OPTICAL FIBER TECHNOLOGY **3,** 162–167 (1997)
ARTICLE NO. OF970212

Theoretical Analysis of the Torsion-Induced Optical Effect in a Plastic Optical Fiber

J. Zubia and J. Arrue

Departamento de Electrónica y Telecomunicaciones, ETSII y IT, Euskal Herriko Unibertsitatea-Universidad del Pais Vasco, Bilbao, Spain

and

A. Mendioroz

Departamento de Física Aplicada I, ETSII y IT, Euskal Herriko Unibertsitatea-Universidad del Pais Vasco, Bilbao, Spain

Received August 21, 1996; revised January 2, 1997

We have analyzed the optical effects induced by a torsion stress in a plastic optical fiber (POF). As a result of the torsion, the POF becomes biaxial and inhomogeneous. The new principal axes of the optical indicatrix and the new indices of refraction referred to them depend on the torsion strength, on the fiber transverse coordinates, and on the difference between the two components of the stress-optical tensor [q]. The changes in the indices of refraction are expected to be slightly smaller than those provoked by tensile or bending stresses.

1. INTRODUCTION

The world of fiber optics has attracted a lot of attention since its discovery as a transmission medium in the 1960s. The reason for this interest lies not only in the application of this medium to communication systems but also in the many fundamental aspects which make this medium attractive from a basic point of view.

Nevertheless, until a few years ago, most of the studies were focused on conventional optical fibers. Recently, a new type of fiber, the plastic optical fiber (POF), was discovered, which, although it displays worse characteristics than its silica counterpart, is easier to handle and cheaper [1–3]; it is suitable for small distance links, even at high speed data rates with the new gradual POFs [4].

For these reasons, many efforts have been devoted to the study of properties of POFs, from both fundamental and practical points of view. However, despite the enormous work, some minor aspects of POFs optical properties, for example, the photoelastic or stress-optical effect, require a systematic study [5].

The purpose of this paper is to give some insight into the stress-optical effects with POFs. More particularly, we have made a theoretical analysis of the torsion-induced optical effect. When a POF is subjected to a torsion, an anisotropy of its optical properties is generated. This anisotropy depends on the structure, geometry, and physical properties of the fiber, as well as on the torsion force. Second-order effects associated with changes in temperature inside the fiber have been disregarded. The optical effects caused by bending and tensile stresses on the fiber have also been included in this paper for the sake of completeness.

This paper begins with an introductory theoretical analysis in which the basis of the photoelastic effect in POFs is explained. This analysis will be particularized to three different kinds of stresses, namely, torsion, tensile or uniaxial stress, and bending. Some tedious and trivial calculus will be omitted in this section and will be exposed more carefully in the Appendix, in order to maintain the fluency of the exposition. In the next section we will discuss the most relevant features of this analysis, comparing the differences between and similarities of those stresses. Finally, the main conclusions of this work will be summarized.

2. THEORETICAL BACKGROUND

The theoretical analysis in this section is based on some essential assumptions about the structure and properties of a POF. We will suppose that an undisturbed POF is an isotropic, transparent, and homogeneous medium, so we will not consider the cladding. Then, our results will be valid only for step-index POFs. It is obvious that the production process induces anisotropy [6], as we will show later, with some minor differences between the optical behavior along the axis of the fiber and that along the axis perpendicular to it. However, these differences will be neglected at this stage.

1068-5200/97 $25.00

As is well known, the stress can be characterized by a symmetric second-rank tensor $[\sigma]$ with six independent components σ_i $(i = 1, \dots, 6)$. We will use the common contracted notation [7] to number the elements of $[\sigma]$. The photoelastic or stress-optical effect is then described up to first order in the stresses by the relation

$$\Delta B_i = q_{ij}\sigma_j, \tag{1}$$

where ΔB_i $(i = 1, \dots, 6)$ is a symmetric second-rank tensor which represents the changes of coefficients in the optical indicatrix or index ellipsoid under the action of applied stresses. The stress-optical tensor, q_{ij}, is a fourth-rank tensor with 36 components $(i, j = 1, \dots, 6)$. As before, an abbreviated notation has been used. A more detailed explanation of the meaning and the notation of this tensor can be found in any elementary book (see, for example, Ref. [7]).

