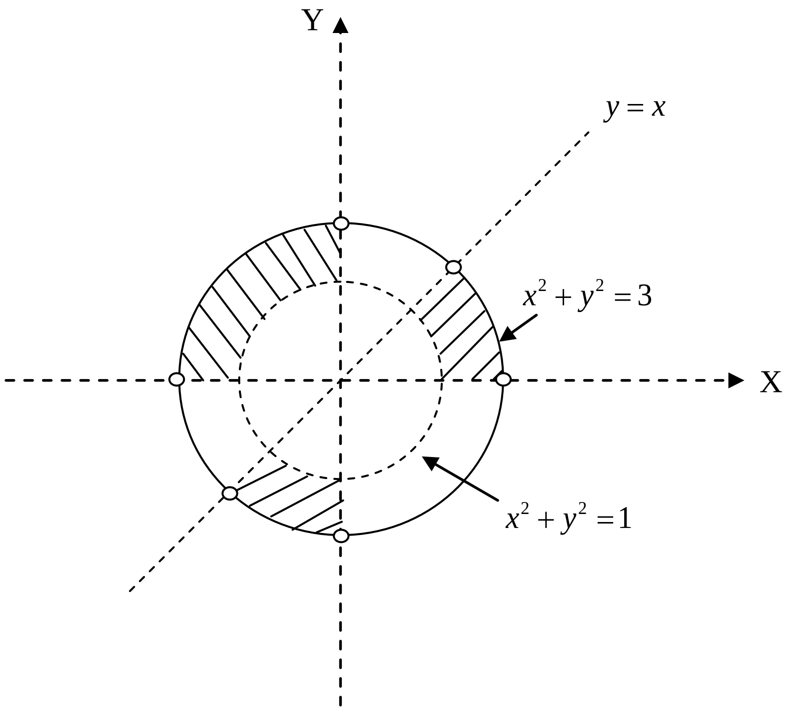


21.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\mathrm{L}\left(\frac{xy}{x-y}\right)+\frac{\sqrt{3-x^2-y^2}}{\sqrt{x^2+y^2-1}}$$

$$D=\left\{(x,y)\in\mathbb{R}^2 / \frac{xy}{x-y}>0, x-y\neq 0, 3-x^2-y^2\geq 0, x^2+y^2-1>0\right\}$$

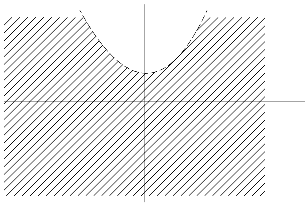
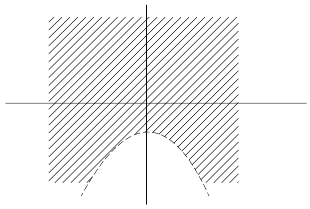
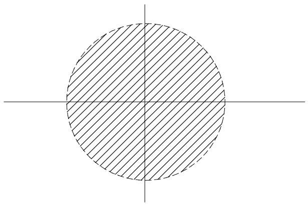
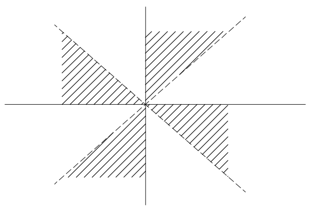
- $\dfrac{xy}{x-y}>0 \iff \begin{cases} x-y>0 \quad \wedge \quad xy>0 \\ \vee \\ x-y<0 \quad \wedge \quad xy<0 \end{cases} \iff \begin{cases} x>y \quad \wedge \quad \begin{cases} x>0\wedge y>0 \\ \vee \\ x<0 \wedge y<0 \end{cases} \\ \vee \\ x<y \quad \wedge \quad \begin{cases} x>0 \wedge y<0\ \# \\ \vee \\ x<0\wedge y>0 \end{cases} \end{cases}$
- $x-y\neq 0 \iff x\neq y$
- $3-x^2-y^2\geq 0 \iff x^2+y^2\leq 3$
- $x^2+y^2-1>0 \iff x^2+y^2>1$



