SEGIDAK

1.- Kalkulatu hurrengo segiden limiteak:

a) $\lim\limits_{n\to\infty}\dfrac{\mathrm{L}\left(\dfrac{10}{n}\right)}{\mathrm{L}(7n^2+8n)}$ b) $\lim\limits_{n\to\infty}\dfrac{\mathrm{L}\left[(n+4)!\right]}{\mathrm{L}(n!)}$

a) $$\lim_{n\to\infty}\frac{\mathrm{L}\left(\frac{10}{n}\right)}{\mathrm{L}(7n^2+8n)}\sim\lim_{n\to\infty}\frac{\mathrm{L}(10)-\mathrm{L}(n)}{\mathrm{L}(n^2)}=\lim_{n\to\infty}\frac{\mathrm{L}(10)}{2\mathrm{L}(n)}-\lim_{n\to\infty}\frac{\mathrm{L}(n)}{2\mathrm{L}(n)}=0-\frac{1}{2}=-\frac{1}{2}$$

b) $$\lim_{n\to\infty}\frac{\mathrm{L}\left[(n+4)!\right]}{\mathrm{L}(n!)}\overset{(*)}{=}\lim_{n\to\infty}\frac{\mathrm{L}\left[(n+4)!\right]-\mathrm{L}\left[(n+3)!\right]}{\mathrm{L}(n!)-\mathrm{L}\left[(n-1)!\right]}=\lim_{n\to\infty}\frac{\mathrm{L}(n+4)}{\mathrm{L}(n)}=1$$

(*) $\{\mathrm{L}(n!)\}$ segida gorakorra eta dibergentea da, beraz Stolz aplika daiteke.

2.- Kalkulatu $\lim\limits_{n\to\infty}\sqrt[n]{\dfrac{e-1}{\arcsin\left(\frac{1}{2}\right)}\cdot\dfrac{e^{1/2}-1}{\arcsin\left(\frac{1}{4}\right)}\cdot\dfrac{e^{1/3}-1}{\arcsin\left(\frac{1}{6}\right)}\cdot\ldots\cdot\dfrac{e^{1/n}-1}{\arcsin\left(\frac{1}{2n}\right)}}$

$$\lim_{n\to\infty}\sqrt[n]{\frac{e-1}{\arcsin\left(\frac{1}{2}\right)}\cdot\frac{e^{1/2}-1}{\arcsin\left(\frac{1}{4}\right)}\cdot\frac{e^{1/3}-1}{\arcsin\left(\frac{1}{6}\right)}\cdot\ldots\cdot\frac{e^{1/n}-1}{\arcsin\left(\frac{1}{2n}\right)}}\overset{(1)}{=}\lim_{n\to\infty}\frac{e^{1/n}-1}{\arcsin\left(\frac{1}{2n}\right)}\sim\lim_{n\to\infty}\frac{\mathrm{L}\left(e^{1/n}\right)}{\frac{1}{2n}}=$$

$$=\lim_{n\to\infty}2n\cdot\frac{1}{n}\cdot\mathrm{L}e=2$$

(1) Bataz besteko geometrikoaren irizpidea.

3.- Kalkulatu $\lim\limits_{n\to\infty}\left(\dfrac{3n+5}{an+2}\right)^{\frac{2n^3+1}{n^2+2}}$ **,** $a>0$ **izanik.**

$$\lim_{n\to\infty}\left(\frac{3n+5}{an+2}\right)^{\frac{2n^3+1}{n^2+2}}=\left(\frac{3}{a}\right)^{\infty}=\begin{cases}\infty & \forall a<3\\ 0 & \forall a>3\\ 1^{\infty} & a=3\end{cases}$$

Baldin $a=3 \;\Rightarrow\; l=1^{\infty}$. Logaritmoak hartuz:

$$\mathrm{L}l=\lim_{n\to\infty}\frac{2n^3+1}{n^2+2}\cdot\mathrm{L}\left(\frac{3n+5}{3n+2}\right)\sim\lim_{n\to\infty}\frac{2n^3}{n^2}\left(\frac{3n+5}{3n+2}-1\right)=\lim_{n\to\infty}2n\frac{3n+5-3n-2}{3n+2}=\lim_{n\to\infty}\frac{6n}{3n+2}=2\Leftrightarrow$$

$$\Leftrightarrow l=e^2$$

4.- Kalkulatu $\lim_{n\to\infty} 2^n \cdot \left(\frac{n+1}{2n+1}\right)^n$ **.**

