Generalization of Lee Method for the analysis of the signal variability


Abstract—The Lee Method, recommended by ITU and CEPT, for obtaining the local mean values of the received signal along a route, was developed for a Rayleigh distribution in UHF band. This paper describes the generalization of this method to any propagation channel and frequency band, and describes the methodology to obtain the parameters involved. The Generalized Lee Method is based on field data samples, which allows estimating the mean values without the requirement of a priori knowing the distribution function that better fits to the propagation channel. The accuracy in obtaining the averaging interval is improved too. The Generalized Lee Method is solved for ground wave propagation at MW band, taking data from field trials of a DRM transmission. The results show that the values considerably differ from those obtained for a Rayleigh channel and prove that the results allow the adequate differentiation of long-term and short-term signals. The Generalized Lee Method completes the results obtained by Lee and Parsons, and makes possible a better characterization of the spatial variability.

Index Terms—Signal variability, Propagation models, Channel estimation, Coverage prediction techniques

I. INTRODUCTION

The signal variability in the broadcasting and mobile radio services can be analyzed considering two contributing factors. One of them is the variation of the field strength mean value, as the receiver location changes. This is usually called “slow fading” or “long-term” variation. Superimposed on this slow fading, there are local instantaneous variations in the signal strength around this mean level, caused by multipath propagation in the nearby surrounding area of the receiver location. This local variation is termed “fast fading” or “short-term” variation [1], [2].

The variation of the mean value is due to large-scale terrain variations along the path profile, to the type of the environment where the receiver is located, and, in ground wave propagation, to the electrical properties of the terrain. The short-term variation of the signal is caused by scattering from man-made and natural obstacles, and is very dependent on the type of environment where the receiver is located.

The Lee Method [3]-[6] is the reference method for estimating the local mean values that form the long-term signal along a route. It is based on an averaging process applied to the envelope of the received signal, and it is recommended by the ITU [1] and CEPT [7] organizations. The short-term is obtained by taking away the long-term from the envelope of the received signal. The use of this method allows systematizing spatial variability analysis in an effective manner. Lee solved the method for radio mobile services in Rayleigh channel and UHF band.

This paper describes the generalization of the calculation of the parameters used in the Lee Method for any propagation condition and frequency band. In Section II, the Lee Method is summarized, and in Section III a bibliographic summary is presented. The Generalized Lee Method proposed in this paper is described in sections IV and V, and it is applied to a specific case in Section VI.

II. COVERAGE AREA ESTIMATION

In mobile reception, the path loss varies continuously with location. The influences that affect this variation can be classified into three main categories, and consequently, the field strength prediction in a given area can be structured in three steps [8].

First, a representative value of the field strength at the area under study is assessed by considering the distance to the transmitter and the path profile. This is the median field strength value, related to the 50% of locations within the area under study.

Then, the location variability is calculated as the percentage of locations that exceed that median field strength value within the area under study. The location variability is defined as the standard deviation of the local means in the area of study [8].

Finally, the third step should include signal variations that will occur over scales of the order of a wavelength due to the phasor addition of multipath effects [8], [9].

The procedure for estimating the mean signal strength is a critical step to develop appropriate techniques for coverage prediction and network planning. Furthermore, a quantitative measure of the signal variability around the local mean values is also essential for evaluating coverage within any given area. This is particularly relevant for digital services, where a small change in the field strength value can produce loss of availability, when the signal level is close to the threshold for good quality [10], [11].

Accordingly, a correct analysis of the spatial variability of
the signal requires a precise differentiation between the long-term variation and the short-term fading. Being the factors affecting each component different in nature, both of them can be modeled using different statistical distributions.

III. LEE METHOD FOR OBTAINING THE LOCAL MEANS OF A MOBILE RADIO SIGNAL

The mobile radio signal level is analyzed as the combined effect of the mean power level and the local variability around that mean level. A radio mobile signal envelope \( r(y) \) is composed of a short-term fading \( r_d(y) \) (or fast variation in the received signal) superimposed on a long-term fading \( m(y) \) (or local average), where \( y \) is the distance run by the receiver along the route. Relation between them can be expressed as follows:

\[
r(y) = m(y) \cdot r_d(y)
\]

(1)

The long-term signal is obtained as a series of local average values along the route, and represents the large-scale variations of the received signal. These local mean values are computed at each location by averaging the signal samples located within an interval around this location.

Once the long-term fading has been calculated, this variation is removed from the signal envelope, to obtain the short-term fading. In fact, the short-term fading \( r_d(y) \) is the signal envelope \( r(y) \) normalized with respect to the mean level \( m(y) \). By this way, slow and fast variations can be differentiated, and separately analyzed, as illustrated in Fig. 1 (a) and Fig. 1 (b).

Clarke suggested the technique of normalizing the data by way of its running mean, dividing each data point by a local mean, obtained from averaging the points symmetrically adjacent to it [12], [2].