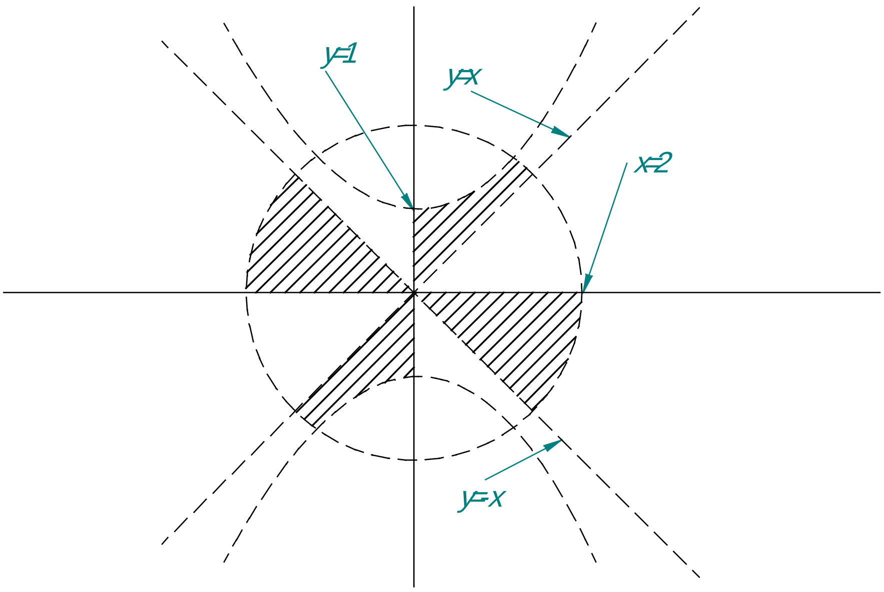
22.- Aurkitu analitiko eta grafikoki honako funtzio honen definizio-eremua:

$$f(x,y)=\frac{\mathrm{L}(x^2-y+1)+\mathrm{L}(x^2+y+1)}{\sqrt{4-x^2-y^2}}+\mathrm{L}\left(\frac{|y|-|x|}{xy}\right)$$

$$D=\left\{(x,y)\in\mathbb{R}^2 \,/\, x^2-y+1>0,\ x^2+y+1>0,\ 4-x^2-y^2>0,\ \frac{|y|-|x|}{xy}>0\right\}$$

a) $x^2-y+1>0 \Rightarrow y<x^2+1 \Rightarrow$ 	b) $x^2+y+1>0$ 
c) $4-x^2-y^2>0$ 	d) $\frac{\|y\|-\|x\|}{xy}>0 \Rightarrow \begin{cases}\begin{cases}\|y\|-\|x\|>0 \\ xy>0 \Rightarrow \begin{cases} x>0 \;\; y>0 \\ x<0 \;\; y<0\end{cases}\end{cases} \\ \begin{cases}\|y\|-\|x\|<0 \\ xy<0 \Rightarrow \begin{cases} x>0 \;\; y<0 \\ x<0 \;\; y>0\end{cases}\end{cases}\end{cases} \Rightarrow$ 

Aurreko zatien ebakidurak, beraz, definizio-eremua ematen digu:

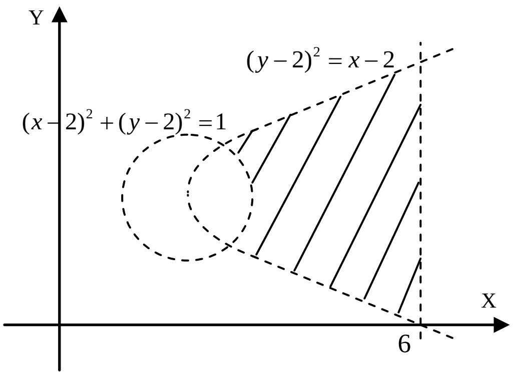


23.- Aurkitu analitiko eta grafikoki honako funtzio honen definizio-eremua:

$$f(x,y)=\frac{L(x^2+y^2-4x-4y+7)+L(6-x)}{\sqrt{x-2-(y-2)^2}}$$

$$D=\left\{(x,y)\in\mathbb{R}^2 \,/\, x^2+y^2-4x-4y+7>0,\ 6-x>0,\ x-2-(y-2)^2>0\right\}$$