When the symmetry of an isotropic medium such as a POF is taken into account, the number of independent elements in the tensor q_{ij} is finally reduced to two different ones, q_{11} and q_{12}, arranged as

$$[q] = \begin{bmatrix} q_{11} & q_{12} & q_{12} & 0 & 0 & 0 \\ q_{12} & q_{11} & q_{12} & 0 & 0 & 0 \\ q_{12} & q_{12} & q_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{11}-q_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & q_{11}-q_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & q_{11}-q_{12} \end{bmatrix}. \tag{2}$$

Next, we will calculate the relative distortion of the optical indicatrix ΔB_i in our POF, originated by three different stresses: torsion, uniaxial or tensile stress, and bending.

2.1. Effect of a Fiber Torsion

The geometry of the problem is illustrated in Fig. 1. The light travels along the fiber by means of the total refraction at the core–cladding interface. Shear forces are applied at both ends of the fiber section to produce a torsion on it. In this situation, the components of the stress tensor will be [8]

$$\begin{aligned} \sigma_1 = \sigma_2 = \sigma_3 = \sigma_6 = 0; \\ \sigma_4 = \mu\tau x; \quad \sigma_5 = -\mu\tau y, \end{aligned} \tag{3}$$

where τ is the torsion angle, which is defined as $\tau = d\varphi/dz$, and μ is the shear modulus. With these components introduced in Eq. (2), only two nonzero components of the tensor ΔB_i are obtained. Those components are

$$\Delta B_4 = (q_{11} - q_{12})\mu\tau x; \quad \Delta B_5 = -(q_{11} - q_{12})\mu\tau y. \tag{4}$$

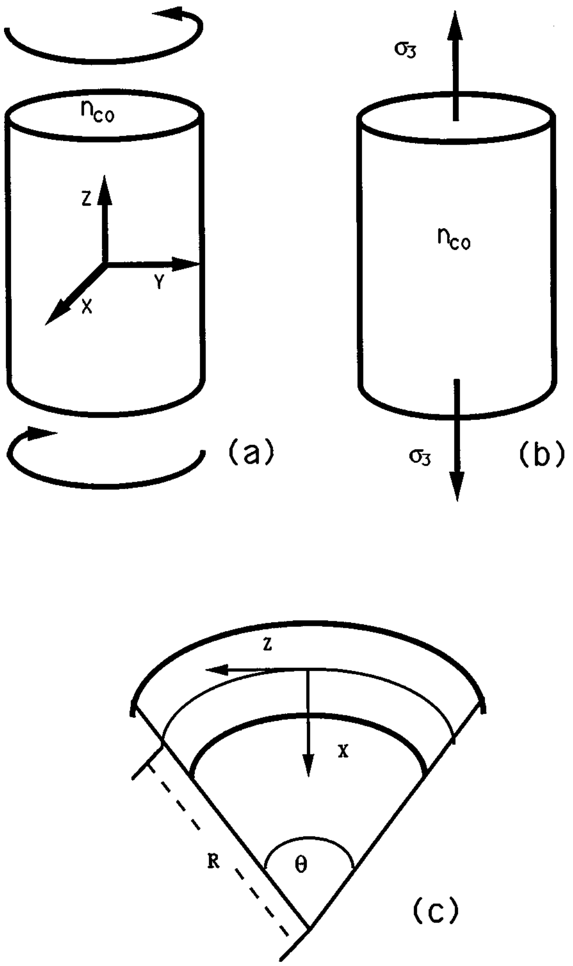

FIG. 1. Geometry of the POF under three different stress conditions. (a) Torsion of the POF by applying pure shear forces at both ends of the fiber. (b) POF subjected to a tensile stress along the fiber axis. (c) Bending of the POF. The arrows indicate the directions of the forces exerted on the POF. In the figure, the reference systems used in the text are also shown.

As is well known, an initially isotropic fiber will have a spherical indicatrix, which follows the equation

$$B(x^2 + y^2 + z^2) = 1 \quad \text{where } B = n_{co}^{-2}. \tag{5}$$

n_{co} is the uniform index of refraction of the fiber core in the absence of stress. Under a torsion stress, the optical indicatrix becomes

$$B(x^2 + y^2 + z^2) + 2xy\,\Delta B_4 + 2xz\,\Delta B_5 = 1, \tag{6}$$

so the presence of a torsion stress promotes the appearance of privileged reference systems.