Lee determined the necessary parameters and the methodology to calculate the local means. These parameters are: the proper length of the running average window \( (2L) \), the minimum number of samples \( (N) \), and the necessary distance between samples to be uncorrelated \( (d) \) [3]-[5]. The local means are obtained by averaging at least \( N \) field strength instantaneous values, separated a distance \( d \), and located within the window \( 2L \). The running mean provides mean values along the route, separated a distance \( d \).

The estimation of the local mean values with a delimited degree of error requires the a priori outlining of the proper values of the parameters involved in the estimation. The correct calculation of the values for these parameters will determine the accuracy in the differentiation of both types of variation (long-term and short-term signals).

A. Determining the proper length of the averaging interval

The mean value is estimated by an integration of the linear values of the signal envelope \( r(y) \) over a suitable length \( 2L \), such that the fast fluctuations within this distance are averaged.

The proper selection of the \( 2L \) value is critical in Lee Method. If \( 2L \) value is chosen too short, rapid variations of the signal \( r(y) \) will still be present in the long-term signal after the averaging process; but if \( 2L \) length is too long, long-term signal will be smoothed out and, consequently, the short-term will include part of the slow variation. Fig. 2 illustrates the influence of the averaging interval \( 2L \) on the accuracy of the estimated mean. This example shows the true mean values \( m(y) \) and the estimated values using interval lengths that are too short (\( \hat{m}_2 \)) or excessively long (\( \hat{m}_3 \)).

![Fig. 1. (a) Estimation of the long-term component of the received signal \( r(y) \). (b) Calculation of the short term component.](image1)

![Fig. 2. Influence of the averaging interval \( 2L \) on the accuracy of the estimated mean.](image2)
Estimated mean $\hat{m}(x)$ will approach to the true mean $m(x)$ when the length $2L$ is properly chosen.

$$\hat{m}(x) \to m(x)$$ (3)

Equation (3) implies that when the $2L$ length is properly chosen, the averaging of the short-term fading will be:

$$\frac{1}{2L} \int_{x-L}^{x+L} r_0(y) \cdot dy \to 1$$ (4)

Being $\hat{m}(x)$ an estimation of the true mean, the fluctuations of $\hat{m}(x)$ around $m(x)$ can be evaluated by means of the variance of the estimated mean values, as a function of the parameter $2L$. Lee assumes that $\hat{m}(x)$ is a Gaussian random variable, with mean $m(x)$, and variance $\sigma^2_m$ [5], [6]. Accordingly, the $68\%$ of the $\hat{m}(x)$ values are within the interval $(m - \sigma_m, m + \sigma_m)$ around the true mean $m(x)$ of a Gaussian distribution.

In order to choose a correct value for $2L$, Lee proposed the use of the parameter called $1\sigma_{\hat{m}}$-Spread:

$$1\sigma_{\hat{m}}\text{-}\text{Spread} = 20 \cdot \log \left( \frac{m + \sigma_m}{m - \sigma_m} \right) \text{ (dB)}$$ (5)

This parameter determines the range relative to the $68\%$ of the $\hat{m}(x)$ values, in a logarithmic scale [6]. Therefore, it allows a numerical evaluation of the fluctuations of the estimated mean, as a function of the value of $2L$. If a value of $1\sigma_{\hat{m}}$-Spread is fixed, the associated $2L$ value will be obtained, and the fluctuations of the estimated mean will remain below a certain dispersion limit. According to the proposal made by Lee, the variables $m$ and $\sigma_m$ are replaced in equation (5) by their theoretical expressions, stating a Rayleigh distribution.

In [3], Lee previously normalizes $\hat{m}(x)$ values with respect to its mean value. The normalization makes the dispersion measurement independent of the mean value. Then, the parameter $1\sigma_{\hat{m}}$-Spread can be expressed as

$$1\sigma_{\hat{m}}\text{-}\text{Spread} = 20 \cdot \log \left( \frac{1 + \sigma_{\hat{m} \text{ norm}}}{1 - \sigma_{\hat{m} \text{ norm}}} \right) \text{ (dB)}$$ (6)

The criterion adopted by Lee [3] for obtaining the proper length of $2L$ is based on the condition that the $68\%$ of the estimated mean values fall within a range of $1 \text{ dB}$ around the true mean:

$$1\sigma_{\hat{m} \text{ norm}} = 1 \text{ dB}$$ (7)

The use of the parameter called $1\sigma_{\hat{m} \text{ norm}}$-Spread has been proposed by Lee [3] for obtaining the proper length of $2L$ is based on the condition that the $68\%$ of the estimated mean values fall within a range of $1 \text{ dB}$ around the true mean. Lee determined the necessary number of samples $N$ required to estimate the mean value, within a range of $\pm 1 \text{ dB}$ around the true mean, with a $90\%$ of confidence [3]:

$$1.65 \frac{\sigma_r}{\sqrt{N}} \leq 1 \text{ dB}$$ (8)