$x^2+y^2-4x-4y+7>0 \Leftrightarrow (x-2)^2+(y-2)^2>1$
$6-x>0 \Leftrightarrow x<6$
$x-2-(y-2)^2>0 \Leftrightarrow (y-2)^2<x-2$



24.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\sqrt{2-|x|-|y|}+\sqrt{\sin\left[\pi\left(x^2+y^2\right)\right]}$$

$$D=\left\{(x,y)\in\mathbb{R}^2 / 2-|x|-|y|\geq 0 \wedge \sin\left[\pi\left(x^2+y^2\right)\right]\geq 0\right\}$$

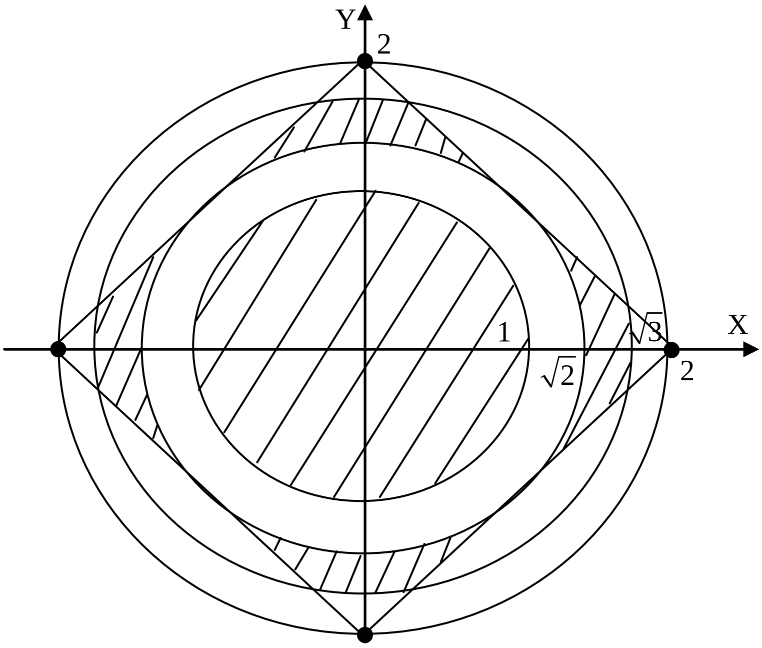
$$2-|x|-|y|\geq 0 \Leftrightarrow |x|+|y|\leq 2$$

$$\sin\left[\pi\left(x^2+y^2\right)\right]\geq 0 \Leftrightarrow \pi\left(x^2+y^2\right)\in \bigcup_{n\in\mathbb{N}\cup\{0\}}\left[2n\pi,(2n+1)\pi\right] \Leftrightarrow x^2+y^2 \bigcup_{n\in\mathbb{N}\cup\{0\}}\left[2n,2n+1\right]$$

Hau da, baldin $n=0\Rightarrow 0\leq x^2+y^2\leq 1$

baldin $n=1\Rightarrow 2\leq x^2+y^2\leq 3$

baldin $n=2\Rightarrow 4\leq x^2+y^2\leq 5$ (eskualde hau ronbotik kanpo dago, (2,0), (-2,0), (0,2) eta (0,-2) puntuak izan ezik)



Marrazturiko eskualdeak eta (2,0), (-2,0), (0,2) eta (0,-2) puntuek osatzen dute definizio-eremua.

Oharra: $\left\{\begin{array}{l} x+y=2\Rightarrow y=2-x \\ x^2+y^2=2\end{array}\right\rangle \Rightarrow x^2+(2-x)^2=2\Leftrightarrow 2x^2-4x+2=0\Leftrightarrow$

$\Leftrightarrow x=\dfrac{4\pm\sqrt{16-16}}{4}=1\Rightarrow y=1\Rightarrow(1,1)$ zuzenaren eta zirkunferentziaren arteko ukitze-puntua da.