2.2. *Effect of a Uniaxial Stress*

Next we will analyze the effect of a tensile stress applied parallel to the fiber axis. In this case, only the element σ_3 will be nonzero. The components of the B_{ij} tensor will change in the following way:

$$\Delta B_1 = \Delta B_2 = q_{12}\sigma_3; \quad \Delta B_3 = q_{11}\sigma_3; \quad \text{and} \quad \Delta B_4 = \Delta B_5 = \Delta B_6 = 0. \tag{7}$$

Then, the indices of refraction will increase by

$$\Delta n_x = \Delta n_y = -n_{co}^3 q_{12}\sigma_3/2; \quad \Delta n_z = -n_{co}^3 q_{11}\sigma_3/2, \tag{8}$$

which means that the POF will become birefringent, with the principal axes located parallel and perpendicular to the direction of the tensile stress. As this tensile stress is usually present in any POF-production process, POFs will be birefringent, with a preferred direction (the optical axis) along the fiber axis. Even if tensile stresses were absent, the temperature changes along the fiber in the production process would give rise to a similar effect [6].

These arguments do not significantly modify the results obtained in the previous paragraph. Equation (5) would have to be rewritten as

$$B_1(x^2 + y^2) + B_2 z^2 = 1 \quad \text{where } B_1 = (n_{co} + \Delta n_x)^{-2} \quad \text{and} \quad B_2 = (n_{co} + \Delta n_z)^{-2}. \tag{9}$$

The first term in Eq. (6) would have to be changed in the same way. Because the inclusion of these results does not contribute to clarify but rather hides the effect of a fiber torsion, we will not consider these adjustments. However, we will point out that the anisotropy induced by a tensile stress is high, around 10^{-3}–10^{-4}, which prevents us from using polarized light [5, 9].

2.3. *Effect of Fiber Bending*

The case of fiber bending is very similar to the situation previously treated. In fact, a bending stress is a uniaxial stress with all the components of the stress tensor equaling zero except for the component σ_3 [8]. If R denotes the bend radius, this stress component will be

$$\sigma_3 = -Ex/R \tag{10}$$

and the changes in the indexes of refraction

$$\Delta n_x = \Delta n_y = n_{co}^3 q_{12} xE/2R; \quad \Delta n_z = n_{co}^3 q_{11} xE/2R. \tag{11}$$

E is Young's modulus. This result corresponds to a bending stress on the xz plane as is plotted in Fig. 1. The bending stress varies linearly with the distance from the neutral surface, i.e., from the surface at which the strain is zero. The correction to the index of refraction depends on the bend radius, being insignificant for a large bend radius and becoming important for a bend radius of the order of the fiber diameter.

2.4. *Effect of the Cladding*

As a starting point, at the beginning of this section we assumed that our medium was homogeneous. This means that we are disregarding the effect of the fiber cladding. In the following, we discuss how good such an approximation is.

A commercial POF is composed of three different layers. The external one, called the jacket, serves to protect the fiber against the surrounding environment. Inside, we have the fiber proper, which is composed of a cladding and a core. The thickness of the cladding is 20 μm whereas the core is 980 μm in diameter.

Only these two last layers influence on the optical behavior of the fiber. When a fiber is subjected to a stress, only part of this stress is applied to the core. From the theory of elasticity we know, for instance, that when a torsion is applied to a cylindrical structure composed of two concentric and different materials (core and cladding), the relation between the stresses applied to each part is [10]

$$\sigma_{core}/\sigma_{cladding} = \mu_{core} d_{co}/\mu_{cladding} d_{cl}, \tag{12}$$

where μ_{core} and $\mu_{cladding}$ are the shear modulus corresponding to the core and the cladding, respectively, and d_{co} and d_{cl} are the corresponding diameters. Taking into account that the core and the cladding are made of similar polymers with similar shear modulus, and remembering that $d_{co} = 980$ μm and $d_{cl} = 1000$ μm, expression (12) tells us that the torsion stress applied to the fiber core is the same as the stress applied outside the cladding. Therefore, in commercial POFs, the effect of the cladding can be disregarded when we are studying the photoelastic properties of the fibers.