Accordingly, the calculation procedure of the parameter $N$ uses discrete variables, and the samples $r_i$ of the envelope of the received signal $r(y)$ are considered. The true mean and the standard deviation of the samples $r_i$ are $m$ and $\sigma_r$, respectively. If the estimated mean $\hat{m}(x)$ is obtained from the received signal $r(y)$, the estimated local mean obtained from the samples $r_i$ is defined as $\hat{r}$, and calculated as follows:

$$\hat{r} = \frac{1}{N} \sum_{i=1}^{N} r_i$$ (9)

The ensemble average $r$ is always a Gaussian variable if all $N$ variables are added in linear scale, and the central limit theorem is fulfilled [13]. The mean of this Gaussian variable is $m$, and the standard deviation is $\sigma_r/\sqrt{N}$.

By standardizing $r$, it is possible to relate the minimum number of samples ($N$) and the standard deviation of the signal samples ($\sigma_r$) to the statistical criteria given by the precision of the estimated mean values: the confidence interval around the standardized true mean ($Z_r$, $+Z_r$), with a degree of certainty (2·$P$(Z)), expressed in %) [14].

$$P(-Z_r \leq z \leq +Z_r) = 2 \cdot P(-Z_1)$$ (10)

For a Gaussian variable, the $90\%$ of the values will fall within the range $m \pm 1.65\sigma_r$, and the following expression can be applied [2], [4]:

$$P(m - 1.65 \frac{\sigma_r}{\sqrt{N}} \leq \hat{r} \leq m + 1.65 \frac{\sigma_r}{\sqrt{N}}) = 90\%$$ (11)

Equation (11) is used by Lee to obtain the sample size $N$ required to estimate the mean value, within a range of $\pm 1 \text{ dB}$ around the true mean, with a $90\%$ of confidence [3]:

$$1.65 \frac{\sigma_r}{\sqrt{N}} \leq 1 \text{ dB}$$ (12)

### C. Distance $d$ between uncorrelated samples

Calculation of the local field strength mean values is based on uncorrelated samples [2], [4]. The necessary distance $d$ between uncorrelated samples of a data set corresponds to the first null of its autocorrelation coefficient. Nevertheless, Lee demonstrated that, as long as the autocorrelation coefficient is lower than 0.2, the signals can be considered as uncorrelated [4].

This criterion is assessed theoretically for a Rayleigh distribution and an omnidirectional scattering model, in which all the spatial arrival angles are equally likely. Results show that the theoretical distance $d$ is the corresponding to the first null of the zeroth order Bessel function of the first kind.
between uncorrelated sampling points considered by Lee is set to 50 (down to 36 for lower accuracy conditions) [3]-[5], and the necessary distance between uncorrelated sampling points considered by Lee is 0.8\(\lambda\), empirically obtained in [15].

When the direct wave is present, and the fading follows a Rician distribution, it is not necessary a length \(2L = 40\lambda\), but Lee recommends using the values obtained for Rayleigh environment to handle all situations [5].

Results adopted by ITU and CEPT are \((2L = 40\lambda, d = 0.8\lambda, N = 50)\) [1], [7].

Other authors have also obtained theoretical and empirical values to normalize the received signal. Parsons [2] developed a similar study, but distinguished between samples taken from a receiver with a linear characteristic and samples from a receiver with a logarithmic characteristic, always considering Rayleigh channel. The results were \((2L = 22\lambda, N = 57)\) for a linear receiver, and \((2L = 33\lambda, N = 85)\) for a logarithmic receiver. Parsons considered 0.38\(\lambda\) the necessary distance between uncorrelated samples.

Moreover, Parsons and Ibrahim [10] empirically found the adequate width of the window to normalize experimental data in urban environment. Outcomes of these tests were lengths of 42 m for 168 MHz (23.52\(\lambda\)), and 20 m for 455 MHz and 900 MHz (30.33\(\lambda\) and 60\(\lambda\), respectively).

Okumura [17] analyzed the spatial variability of the signal, with data from field trials in Tokyo, at several frequencies from 453 MHz to 1920 MHz. He divided the data records into intervals of 20 m, which corresponds to 30\(\lambda\) in 453 MHz, and 128\(\lambda\) in 1920 MHz.

Davis and Bogner empirically demonstrated that variations in the local average values appeared as the averaging distance increased above 25 m \((2L = 41.67\lambda\) at 500 MHz) [18]. This is a significant result, because it demonstrated the need of an upper limit in the averaging interval length.

Results for the parameter \(d\) show a significant difference between the theoretical study of Clarke [12], [15] \((d = 0.38\lambda)\), and the experimental analysis of Lee [15] and Rhee [16] \((d = 0.8\lambda)\).