$$\lim_{n\to\infty} 2^n \cdot \left(\frac{n+1}{2n+1}\right)^n = \lim_{n\to\infty}\left(\frac{2n+2}{2n+1}\right)^n = (1^\infty) = A \quad \Leftrightarrow \quad \mathrm{L}A = \lim_{n\to\infty} n\cdot \mathrm{L}\left(\frac{2n+2}{2n+1}\right) \sim$$

$$\sim \lim_{n\to\infty} n\cdot\left(\frac{2n+2}{2n+1}-1\right) = \lim_{n\to\infty} n\cdot\left(\frac{2n+2-2n-1}{2n+1}\right) = \lim_{n\to\infty}\frac{n}{2n+1} = \frac{1}{2} \quad \Leftrightarrow \quad A = e^{1/2}$$

5.- Kalkulatu $\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{\sqrt{n^2+n}-an} \qquad \forall a\in(0,1]$ **.**

$$\lim_{n\to\infty}\left(\sqrt{n^2+n}-an\right) = \lim_{n\to\infty}\frac{n^2+n-a^2n^2}{\sqrt{n^2+n}+an} = \lim_{n\to\infty}\frac{(1-a^2)n^2+n}{n\left(\sqrt{1+\frac{1}{n}}+a\right)} = \begin{cases}\frac{1}{2} & \text{baldin } a=1\\ 1^\infty & \text{baldin } a\in(0,1)\end{cases}$$

Beraz, $\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{\sqrt{n^2+n}-an} = \begin{cases}1^{1/2}=1 & \text{baldin } a=1\\ 1^\infty = A & \text{baldin } a\in(0,1)\end{cases}$

Orduan, $\forall a\in(0,1)$:

$$\mathrm{L}A = \lim_{n\to\infty}\left(\sqrt{n^2+n}-an\right)\cdot \mathrm{L}\left(1+\frac{a}{n}\right) \sim \lim_{n\to\infty}\frac{(1-a^2)n^2+n}{n\left(\sqrt{1+\frac{1}{n}}+a\right)}\cdot\frac{a}{n} \sim \lim_{n\to\infty}\frac{(1-a^2)n^2}{(1+a)n}\cdot\frac{a}{n} = a(1-a) \Leftrightarrow$$

$$\Leftrightarrow A = e^{a(1-a)}$$

6.- Kalkulatu $\lim_{n\to\infty}\frac{a^n\cdot e^{1/a}+e\cdot a^2}{a^n-3}$**,** $a>0$ **izanik.**

$$\forall a>1 \quad \lim_{n\to\infty} a^n = \infty \quad \Rightarrow \quad \lim_{n\to\infty}\frac{a^n\cdot e^{1/a}+e\cdot a^2}{a^n-3} \sim \lim_{n\to\infty}\frac{a^n\cdot e^{1/a}}{a^n} = e^{1/a}$$

$$\forall a<1 \quad \lim_{n\to\infty} a^n = 0 \quad \Rightarrow \quad \lim_{n\to\infty}\frac{a^n\cdot e^{1/a}+e\cdot a^2}{a^n-3} \sim \lim_{n\to\infty}\frac{e\cdot a^2}{-3} = -\frac{e\cdot a^2}{3}$$

Baldin $a=1 \quad \lim_{n\to\infty} a^n = \lim_{n\to\infty} 1 = 1 \quad \Rightarrow \quad \lim_{n\to\infty}\frac{a^n\cdot e^{1/a}+e\cdot a^2}{a^n-3} = \lim_{n\to\infty}\frac{e+e}{1-3} = -e$

7.- Kalkulatu $\lim\limits_{n\to\infty}\left(\frac{-1}{n}+\frac{2-\sqrt{10}}{n}+\ldots+\frac{n-\sqrt{n^2+3n}}{n}\right)$**.**

$$\lim_{n\to\infty}\left(\frac{-1}{n}+\frac{2-\sqrt{10}}{n}+\ldots+\frac{n-\sqrt{n^2+3n}}{n}\right)=\lim_{n\to\infty}\frac{-1+(2-\sqrt{10})+\ldots+(n-\sqrt{n^2+3n})}{n}\overset{(*)}{=}$$

$$=\lim_{n\to\infty}\left(n-\sqrt{n^2+3n}\right)\overset{(**)}{=}\lim_{n\to\infty}\frac{n^2-(n^2+3n)}{n+\sqrt{n^2+3n}}\sim\lim_{n\to\infty}\frac{-3n}{2n}=-\frac{3}{2}$$

(*) $\{n\}$ hertsiki gorakorra eta dibergentea da, beraz Stolz erabil daiteke.
(**) Konjokatuaz biderkatuz eta zatituz