3. RESULTS AND DISCUSSION

As we have mentioned above, the applied torsion destroys, at least partially, the symmetry of the medium, giving rise to an index ellipsoid with reduced symmetry. The optical properties of the medium are easier to handle if the principal axes of the index ellipsoid are known. Then, it would be advisable to diagonalize the new tensor B_{ij}. After some trivial calculus, the eigenvectors of B_{ij} and their corresponding eigenvalues are obtained. The procedure has been included *in extenso* in the Appendix. In this section, we will only show and discuss the results.

The eigenvectors and their corresponding eigenvalues are given in Table 1. It can be seen that the z direction is no longer an optical axis of the medium. Moreover, the directions of the principal axes depend on the polar coordinate θ but not on the r and z coordinates. The eigenvector V_1 lies on the transverse plane and undergoes no radial variation. This principal axis only varies with the polar angle. On the contrary, the eigenvectors V_2 and V_3 have a component along the z direction. As it also happens with the vector V_1, the eigenvectors V_2 and V_3 remain invariant along the radial direction.

In Table 1, the eigenvalues of the index ellipsoid are also shown. They differ from the nonstress index of refraction n_{co} by a correction factor $n_{co}^2(q_{11} - q_{12})\mu\tau r/2$, which is much smaller than the unity and which depends only on the radial coordinate. Along the fiber axis $r = 0$, the index ellipsoid becomes a sphere, that is, $n_1, n_2, n_3 = n_{co}$, so this axis will be isotropic. Going far away from the fiber axis, the index ellipsoid becomes more and more asymmetric and the fiber more birefringent. At the core−cladding interface, the difference between the refractive indices n_2 and n_3 is maximum. This fact leads to a change in the polarization state of a wave travelling along the twisted fiber. Defining the beat length as $L_p^\tau = 2\pi/\delta\varphi$, where $\delta\varphi$ is the optical phase shift of a ray at an angle θ, we find that $L_p^\tau = 4\lambda/[n_{co}^3(q_{11} - q_{12})\mu\tau\rho \tan\theta]$. In this expression ρ stands for the radius of the fiber core. For a PMMA fiber [11], $n_{co} = 1.492$, $q_{11} - q_{12} \sim 10^{-11}$ m^2/N, $\lambda = 600$ nm, $\rho = 980$ μm, $\mu = 1.1 \cdot 10^9$ $\tau = 2\pi$ rad/m, $\theta = 19.43°$, the beat length is about 10^{-2} m. Therefore, disregarding other effects on the polarization state, we can affirm that the polarization state is not maintained along the fiber (in the above expression we have chosen θ to be the maximum possible value, which corresponds to the complementary critical angle; $n_{cl} = 1.407$).

At this point it is interesting to remark on some differences between the optical effects induced by torsion, bending, and tensile stresses. Table 2 shows the results corresponding to these stresses. First, tensile and bending stresses are uniaxial, since they cause the appearance of an optical axis, which in both cases is parallel to the fiber axis. In a plane perpendicular to the fiber axis, the optical response is always the same whatever the direction is. With bending stresses, however, the induced anisotropy is inhomogeneous and is a function of the distance to the neutral surface. When the effect of a torsion stress is examined, a similar conclusion is reached. Nevertheless, in this case, the directions of the principal axes also depend on the position and so the calculus involved in obtaining the ray path is much more complicated.

TABLE 2
Changes in the Indexes of Refraction Due to Tensile and Bending Stresses

Eigenvectors	Tensile stress	Bending stress
$\Delta n_x = \Delta n_y$	$-n_{co}^3 q_{12}\sigma_3/2$	$-n_{co}^3 q_{12} xE/2R$
Δn_z	$-n_{co}^3 q_{11}\sigma_3/2$	$-n_{co}^3 q_{11} xE/2R$

Note. x direction is perpendicular to the fiber axis and in the plane of the bent.