V. DISCUSSION ON THE APPLICATION SCOPE OF THE METHOD PROPOSED BY LEE

The methodology and values summarized in the previous sections II and III, have been used largely over the last two decades of the XX century. Its application has focused largely on field measurement surveys for mobile radiocommunication systems in the VHF and UHF frequency bands, provided an ideal Rayleigh channel condition, characterized by the following conditions:

- The frequencies used by the service under study are within the UHF and VHF bands [2].
- The multipath created by reflection and refraction on different static and moving elements in the reception area is the dominant propagation mechanism [2].
- The interfering waves of multipath must be randomly varying in phase and be of near equal power [9].
- The multipath reaches the receiver with uniformly distributed spatial arrival angle [2], [4], [12]. Subsequent studies have proved that this condition is not always fulfilled, and in those cases the Weibull or Nakagami models describe more accurately the propagation channel [9], [19].
- There must be a minimum of five interfering waves [9].
- The scattering model considers that the incoming waves travel horizontally [12]. This two-dimensional model successfully explains almost all the observed properties of the signal envelope, but in practice there are differences between what is observed and what is predicted [2].

These restrictions are not always applicable when studying the statistics of a radiofrequency signal from a measurement data set.

The application of the Lee Method requires a previous knowledge of the statistical distribution function of the field strength values. In some cases, the distribution itself is precisely the aim of the study when determining the distribution function that best fits the propagation channel and its parameter values. In this case is clear the usefulness of a more generalized method, not dependent on the a priori knowledge of the distribution function.

In order to extend the application of the method proposed by Lee to a more generalized application scenarios, this paper proposes to adapt the calculation of the parameters of the Lee Method based on field data (field strength samples), instead of basing it on theoretical distribution functions. The result will
make possible the calculation of received field strength mean values along a route, independently from the reception conditions: frequency band, reception environment and propagation factors.

VI. GENERALIZATION OF THE LEE METHOD

A. Postulates of the Generalized Lee Method

The generalization of the Lee Method has the main objective of developing a procedure for calculating the adequate values of the parameters involved (averaging window $2L$, minimum number $N$ of samples, and the distance $d$ between uncorrelated samples), regardless of the propagation channel, the frequency band and the reception conditions, using field strength samples.

The determination of the distribution of the received signal is not an a priori requirement. Furthermore, the source data in the calculation process are the field strength samples. Both conditions allow the generalized method to be applicable to any situation.

Therefore, the Generalized Lee Method describes how to obtain the more adequate values for the parameters used in the estimation of the local mean at each point of a route, regardless of the signal distribution. It can be considered as a general procedure proposal to obtain these values. The correct values of these parameters will allow differentiating long-term and short-term fading occurrences of the received signals.

The basis for this study has been obtained from the widely referenced studies made by Lee [3]-[6], Parsons [2] and Clarke [10], and the empirical investigations of Rhee [16] and Davis [18].

In the following sections, the procedure for obtaining the parameters $2L$, $N$, and $d$ is described. A new methodology is proposed for the $2L$ parameter and some application recommendations are highlighted for the $N$ and $d$ parameters as described by Parsons and Lee.

B. Obtaining the distance of the averaging interval $2L$

The averaging process of the field strength samples within a $2L$ interval allows obtaining the local mean values along the route. The original proposal included a normalization using the theoretical true mean $m(x)$ resultant from a Rayleigh behavior. These two restrictions will be discussed in this section along with a proposal for overcoming them irrespective the channel model and frequency.

As detailed in Section II, the calculation proposed by Lee provides a lower limit to the proper value of the $2L$ interval. Otherwise, Davis and Bogner [18] verified empirically that a too long averaging distance causes a greater variation between the local average values. This is due to the superposition of the effect of the varying local mean on the short-term variation. The effect of either underestimating or overestimating the length of $2L$ can be appreciated in Fig. 3 and Fig. 4. In both figures, the estimated local mean values have been obtained along an averaging interval $2L$ around the point under study.

The running mean performs the successive calculation of the local average values. If the $2L$ length is properly chosen, the averaging process will compensate adequately the variations of the instantaneous data. Because of this low variation, the true mean values can be considered constant within each $2L$ interval [5], [6] (see Fig. 3). Thus, the estimated mean values will show certain limited fluctuation around the true mean, constant within $2L$ interval. The criterion to determine the proper length $2L$ is based on the delimitation of the maximum fluctuation of the estimated mean values around the true mean within each $2L$ interval, that is, the criterion to adequately average the short-term variations.

![Fig. 3. Estimation of the local mean value using a proper length $2L$.](image1)

The $2L$ value corresponding to that maximum fluctuation is the criterion used by Lee, and sets up a lower limit to the definition of the $2L$ adequate length. Lower values of the averaging window will supply with very fluctuating mean values, including part of the short-term signal, and consequently, containing a great error (see Fig. 4, where the estimated means $\bar{m}_1$ have been calculated using a too short interval).

