High relative accuracy computations through Newton polynomial bases

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Abstract: The resolution of interpolation or approximation problems usually requires algebra computations related to different types of matrices. Sometimes, these matrices are notoriously ill-conditioned and so, the standard routines implementing best traditional numerical methods cannot keep numerical errors under control, providing inaccurate solutions. So, an important goal in the field of Numerical Linear Algebra is to achieve algorithms to high relative accuracy (HRA), which implies a great accuracy since the relative errors in the computations have the same order as the machine precision and this accuracy is not affected by the dimension or the conditioning of the problem to be solved.

The class of totally positive matrices has been extensively studied (see [1, 2]) and attracts much interest in several fields of mathematics and their applications, including approximation theory, combinatorics, computer-aided geometric design, or economics. Over the past years, many researchers have been concentrating on achieving accurate numerical solutions for ill-conditioned algebraic problems with totally positive matrices. In fact, many efforts have been devoted to finding a bidiagonal factorization of such matrices, since its accurate computation allows us to numerically solve relevant algebraic problems to HRA (see [3], [5], [7]).

In this presentation, we shall focus on the linear transformation between the monomial basis and the Newton basis corresponding to a sequence of interpolation nodes. The total positivity property of the matrices for the change of basis will be examined and efficient algorithms proposed for the HRA resolution of related algebraic problems.

Several interesting applications will be showcased, including the HRA calculation of the divided differences for the computation of the Newton form of the Lagrange interpolant [4]. Furthermore, Stirling matrices can be considered as particular cases of the above mentioned change of basis matrices. The algorithms for the HRA resolution of algebraic problems with collocation and wronskian matrices of Touchard polynomial bases derived in [6] will be also shown.

References:

[1] Ando, T., 1987, Totally positive matrices, Linear Algebra Appl. 90 pp.165–219.

[2] Gasca, M. and Peña, J.M., On Factorizations of Totally Positive Matrices, in: Gasca M and Michelli CA (Eds.), Total Positivity and Its Applications (Jaca, 1994), Kluwer Academic Publishers, Dordrecht, 1996, pp. 109–130.

[3] Delgado, J., Orera, H. and Peña J.M., 2019. Accurate algorithms for Bessel matrices, J. Sci. Comput. **80** 1264–1278.

[4] Khiar, Y., Mainar, E., Royo-Amondarain, E. and Rubio, B., 2024. On the accurate computation of the Newton form of the Lagrange interpolant. Numer Algor.

[5] Demmel, J. and Koev, P., 2005. The accurate and efficient solution of a totally positive generalized Vandermonde linear system, SIAM J. Matrix Anal. Appl. 27 pp.42–52.

[6] Mainar, E., Peña J.M. and Rubio, B., 2024. High relative accuracy through Newton bases. Numer Algor 95, pp.747–772.

[7] Marco, A. and Martínez J.J., 2013. Accurate computations with totally positive Bernstein-Vandermonde matrices, Electron. J. Linear Algebra 26 pp.57–380.