8.- Kalkulatu $\lim\limits_{n\to\infty}\left(\sqrt[n]{n!}\right)^{\frac{7}{3n^2+2}}$**.**

$$\lim_{n\to\infty}\left(\sqrt[n]{n!}\right)^{\frac{7}{3n^2+2}}=\lim_{n\to\infty}\left(n!\right)^{\frac{7}{3n^3+2n}}=\infty^0=A\quad\Leftrightarrow\quad \mathrm{L}A=\lim_{n\to\infty}\frac{7}{3n^3+2n}\cdot\mathrm{L}\left(n!\right)\sim\frac{7}{3}\lim_{n\to\infty}\frac{\mathrm{L}\left(n!\right)}{n^3}=$$

$$\overset{(*)}{=}\frac{7}{3}\lim_{n\to\infty}\frac{\mathrm{L}\left(n!\right)-\mathrm{L}\left((n-1)!\right)}{n^3-(n-1)^3}=\frac{7}{3}\lim_{n\to\infty}\frac{\mathrm{L}\left(\frac{n!}{(n-1)!}\right)}{n^3-(n^3-3n^2+3n-1)}=\frac{7}{3}\lim_{n\to\infty}\frac{\mathrm{L}(n)}{3n^2-3n+1}\sim$$

$$\sim\frac{7}{9}\lim_{n\to\infty}\frac{\mathrm{L}(n)}{n^2}\underset{\mathrm{L}(n)\ll n^2}{=}0\quad\Leftrightarrow\quad A=e^0=1$$

(*) Stolz aplikatuz, $\{n^3\}$ hertsiki gorakorra eta dibergentea baita.

9.- Kalkulatu $\lim\limits_{n\to\infty}\frac{\left(2n+\mathrm{L}n\right)\cdot\left(2^{1/n}-1\right)}{\left(n^2+4\right)\cdot\mathrm{tg}^2\left(\frac{2}{n}\right)}$**.**

$$\lim_{n\to\infty}\frac{\left(2n+\mathrm{L}n\right)\cdot\left(2^{1/n}-1\right)}{\left(n^2+4\right)\cdot\mathrm{tg}^2\left(\frac{2}{n}\right)}\sim\lim_{n\to\infty}\frac{2n\cdot\mathrm{L}\left(2^{1/n}\right)}{n^2\cdot\frac{4}{n^2}}=\frac{1}{2}\cdot\lim_{n\to\infty}n\cdot\frac{1}{n}\mathrm{L}2=\frac{\mathrm{L}2}{2}$$

10.- Kalkulatu hurrengo limiteak:

a) $\lim_{n\to\infty}\left[\log_n(n+2)\right]^{3n\cdot \mathrm{L}n}$

b) $\lim_{n\to\infty}\left[\frac{1}{2n}+\frac{2}{3n}+...+\frac{n}{(n+1)n}\right]$

a) $\lim_{n\to\infty}\left[\log_n(n+2)\right]^{3n\cdot \mathrm{L}n}=\lim_{n\to\infty}\left[\frac{\mathrm{L}(n+2)}{\mathrm{L}n}\right]^{3n\cdot \mathrm{L}n}\underset{(\mathrm{L}(n+2)\sim \mathrm{L}n)}{=}1^{\infty}=A \quad\Leftrightarrow$

$$\Leftrightarrow \mathrm{L}A=\lim_{n\to\infty}3n\cdot \mathrm{L}n\cdot \mathrm{L}\left[\frac{\mathrm{L}(n+2)}{\mathrm{L}n}\right]\sim\lim_{n\to\infty}3n\cdot \mathrm{L}n\cdot\left[\frac{\mathrm{L}(n+2)}{\mathrm{L}n}-1\right]=\lim_{n\to\infty}3n\cdot\left[\mathrm{L}(n+2)-\mathrm{L}n\right]=$$

$$=\lim_{n\to\infty}3n\cdot \mathrm{L}\left(\frac{n+2}{n}\right)\sim\lim_{n\to\infty}3n\cdot\left(\frac{n+2}{n}-1\right)=\lim_{n\to\infty}3n\cdot\frac{n+2-n}{n}=6 \quad\Leftrightarrow\quad A=e^6$$

b) $\lim_{n\to\infty}\left[\frac{1}{2n}+\frac{2}{3n}+...+\frac{n}{(n+1)n}\right]=\lim_{n\to\infty}\frac{\frac{1}{2}+\frac{2}{3}+...+\frac{n}{n+1}}{n}\overset{(STOLZ)}{=}\lim_{n\to\infty}\frac{\frac{n}{n+1}}{n-(n-1)}=$

$$=\lim_{n\to\infty}\frac{n}{n+1}=1$$

STOLZ: $\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{a_n-a_{n-1}}{b_n-b_{n-1}}$ non, kasu honetan, $\{b_n\}=\{n\}$ hertsiki gorakorra eta dibergentea den.