![Fig. 4. Estimation of the local mean value using inappropriate values of $2L$.](image2)

The variation of the long-term signal is not considered by Lee, but it must be also considered for larger values of $2L$, because this criterion sets up an upper limit to the $2L$ length. In these situations, the true mean cannot be stated as constant within the averaging interval, and its variation must be taken...
into account to determine the upper limit to $2L$ distance. That is, larger values of $2L$ will average the fast fading, but in addition will smooth out the long-term signal, and part of the long-term variation will be obtained in the short-term signal. This aspect is illustrated in Fig. 4, where a too long averaging window has been used to assess the estimated means $\bar{r}_2$. The fluctuation of the results is bigger when inappropriate values of $2L$ are used obtained, as it is shown in Fig. 4, and in these situations, the mean value cannot be considered constant within the $2L$ length.

The proposal for the selection of the proper values of $2L$ interval is twofold: the calculation of an upper limit of the parameter $2L$ to account for the correct estimation of the long-term, and a new way of estimating the lower limit of the averaging interval to properly compensate the short-term variation.

The Generalized Lee Method procedure explained here should be carried out for each propagation channel and frequency band. The calculation of the optimum range for $2L$ will be based on having a field strength sample database representative of the frequency and propagation channel. The whole procedure must be carried out for a wide range of $2L$ values.

First, the mean values $\bar{r}$ of the received signal under study must be computed, by way of the running mean. These estimated mean values $\bar{r}$ must be normalized to its corresponding true mean values $m$, before estimating the variability. Consequently, it is necessary to know the value of the true mean $m$ for each $2L$ interval. Lee solved this question in a theoretical way, considering continuous Rayleigh distribution functions, and using the related statistical parameters.

In the Generalized Lee Method, where it is not necessary to know the statistical distribution that better fits the spatial variability of the signal, this matter cannot be tackled theoretically. Therefore, an approximate true mean will be calculated and used for normalizing the estimated mean $\bar{r}$. The approximate true mean ($m'$) will be computed by a running mean of the estimated mean values obtained within each $2L$ interval. The basis of this way of calculation is that, if the integration interval is small with regard to the $2L$ distance, the estimated mean $\bar{r}$ is a Gaussian variable, and the true mean $m$ is the mean of that Gaussian variable $\bar{r}$ [6]. This reasoning can be represented as:

$$\bar{r} = m(1+\delta)$$

Where $\delta$ is a zero mean Gaussian variable. Figures 5 and 6 show an example of the calculation of the approximate true mean $m'$ for different values of $2L$ interval. As illustrated, the approximate true mean is almost identical to the true mean when $2L$ length is right. In this case, the spread of the estimated mean values is low, and the true mean $m$ can be considered constant within $2L$ (see Fig. 5) [5], [6]. On the contrary, if the window length is not adequate, the spread is greater, because of the short-term variation ($2L$ too short), or because the long-term variation takes place in the excessively long $2L$ intervals (see Fig. 6). However, although the approximate true means differ from the true means, they are a helpful reference to estimate the greater spread of the estimated means.

![Fig. 5. Calculation of the approximate true mean ($m'$) using a proper value for the length $2L$.](image1)

![Fig. 6. Calculation of the approximate true means using inappropriate values of $2L$.](image2)

Once the estimated means $\bar{r}$ have been calculated, they are normalized by its approximate true mean values $m'$.

Finally, the spread of the set of the normalized mean values that compose the sample database must be evaluated by means of $1\sigma_{\bar{r}}$Spread, using (6). The values of $1\sigma_{\bar{r}}$Spread obtained using this procedure are function of the $2L$ parameter, and therefore it is possible to represent the results of $1\sigma_{\bar{r}}$Spread in a graph for a wide range of $2L$ values. An example of this representation is illustrated in the Fig. 7, where the Generalized Lee Method has been applied to ground wave propagation in the Medium Wave band.

The graph of $1\sigma_{\bar{r}}$Spread as a function of $2L$ will show the spread of the estimated means for every $2L$. The optimum $2L$ length will offer a minimum value for $1\sigma_{\bar{r}}$Spread (a minimum spread of the estimated local means), according to the basis of the Generalized Lee Method. The proper $2L$ distance will be obtained directly from this graph.
It is possible to define a range of valid values of $2L$ length, as in (16), delimited by a lower limit, from which short-term will be adequately averaged, up to an upper limit, which will ensure a right long-term signal.

When applying the method to the real sample data, in some routes, the slow variation may be much less relevant than the fast, and only the lower limit will be obtained. In those cases, the result will be a curve analogous to that obtained by Lee [3], and the accuracy of the results will be similar. The rest of the cases will present both limits, and consequently, more accurate results.