We dedicate the last part of this section to calculating the orders of magnitude of these different stresses, in order to elucidate their relative importances. Taking a typical value $q \sim 10^{-11}$ m^2/N for polymers [8], the induced change of the refractive index turns out to be $\Delta n_z = 10^{-4}$–10^{-5} when a tensile stress of magnitude $\sigma_3 = 10^7$ N/m^2 (10 Kg/mm^2, the actual section of POFs) is applied to the POF. For the bending correction to be of the same order of magnitude as the tensile stress correction, the equation $xE/R = E\delta/L$ must hold, where δ is the change in the fiber length caused by the tensile stress and L is the undisturbed length of the fiber. This equation means that for tight bends for which $x = R$, bending stresses should be given more importance since changes δ of the same order as the fiber length are unrealistic. Anyway, for the same stresses σ_3, the effect will be equal. A rough estimation of the change in the index of refraction, due to the fiber torsion, can be obtained by taking the ratio

$$\Delta n_{torsion}/\Delta n_{tensile} = \left[n_{co}^3(q_{11} - q_{12})\mu\tau\rho/2\right]/\left[n_{co}^3 q_{12}\sigma_3/2\right],$$

which, simplifying and assuming equal stresses $\mu\tau\rho = \sigma_3$, leads to $\Delta n_{torsion}/\Delta n_{tensile} = (q_{11} - q_{12})/q_{12}$. Therefore, the change in the index of refraction is expected to be one or two orders of magnitude smaller than the change caused by a uniaxial stress.

TABLE 1
Principal Axis Vectors of the Index Ellipsoid and Their Corresponding Indexes of Refraction for a POF Subjected to a Torsion Stress

Eigenvectors:	V_1	V_2	V_3
Components:	$(\cos\theta, \sin\theta, 0)$	$1/\sqrt{2}(\sin\theta, -\cos\theta, 1)$	$1/\sqrt{2}(\sin\theta, -\cos\theta, -1)$
Refractive index:	n_{co}	$n_{co}[1 - n_{co}^2(q_{11} - q_{12})\mu\tau r/2]$	$n_{co}[1 + n_{co}^2(q_{11} - q_{12})\mu\tau r/2]$

Note. r and θ stand for the usual polar coordinates, $r = (x^2 + y^2)^{1/2}$ and $\theta = tan^{-1}(y/x)$.

4. CONCLUSIONS

We have analyzed the optical effects induced by three different stresses. Uniaxial, tensile, and bending stresses turn the isotropic POF into a uniaxial optical medium with its optical axis parallel to the fiber axis. In addition, bending the fiber converts it into an inhomogeneous medium as the refractive index depends on the position as well as on the direction of the ray path through the POF. Only at the neutral surface is the optical anisotropy null. Finally, the torsion on the fiber makes the POF become a biaxial medium, with an optical anisotropy that is higher as we move out from the center of the fiber. The optical anisotropy, which is zero on the axis of the POF, depends, among other things, on the difference between the values of the tensor elements $(q_{11} - q_{12})$. So, in order to reach a better understanding of this stress-optical effect, measurements of these parameters would be interesting. For example, one tensile stress experiment would allow us to infer the parameters q_{11} and q_{12} assuming that Young's modulus E is known.

As the main conclusion, we can affirm that the torsion-induced optical anisotropy is expected to be slightly smaller than that provoked by tensile and bending stresses.

APPENDIX

The index ellipsoid follows the equation

$$B_{ij}x_ix_j = 1, \tag{13}$$

where $x_i, x_j = x, y, z$ and

$$[B] = \begin{bmatrix} \frac{1}{n_{co}^2} & 0 & -y\mu\tau(q_{11}-q_{12}) \\ 0 & \frac{1}{n_{co}^2} & x\mu\tau(q_{11}-q_{12}) \\ -y\mu\tau(q_{11}-q_{12}) & x\mu\tau(q_{11}-q_{12}) & \frac{1}{n_{co}^2} \end{bmatrix}. \tag{14}$$