11.- Kalkulatu:

a) $\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n$**, non** $a\in\mathbb{R}$ **.**

b) $\lim_{n\to\infty}\frac{\mathrm{L}(1\cdot 2\cdot 3\cdot\ldots\cdot n)}{1+2+3+......+n}$

a) Baldin $a=0 \quad\Rightarrow\quad \lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}1^n=1$

Baldin $a\neq 0 \quad\Rightarrow\quad \lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=1^{\infty}=A \quad\Leftrightarrow\quad \mathrm{L}A=\lim_{n\to\infty}n\cdot \mathrm{L}\left(1-\frac{a}{n}\right)\sim$

$$\sim \lim_{n\to\infty} n\cdot\left(-\frac{a}{n}\right)=-a \quad \Leftrightarrow \quad A=e^{-a}$$

b) $\lim_{n\to\infty}\frac{\mathrm{L}(1\cdot 2\cdot 3\cdot\ldots\cdot n)}{1+2+3+......+n}=\lim_{n\to\infty}\frac{\mathrm{L}1+\mathrm{L}2+\mathrm{L}3+\ldots+\mathrm{L}n}{1+2+3+......+n}=\lim_{n\to\infty}\frac{a_n}{b_n}\overset{(*)}{=}$

(*) $\{b_n\}=\{1+2+3+......+n\}$ hertsiki gorakorra eta dibergentea, orduan Stolz erabil daiteke.

$$\overset{(*)}{=}\lim_{n\to\infty}\frac{a_n-a_{n-1}}{b_n-b_{n-1}}=\lim_{n\to\infty}\frac{\mathrm{L}n}{n}\overset{(\mathrm{L}n\ll n)}{=}0$$

12.- Kalkulatu $\lim_{n\to\infty}\sqrt[n]{1^3+3^3+\ldots+(2n+1)^3}$

Bi eratan egin daiteke.

1. modura:

$$\lim_{n\to\infty}\sqrt[n]{1^3+3^3+\ldots+(2n+1)^3}\overset{(Z-E)}{=}\lim_{n\to\infty}\frac{1^3+3^3+\ldots+(2n+1)^3}{1^3+3^3+\ldots+(2n-1)^3}\overset{(STOLZ)}{=}\lim_{n\to\infty}\frac{(2n+1)^3}{(2n-1)^3}=1$$

Z-E: $a_n=1^3+3^3+\ldots+(2n+1)^3>0 \quad \forall n \Rightarrow \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\frac{a_n}{a_{n-1}}$

Stolz: $b_n=1^3+3^3+\ldots+(2n-1)^3>0 \quad \forall n,$ hertsiki gorakora eta dibergentea $\Rightarrow$

$$\Rightarrow \quad \lim_{n\to\infty}\frac{a_n}{b_b}=\lim_{n\to\infty}\frac{a_n-a_{n-1}}{b_b-b_{n-1}}$$

2. modura:

$$\lim_{n\to\infty}\sqrt[n]{1^3+3^3+\ldots+(2n+1)^3}=\infty^0=A \quad \Leftrightarrow \quad \mathrm{L}A=\lim_{n\to\infty}\frac{\mathrm{L}\left(1^3+3^3+\ldots+(2n+1)^3\right)}{n}\overset{(STOLZ)}{=}$$

$$=\lim_{n\to\infty}\left[\mathrm{L}\left(1^3+3^3+\ldots+(2n+1)^3\right)-\mathrm{L}\left(1^3+3^3+\ldots+(2n-1)^3\right)\right]=$$

$$=\lim_{n\to\infty}\mathrm{L}\left(\frac{1^3+3^3+\ldots+(2n+1)^3}{1^3+3^3+\ldots+(2n-1)^3}\right)=\mathrm{L}\left(\lim_{n\to\infty}\frac{1^3+3^3+\ldots+(2n+1)^3}{1^3+3^3+\ldots+(2n-1)^3}\right)\overset{(*)}{=}\mathrm{L}(1)=0 \quad \Leftrightarrow \quad A=e^0=1$$

Stolz: Kasu honetan $b_n=n>0 \quad \forall n,$ hertsiki gorakora eta dibergentea.

(*) Hemendik aurrera 1. modura bezala jarraitzen dugu.