Era berean, $\left\{\begin{array}{l} x+y=2\Rightarrow y=2-x \\ x^2+y^2=3\end{array}\right\rangle \Rightarrow x^2+(2-x)^2=3\Leftrightarrow 2x^2-4x+1=0\Leftrightarrow$

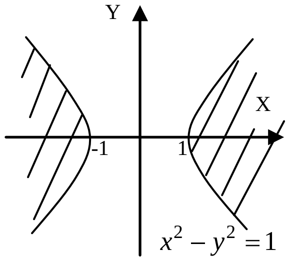
$\Leftrightarrow x=\dfrac{4\pm\sqrt{16-8}}{4}=1\pm\dfrac{\sqrt{2}}{2}\Rightarrow y=1\mp\dfrac{\sqrt{2}}{2}\Rightarrow\left(1\pm\dfrac{\sqrt{2}}{2},1\mp\dfrac{\sqrt{2}}{2}\right)$ zirkunferentziaren eta zuzenaren arteko ebaki-puntuak diren.

25.Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\frac{\sqrt{x^2-y^2-1}}{\mathrm{L}\left(2-|x|-|y|\right)}+\arccos\left(\frac{y}{|x|}\right)$$

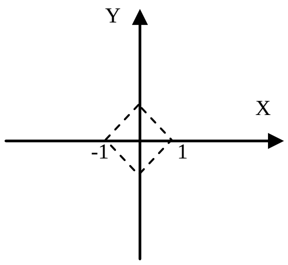
$$D=\left\{(x,y)\in\mathbb{R}^2 / x^2-y^2-1\geq 0,\ \mathrm{L}\left(2-|x|-|y|\right)\neq 0,\ 2-|x|-|y|>0,\ |x|\neq 0,\ -1\leq\frac{y}{|x|}\leq 1\right\}$$

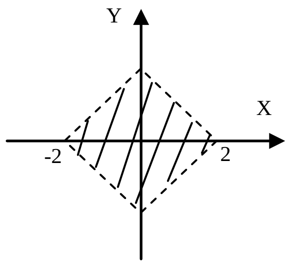
$x^2-y^2-1\geq 0 \Leftrightarrow x^2-y^2\geq 1$



$\mathrm{L}\left(2-|x|-|y|\right)\neq 0 \Leftrightarrow 2-|x|-|y|\neq 1 \Leftrightarrow |x|+|y|\neq 1$

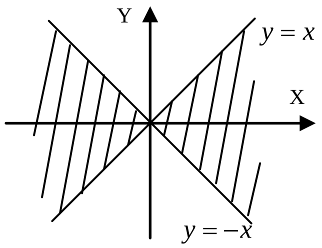
$2-|x|-|y|>0 \Leftrightarrow |x|+|y|<2$



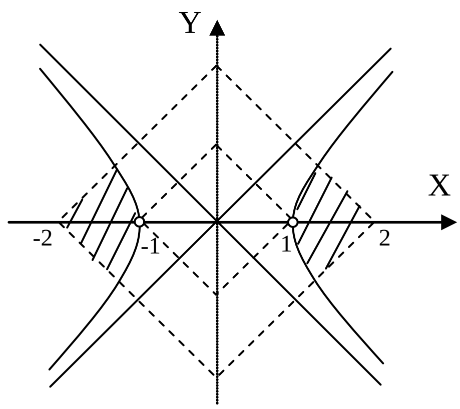


$|x|\neq 0 \Leftrightarrow x\neq 0$

$-1\leq\frac{y}{|x|}\leq 1 \Leftrightarrow -|x|\leq y\leq |x|$



Eta zati guztien ebakidura:



26.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

$$f(x,y)=\frac{\mathrm{L}(4-4x^2-y^2)+\sqrt{1-xy}}{\sqrt{-xy}}$$

$D\left\{(x,y)\in\mathbb{R}^2 \,/\, 4-4x^2-y^2>0, 1-xy\geq 0, -xy>0\right\}$

$4-4x^2-y^2>0 \Leftrightarrow x^2+\frac{y^2}{4}<1$ (elipseak mugaturiko eskualdea. Elipse bera ez dago definizio-eremuan)

$1-xy\geq 0 \Leftrightarrow xy\leq 1$ (hiperbolak eta euren bi adarren arteko eskualdea)

$-xy>0 \Leftrightarrow xy<0 \Leftrightarrow \begin{cases} x>0 \text{ eta } y<0 \text{ (4. koadrantea)} \\ y>0 \text{ eta } x<0 \text{ (2. koadrantea)} \end{cases}$