CEDYA 2024 - Communication proposal Section: PLENARY TALK

On compressible fluid flows driven by random data

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Abstract:

In computational fluid dynamics, stochastic collocation or the Monte Carlo methods are typically used to quantify the propagation of data uncertainty. Despite the large popularity of these methods, their rigorous convergence analysis for compressible fluid flows was missing in general.

In this talk, I will review our recent results obtained for the random compressible Euler and Navier-Stokes systems, [1]-[6]. We suppose that the initial and boundary data as well as model parameters, such as the viscosity coefficients, are random variables. Consequently, a solution of the PDE system will be a random process. The stochastic collocation or the Monte Carlo methods are combined with a suitable deterministic discretization scheme, such as a finite volume method. Since the compressible Navier–Stokes and the Euler equations are not uniquely solvable in the class of global weak solutions, we cannot apply pathwise arguments to analyze the random equations. Instead, we apply stochastic compactness arguments via the Skorokhod representation theorem and the Gyöngy–Krylov method. We study both the statistical convergence rates as well as the approximation errors.

The convergence of the deterministic Navier-Stokes or Euler system is realized via dissipative solutions [7],[8],[9]. Assuming that numerical solutions satisfy in probability suitable conditions leading to a global regular solution, we prove that the Monte Carlo finite volume method as well as the stochastic collocation finite volume method converge to a statistical strong solution. The convergence rates of the finite volume and statistical methods are discussed as well [3],[9]. Numerical experiments will illustrate theoretical results [4, 5, 6].

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