C. Distance $d$ between uncorrelated samples

The results of Lee [15] and Rhee [16], described above, cannot be extrapolated to other propagation conditions, since the autocorrelation coefficient is very dependent on the channel model [16]. The generalization of the Lee Method, because of its generic nature, requires the definition of a computation methodology for estimating the distance $d$, based on field data and valid for any propagation condition. The methodology proposed here is based on the mentioned studies of Lee and Rhee.

The first step consists of normalizing the sample, because the variation of the local means along the distance influences on the autocorrelation coefficient [16]. For a right normalization, $2L$ length must be previously calculated. Results of parameter $d$ will enable the validation of the $2L$ value, as it will be explained bellow.

Subsequently, the autocorrelation coefficient of the normalized data must be calculated, ensuring that the number of data samples is enough for a statistical meaningful correlation [16], [21]. The distance relative to the first null (or the distance that matches the 0.2 value as a less restrictive criterion) from the data pool of records must be computed.

If the normalization has been adequately realized, the separation values will be similar for the most of the data records of the same environment, and that will be the value for de distance $d$. For this reason, it is necessary a thorough classification of the reception conditions of the routes involved in the analysis.

D. Necessary number $N$ of measuring points

Parsons [2] solved the last stage of the calculation process of the parameter $N$ in a different way than the previously realized by Lee [3]. The procedure for obtaining the parameter $N$ proposed in the Generalized Lee Method is that defined by Parsons.

From (10), and considering a receiver with a linear characteristic, a maximum error within ±1dB around the true mean, and a 90% of confidence level, Parsons obtained the sample size required in the estimation as

$$\frac{1.65 \cdot (10^{0.1} + 1) \cdot \sigma_r}{(10^{0.1} - 1) \cdot m} \leq \sqrt{N} \quad (14)$$

$$207.22 \left( \frac{\sigma_r}{m} \right)^2 \leq N \quad (15)$$

The minimum number of samples necessary for the average estimations depends on the standard deviation, on the true mean of the data, and on the level of confidence.

The calculation of $N$ requires the previous computation of the length of the window for averaging and the necessary distance between uncorrelated samples.

VII. GENERALIZED LEE METHOD APPLIED TO MW BAND

The recently developed digital broadcasting standards HD-RADIO/IBOC [22] and DRM [23], have renewed the interest in the MW band and one of the associated propagation mechanisms which is ground wave. New commercial transmissions have been initiated worldwide, and several studies aim to get a better characterization of the propagation and reception conditions for these services [24]-[27]. The propagation channel is completely different from the studies for mobile services in the VHF and UHF bands, which originated the studies from Lee and Parsons. Furthermore, a distance of 40λ, at 1 MHz corresponds to 12 Km, which results in a too long interval for averaging the measured samples and consequently separating short and long-term variations. Consequently when analyzing the results from measurement campaigns, the application of the traditional methods lead to erroneous results and made clear the need for the so called, Generalized Lee Method.

The proposals in previous sections are solved in this section for ground wave propagation in the MW band [28]. This exercise will allow contrasting the results with those obtained by Lee and Parsons for a Rayleigh distribution, and verifying if the generalized method is valid for other conditions than Rayleigh channels.

A. Measurement Campaign and Data Selection

DRM field strength measurements were carried out in Spain, in the coverage area of an experimental DRM transmission in 1359 kHz with 9 kHz channel bandwidth. The transmitted digital power (EIRP) was 4 kW RMS, and the radiating system was composed by a 1.1 dBi vertical monopole of 30 m height. The antenna and time schedule ensured ground-wave propagation, without relevant ionospheric interference. At the mobile unit, a 40-MHz low-pass filter was included to avoid undesired out-of-band interference.

Power measurements were done integrating the power spectral density over the 9 kHz bandwidth every DRM frame (400 ms). A GPS receiver and a wheel tachometer provided the positioning and distance information.

The broadcast system, transmitting parameters and
A set of 142 data records of routes between 2 km and 6 km length, in rural/suburban environment, was selected from the whole field data collected in the measurement campaign, to obtain the parameters of the Generalized Lee Method.

B. Results

The distance $2L$ has been obtained as proposed in Section V, using the estimation of the true mean ($m'$) instead of the theoretical Rayleigh based $m(x)$ for the normalization of the data records of the field trials. The normalization allows to process the group of the data records as a whole [2], [10].

Fig. 7 shows the values of $1\sigma_m$Spread for a wide range of $2L$ distances in rural environment. It is clearly observed that the spread of the estimated local means has a minimum for a delimitated set of $2L$ values. Hence, it is possible to consider the upper limit included in the Generalized Method, due to the spread of the mean values when the averaging interval is too long.

![Graph showing $1\sigma_m$Spread vs Averaging interval 2L](image)

Fig. 7. Generalized Lee Method applied to MW band: spread of the estimated local means as a function of $2L$ parameter. The minimum values of the curve are the optimum for $2L$ length.