Then it is seen that the torsion induces the appearance of nondiagonal elements in the index ellipsoid for our reference system. Diagonalizing this matrix we find the new principal axes. For this purpose we form the determinant

$$\begin{vmatrix} \frac{1}{n_{co}^2} - \lambda & 0 & -y\mu\tau(q_{11}-q_{12}) \\ 0 & \frac{1}{n_{co}^2} - \lambda & x\mu\tau(q_{11}-q_{12}) \\ -y\mu\tau(q_{11}-q_{12}) & x\mu\tau(q_{11}-q_{12}) & \frac{1}{n_{co}^2} - \lambda \end{vmatrix} = 0, \tag{15}$$

from which we will deduce the eigenvalues. Reducing the determinant, we obtain

$$\left(\frac{1}{n_{co}^2} - \lambda\right)\left[\left(\frac{1}{n_{co}^2} - \lambda\right)^2 - \left(\left(\mu\tau(q_{11} - q_{12})\right)^2 r^2\right) = 0\right], \tag{16}$$

and solving for λ we find three different roots of this equation which are given by

$$\begin{aligned} \lambda_1 &= \frac{1}{n_{co}^2} \\ \lambda_2 &= \frac{1}{n_{co}^2} + \mu\tau r(q_{11} - q_{12}) \\ \lambda_3 &= \frac{1}{n_{co}^2} - \mu\tau r(q_{11} - q_{12}). \end{aligned} \tag{17}$$

The next step is to obtain the associated eigenvectors by introducing the eigenvalues in the equation $B_{ij}x_j = \lambda x_i$. For example, for the first eigenvalue this relation means

$$\begin{bmatrix} \frac{1}{n_{co}^2} & 0 & -y\mu\tau(q_{11}-q_{12}) \\ 0 & \frac{1}{n_{co}^2} & x\mu\tau(q_{11}-q_{12}) \\ -y\mu\tau(q_{11}-q_{12}) & x\mu\tau(q_{11}-q_{12}) & \frac{1}{n_{co}^2} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{n_{co}^2}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \tag{18}$$

from which it follows that the corresponding unitary eigenvector is $V_1 = (\cos\theta, \sin\theta, 0)$. The eigenvectors V_2 and V_3 are calculated in the same way. Their components are given in Table 1. Finally, referring to the last three eigenvectors, we can rewrite the equation of the index ellipsoid in the principal coordinate system as

$$\frac{x_1^2}{n_{co}^2} + \left[\frac{1}{n_{co}^2} - \mu\tau r(q_{11} - q_{12})\right]x_2^2 + \left[\frac{1}{n_{co}^2} + \mu\tau r(q_{11} - q_{12})\right]x_3^2 = 1. \tag{19}$$

ACKNOWLEDGMENT

This work is supported by the Departamento de Educación, Universidades e Investigación del Gobierno Vasco under Project UE96/38.

REFERENCES

[1] T. Kaino and K. Katayama, ''Polymer for optoelectronics,'' *Polymer Eng. Sci.*, vol. 29, 1209 (1989).

[2] D. Hanson, ''Wiring with plastic,'' *IEEE Lightwave Technol. Sci.*, 34 (1992).

[3] T. Kaino, *Polymers for Lightwave and Integrated Optics*, Dekker, New York, 1992.

[4] T. Ishigure, E. Nihei, Y. Koike, C. E. Forbes, L. LaNieve, R. Straff, and H. A. Deckers, ''Large core, high-bandwidth polymer optical fiber for near infrared use,'' *IEEE Photon. Technol. Lett.*, vol. 7, 403 (1995).

[5] J. Dugas, I. Pierregeau, J. Farenc, and J. P. Peichst, ''Birefringence and material stress in polystyrene optical fibers,'' *Appl. Opt.*, vol. 33, 3545 (1994).

[6] S. A. Miller and I. P. Kaminow, *Optical Fiber Communications, II*, p. 85, Academic Press, San Diego, 1988.

[7] J. F. Nye, *Physical Properties of Crystals*, p. 253, Oxford Univ. Press, Oxford, 1957.

[8] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity*, p. 93, Editorial Reverté, Barcelona, 1969.

[9] J. Zubia and J. Arrue, unpublished manuscript.

[10] Gere-Timoshenko, *Mechanics of Materials*, PWS−Kent, Boston, 1994.

[11] D. W. Van Krevelen, *Properties of Polymers*, Elsevier, Amsterdam, 298, 1990.