The optimum values for the parameter $2L$ are those included around this minimum. For $1\sigma_m$Spread less than 1.05 dB:

$$0.9\lambda \leq 2L \leq 2.1\lambda$$  \hspace{1cm} (16)

For shorter lengths, the spread of the estimated local means is higher, because the short-term is not properly averaged. When the $2L$ length is longer than $3\lambda$, the long-term signal varies inside the averaging window, and the variation of the estimated means within the $2L$ intervals increases considerably.

As demonstrated in [16] and [31], the separation between uncorrelated samples will be similar for the most of the data records, if the distance $2L$ is properly chosen and the normalization process adequately realized. The necessary separation between measuring points for obtaining uncorrelated samples ($d$) was found to be $0.17\lambda$. The distance between uncorrelated samples, is calculated after normalizing the field strength samples using a specific $2L$ value. Fig. 8 shows this relationship. The horizontal axis on Fig. 8 represents the candidate values of the averaging window length $2L$. The curve represents, for each $2L$ value, the statistical mode value of $d$ for the whole record set. For instance, the first point in the curve, $d = 0.15$, is obtained when applying a normalization window $2L$ of $1\lambda$, to the database and then calculating the distance between uncorrelated samples (following the methodology explained in Section VI). It can be observed that the most frequent outcome for the data records is $d = 0.17\lambda$, when the $2L$ distance takes values included in (16). This fact validates the results obtained for the parameter $2L$.

![Graph showing Distance d vs Averaging interval 2L](image)

Fig. 8. Generalized Lee Method applied to MW band: statistical mode of the distance $d$ between uncorrelated samples, for varying $2L$ length.

The necessary number of measuring points within the averaging interval for estimating within $\pm 1$ dB around the true mean has been also computed for MW band. Table I shows the results for two degrees of certainty (90% and 95%).

As it can be observed, it is necessary to consider a larger number of samples for a higher degree of certainty. The spatial variability in the suburban environment was found to be higher than in rural environments. For this reason, the necessary number of measuring points is higher.

| TABLE I |
|-----------------|--------|--------|
| NEEDED NUMBER OF MEASURING POINTS $N$ FOR MW BAND |
| Precision | Rural | Suburban |
| $\pm 1$ dB, 90% | 8 | 11 |
| $\pm 1$ dB, 95% | 11 | 15 |

Finally, the last step consists on revising the coherence of the results for $N$, $d$ and $2L$, since the number $N$ of measuring points, separated a distance $d$, must fit within the $2L$ length:

$$N \cdot d \leq 2L$$  \hspace{1cm} (17)

In short, the values of the parameters using the Generalized Lee Method for ground wave at MW band are depicted in Table II. It is significant the great difference between the values obtained for ground wave at MW band and the values obtained by Lee for a Rayleigh distributed signal at the UHF band ($2L = \lambda_0$, $d = 0.8\lambda$, $N = 50$).

The results are consistent with the expectations before analyzing the problem. A distance of $2.1\lambda$ at the frequency used in the field trials (1359 kHz) corresponds to 464 m, which results an adequate value for the analysis of the location variability [28], [29].
TABLE II
SUMMARY OF RESULTS

<table>
<thead>
<tr>
<th>MW BAND</th>
<th>RESULTS OBTAINED BY LEE AT UHF BAND</th>
</tr>
</thead>
<tbody>
<tr>
<td>RURAL</td>
<td>0.17λ</td>
</tr>
<tr>
<td>SUBURBAN</td>
<td>8</td>
</tr>
<tr>
<td>2L</td>
<td>2.0λ</td>
</tr>
</tbody>
</table>

C. Achieving the long-term and the short-term components

The results in Table II have been used to distinguish between slow and fast variations of the received field strength samples from the field measurements. Fig. 9 and Fig. 10 illustrate a representative example of the calculation of the long-term (variation of the local mean values) and short-term (local variations in the signal strength around the mean values) signals, with the values depicted in Table II.

![Fig. 9](image)

Fig. 9. Generalized Lee Method applied to MW band. Instantaneous field strength and long-term component.

![Fig. 10](image)

Fig. 10. Generalized Lee Method applied to MW band. Short-term component.

The example shown in Fig. 9 and Fig. 10 shows that the long-term and short-term are correctly differentiated, and that can be analyzed separately. The fadings higher than 10 dB of the short-term signal have been located and identified with bridges. Small fading occurrences of the short-term, from 3dB up to 8 dB, correspond to large objects located near the receiver that obstruct the signal reception.

VIII. Conclusion

The analysis of the spatial variability is a basic study to find out the coverage area and the signal quality of broadcasting services. This analysis is closely related to the statistical calculation process of the network planning parameters for digital radiocommunication systems that require reception in almost 100% of the locations within the coverage area.

This analysis requires examining accurately the behavior of the long-term and short-term signal components, which are influenced by the path profile, the type of environment, and the obstacles located in the vicinity of the receiver, among other factors.

The correct differentiation of the log-term and the short-term signals is the main point for that purpose. The method proposed by Lee allows obtaining the proper values of the necessary parameters to achieve the local mean values: the minimum number of samples $N$, separated a necessary distance $d$ to be uncorrelated samples, within an averaging window that has an adequate length $2L$ to properly average the samples. The values for those parameters were obtained by Lee for a Rayleigh distribution in UHF band, and have been accepted and used widely by ITU and CEPT studies and reports.

This paper describes a proposal to enhance the existing procedures. This proposal, called by the authors the Generalized Lee Method, is intended for obtaining the proper values of the parameters defined by the original method proposed by Lee, but making the process independent from the propagation channel, the frequency band and the reception conditions. The calculation is based on using field strength samples that are representative of the frequency band and the propagation channel. This generalized method does not require the prior determination of the signal distribution, and improves the accuracy in the delimitation of the averaging interval.

The results from this study provide a general tool for studying the spatial distribution of field strength samples. If a field strength sample database is available for a certain band and propagation channel, the results presented in this paper will allow to calculate the relevant $2L$, $N$ and $d$ irrespective of the frequency and propagation channel statistics.

This paper has illustrated the use of this Generalized Lee Method applying it to ground wave propagation in the MW band. The values obtained for the parameters $2L$ and $N$ are considerably smaller than those obtained by Lee and Parsons. This is probably due, in the case of $2L$, to the relationship between wavelength and the size of the obstacles that generate variations in the long-term signal, and in the case of $N$, to the lower signal variability in this band, especially in rural and suburban environments. The application of this method at MW band shows that the short-term and the long-term components of the signal are adequately differentiated, and associated to the fast variation and to the local average values of the spatial variability, respectively.
The results complete those obtained by Lee for higher frequencies in a Rayleigh environment. Furthermore, results make possible comparative analysis of field strength values from mobile reception in different environments.

APPENDIX. BLOCK DIAGRAM SUMMARIZING THE GENERALIZED LEE METHOD

\[ r(y) = m(y) \cdot r_0(y) \]

\[ m(y) \text{ is composed of local mean values, which are estimated by using the running mean of the instantaneous field strength values } (r_i) \]

\[
\begin{align*}
2L_1 & \Rightarrow \bar{r}_1 = \frac{\sum_{i=1}^{N_1} r_i}{N_1} \Rightarrow \bar{r}_{1\text{norm}} = \frac{\bar{r}_1}{\sum_{i=1}^{N_1} r_i / N_1} \Rightarrow 1\sigma_{\bar{r}_1}^{\text{Spread}} = 20\log \left( \frac{1 + \sigma_{\bar{r}_1}^{\text{norm}}}{1 - \sigma_{\bar{r}_1}^{\text{norm}}} \right) \\
& \Rightarrow \text{(} \sigma_{\bar{r}_1}^{\text{norm}} \text{ is calculated using all the routes of the database)} \\
2L_n & \Rightarrow \bar{r}_n = \frac{\sum_{i=1}^{N_n} r_i}{N_n} \Rightarrow \bar{r}_{n\text{norm}} = \frac{\bar{r}_n}{\sum_{i=1}^{N_n} r_i / N_n} \Rightarrow 1\sigma_{\bar{r}_n}^{\text{Spread}} = 20\log \left( \frac{1 + \sigma_{\bar{r}_n}^{\text{norm}}}{1 - \sigma_{\bar{r}_n}^{\text{norm}}} \right) \\
\end{align*}
\]

\[ P(m - 1.65 \frac{\sigma_m}{\sqrt{N}} \leq \bar{r} \leq m + 1.65 \frac{\sigma_m}{\sqrt{N}}) = 90\% \]

\[ 20\log \left( m + 1.65 \frac{\sigma_m}{\sqrt{N}} \right) - 20\log \left( m - 1.65 \frac{\sigma_m}{\sqrt{N}} \right) \leq 2\text{dB} \]

\[ \Rightarrow \text{Estimation of empirical } m, \sigma_m, \text{ and estimation of } N \text{ of each route} \Rightarrow \text{Selection of the most frequent value of } N \]

\[ d \]

\[ \text{Normalization of the field strength samples} \Rightarrow \text{Estimation of the first null of the autocorrelation coefficient} \Rightarrow \text{Verification of the convergence of } d \text{ values (proper normalization in step 1)} \Rightarrow \text{Proper value of } d \]

REFERENCES


David de la Vega received the M.S. Degree in Telecommunications Engineering from the University of the Basque Country, Spain, in 1996. In 1998 he joined the Radiocommunications and Signal Processing research group at the Department of Electronics and Telecommunications of the University of the Basque Country.

He is currently a professor at the University of the Basque Country, teaching on circuit theory and radar and remote sensing systems. His research interests focuses on propagation channel models and prediction methods for the new digital TV and radio broadcasting services.